Bequest Session

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11th November 2014 (Tenth Meeting of Working Group on
Macroeconomic Aspects of Intergenerational Transfer: International
Symposium on Demographic Change and Policy Response)
Goals

1. Previous model for estimating bequest
   - **Drawback #1**: The bequest model was deterministic
   - **Drawback #2**: Unrealistic profiles early in life

2. Proposing a new model for estimating bequest
   - The new model should be consistent with economic theory and rigorous with the demographic setup
     - Dynamic General Stochastic Economic (DGSE) model populated by overlapping generations
     - Stochasticity comes from the risk of mortality rather than through productivity or income shocks

3. Pending research questions
   - Assessment of the role of bequests vs inter-vivos intergenerational transfers as sources of wealth
   - Relation to annuitization of wealth
Model background

- Time is discrete

- Individuals are assumed to receive a stream of income over their lifecycle \( \{y_x\}_{x=0}^\omega \) and to make decisions about consumption/saving.

- Individuals face mortality risk.

- Let \( \pi_x \) be the conditional probability of surviving to age \( x \) and \( \ell_x = \prod_{u=0}^{x-1} \pi_u \) be the probability of surviving from birth to age \( x \).

- Let \( \theta_x \) be a random variable that denotes whether the parent of an individual of age \( x \) is alive (s) or dead (d), i.e. \( \theta_x \in \{s, d\} \).

- Let \( \theta^x = (\theta_0, \theta_1, \ldots, \theta_x) \) represent the history of the variable \( \theta \) up to age \( x \).
Transition probabilities: $P\left(\theta_{x+1} \mid \theta^x \right)$

In a stable population the probability that the parent of an individual of age $x$ dies is characterized by the following Markovian process:

$$
\begin{pmatrix}
\ell^{\theta}_{x+1} \\
1 - \ell^{\theta}_{x+1}
\end{pmatrix}
= 
\begin{pmatrix}
\pi^\theta_x & 0 \\
1 & 1 - \pi^\theta_x
\end{pmatrix}
\cdot 
\begin{pmatrix}
\ell^\theta_x \\
1 - \ell^\theta_x
\end{pmatrix}
$$

with $\ell^\theta_0 = 1$ and $\pi^\theta_x = \sum_{u=x}^{\omega} \frac{e^{-nu} f_u \ell_{u+x+1}}{\sum_{u=0}^{\omega-x} e^{-nu} f_u \ell_{u+x}}$,

where

- $\ell^\theta_a$ is the survival probability of the parent of an individual at age $a$
- $\ell_a$ is the survival probability to age $a$
- $\omega$ is the maximum longevity
- $n$ is the population growth rate
- $f_a$ is the fertility rate at age $a$
Average bequest received

Per capita bequest received

\[ b_x = \ell_x^\theta (1 - \pi_x^\theta) E[B_x] \]

In a stable population the average bequest received at age \( u \) is given by

\[ E[B_u] = \sum_{x=0}^{\omega-u} P(A_u = x) E(B|A_u = x) \]

with

\[ P(A_u = x) = \frac{e^{-nx}f_x d_{x+u}}{\sum_{x=0}^{\Omega-u} e^{-nx}f_x d_{u+x}}, \]

\[ E(B|A_u = x) = \sum_{h=0}^{\infty} \frac{E[a(\theta^{x+u})]}{1 + h} P(\mathcal{H}_{x+u} = h), \]

where

- \( A_u \) is the random variable ‘Age of death of a parent of an individual of age \( u \)’
- \( d_x = \ell_x - \ell_{x+1} \) is the fraction of deaths between ages \( x \) and \( x + 1 \)
- \( \mathcal{H}_x \) is the number of additional heirs at age \( x \)
- \( E[a(\theta^x)] = \int a(\theta^x) dP(\theta^x) \) is the mean financial wealth at age \( x \)
A household head of age $x > x_0$ maximizes the following conditional expected utility with respect to consumption ($c$):

