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Transfers, Capital, and Consumption over the Demographic Transition

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Introduction

Economic development and demographic transition are linked in complicated and reciprocal ways. Economic development leads to declines in fertility and mortality while changing population size, growth rate, and age distribution influence the pace of economic growth. It is this second causal direction that we will consider here: how the transition shapes development.

Recent studies suggest that many countries benefit from age structure changes that raise the support ratio and concentrate the population in the working ages (Bloom and Williamson 1998; Lee, Mason et al. 2000; Bloom and Canning 2001; Lee, Mason et al. 2001; Mason 2001; Bloom, Canning et al. 2002; Lee, Mason et al. 2003; Mason and Lee 2004; Mason 2005). Such gains appear to be transitory, however, because in later phases of the demographic transition, low fertility and rising life expectancy lead to population aging and a decline in the support ratio. The inceptions of fertility decline and of population aging bound the "window of opportunity" during which conditions for sustained economic growth may – or may not – be realized.

As a general proposition transitory gains can be transformed into sustainable gains by creating human or physical capital. The emphasis here is on physical capital and the proposition advanced is that the demographic transition leads to large increases in the demand for wealth relative to income or labor. The potential result, which we call the "second demographic dividend", is a period of more rapid economic growth leading to a permanently higher level of per capita income and consumption.

The societal demand for wealth rises in part because the population grows older and in part because people may save more at younger ages. Because the elderly in any population typically hold the most wealth, an increase in their share in the population leads to an increase in average wealth holdings. In addition, the decline in the number of surviving children and the rise in longevity lead working age people to save more, which also increases the accumulation of wealth. Policy and institutional context influence the extent to which these potential gains from the second demographic dividend are realized. In particular, heavy reliance on either public PAYGO pension programs or familial transfers to provide financial support for the elderly undermines the effects of the demographic transition on capital accumulation.

In other work, we have explored these processes by simulating a model of life cycle savings applied to Taiwan and the US over the course of their demographic transitions. The present study extends this earlier work in two important ways. First, we make use of detailed and comprehensive estimates of the economic lifecycle (Lee, Lee et al. 2005) and the economic support system (Mason, Lee et al. forthcoming) that were not previously available. Second, in response to the many questions raised about the life cycle saving hypothesis, we take a different approach to modeling consumer behavior. The life cycle saving hypothesis tells us individuals consume and save at each age governed by the wish to smooth consumption over the life cycle constrained by their lifetime earnings. The model used here acknowledges the pervasive nature of public and familial transfers and, in our view, is more realistic. Consumption and age profiles are governed by tastes and perceptions about "needs", but constrained by