$$V[a(\theta^x)] = U[c(\theta^x)] + \beta \pi_x \sum_{\theta_{x+1} \in \{s, d\}} V[a(\theta^{x+1})] P(\theta_{x+1} | \theta^x),$$

s.t. the budget constraint

$$\begin{cases} a(\theta^{x+1}) &= R[a(\theta^x) + y_x + \tau_x - c(\theta^x)] + R\mathbb{E}[B_x] \quad \text{If } (\theta_{x+1}, \theta_x) = (d, s), \\ a(\theta^{x+1}) &= R[a(\theta^x) + y_x + \tau_x - c(\theta^x)] \quad \text{Otherwise}. \end{cases}$$

$a(\cdot) \geq 0$ is the financial wealth (borrowing constraint)
$R > 1$ is the capitalized interest rate
$\mathbb{E}[B_x]$ is the average bequest received at age $x$
y$_x$ is the endowment at age $x$
$\tau_x$ is the transfer at age $x$
The optimal consumption path is characterized by the following Euler equation:

\[
\begin{align*}
U_c [c(\theta^x)] &= R\beta\pi_x (\pi_x^\theta U_c [c(s, \theta^x)] + (1 - \pi_x^\theta) U_c [c(d, \theta^x)]) \quad \text{if } \theta_x = s, \\
U_c [c(\theta^x)] &= R\beta\pi_x U_c [c(d, \theta^x)] \quad \text{if } \theta_x = d,
\end{align*}
\]

Note that there exists saving for precautionary motive when $\theta_x = s$ (Jensen’s inequality)
Assumptions (target: maximum bequest-output ratios)

1. Stable populations that differ by their fertility and mortality schedules (data collected from UNPD, *World Population Prospects: The 2012 Revision*)

2. The consumption of children is supported by parents

3. The labor income profile is that of a developed country (*to maximize savings for retirement motive*)

4. The financial wealth of an individual without offspring is taxed at 100% and distributed ($\tau_x$) according to the expected bequest profile

5. Two strong demographic assumptions: At the aggregate level the total number of offspring at age $x$ of an individual at age $u$ is assumed to be given by the sum of $N_{u-x}$ independent Bernoulli random variables, where $N_{u-x}$ is assumed to be distributed according to a Poisson of parameter $f_{u-x}$

6. Fixed interest rate $r = 5\%$ and no productivity growth $g = 0\%$
Underlying demographic data

Figure: Fertility ($f_x$) and survival ($\ell_x$) profiles. Source: UNPD, *World Population Prospects: The 2012 Revision*. 
High mortality prevents splitting the wealth among too many heirs

**Figure:** Expected number of offspring of a parent at age $x$ (conditional on being one of the offspring)
The per capita bequest inflow shifts to the right the higher the proportion of bequest given to spouses
Per capita bequest inflows and outflows could be as large as the labor income earned in advanced countries.

**Figure:** Per capita bequest inflows across the lifecycle by fraction of bequest given to offspring.
The bequest received increases more than proportionally with declines in TFR due to uncertainty.

**Figure:** Bequest received at death of the parent, by marital status, $E[B_x]$. 
Strong effect of mortality decline on the bequest-output ratio

Table: Stochastic Model: Maximum bequest-to-output ratios for \( r = 5\% \), \( g = 0\% \), \( \alpha = 100\% \) under a stable-population structure (Results in %)

<table>
<thead>
<tr>
<th>Life expectancy</th>
<th>7.49</th>
<th>6.57</th>
<th>5.57</th>
<th>4.49</th>
<th>3.47</th>
<th>2.46</th>
<th>1.63</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>46.9</td>
<td>4.05</td>
<td>4.63</td>
<td>5.87</td>
<td>8.20</td>
<td>12.12</td>
<td>21.36</td>
<td>48.32</td>
<td>89.07</td>
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<td>47.5</td>
<td>4.01</td>
<td>4.57</td>
<td>5.84</td>
<td>8.08</td>
<td>11.88</td>
<td>21.04</td>
<td>47.10</td>
<td>86.23</td>
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<td>53.8</td>
<td>3.67</td>
<td>4.25</td>
<td>5.37</td>
<td>7.42</td>
<td>10.92</td>
<td>19.24</td>
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<td>62.3</td>
<td>3.29</td>
<td>3.87</td>
<td>4.91</td>
<td>6.82</td>
<td>10.02</td>
<td>17.27</td>
<td>36.71</td>
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<td>66.6</td>
<td>3.07</td>
<td>3.63</td>
<td>4.62</td>
<td>6.47</td>
<td>9.67</td>
<td>16.57</td>
<td>34.56</td>
<td>55.22</td>
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<td>70.4</td>
<td>2.90</td>
<td>3.45</td>
<td>4.44</td>
<td>6.29</td>
<td>9.31</td>
<td>16.26</td>
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<td>74.7</td>
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<td>42.97</td>
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\(^1\) We have used a CES production function \( Y_t = \left( aK_t^{\frac{\sigma}{\sigma-1}} + (1 - a)H_t^{\frac{\sigma}{\sigma-1}} \right)^{\frac{\sigma-1}{\sigma}} \) with \( \sigma = 1.2 \) and \( a = .25 \), so that higher capital/output ratios lead to higher capital shares \( \alpha_t = a \left( \frac{K_t}{Y_t} \right)^{1-\frac{1}{\sigma}}. \)

\(^2\) We have assumed the following instantaneous utility function at any age \( x \), \( U(c_x) = \eta_x (c_x / \eta_x)^{1-\sigma-1} \) with \( \sigma = 2 \).
Strong effect of the mortality decline on the bequest-output ratio

Table: Deterministic Model: Maximum bequest-to-output ratios for $r = 5\%$, $g = 0\%$, $\alpha = 100\%$ under a stable-population structure (Results in %)

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1 We have used a CES production function $Y_t = \left(ak_t^{\frac{\sigma}{\sigma - 1}} + (1 - a)H_t^{\frac{\sigma}{\sigma - 1}}\right)^{\frac{\sigma - 1}{\sigma}}$ with $\sigma = 1.2$ and $a = .25$, so that higher capital/output ratios lead to higher capital shares $\alpha_t = a \left[\frac{K_t}{Y_t}\right]^{1 - \frac{1}{\sigma}}$. 2 We have assumed the following instantaneous utility function at any age $x$, $U(c_x) = \eta_x \frac{(c_x/\eta_x)^{1-\sigma} - 1}{1-\sigma}$ with $\sigma = 2.$
**Strong effect of savings for retirement motive on the wealth-output ratio**

**Table: Deterministic Model:** Maximum wealth-to-output ratios for \( r = 5\% \), \( g = 0\% \), \( \alpha = 100\% \) under a stable-population structure (Results in \%) 

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Pros

- The assumptions of the model allow us to widely use available demographic data
- We can model several alternatives of transfers between parents and children and between spouses

Cons

- It is very difficult to theoretically justify the introduction of an annuity market
- Saving for precautionary motive does not lead to higher wealth unless uncertainty is very high (high mortality and wealth)

Future work

- Introduction of a housing market (additional savings still needed)
- Assessment of the contribution of savings for precautionary motive to total savings
• **Bequest wealth** at age $x$ for the cohort born in year $s$:

$$w_{x,s} = \sum_{z=x}^{\omega} (\text{bequest}_{z,s}^{\text{inflow}} - \text{bequest}_{z,s}^{\text{outflow}}) \left( \prod_{u=x}^{z} \frac{\pi_{u,s}}{1 + r} \right),$$

where $\pi_{x,s}$ is the conditional probability of surviving from age $x$ to $x + 1$ for the cohort born in year $s$.

• **Aggregate wealth** in year $t$:

$$W_t = \sum_x w_{x,t-x} N_{x,t-x}$$
Figure: Bequest wealth
Figure: Bequest wealth (fixed fertility)
Figure: Bequest wealth (fixed mortality)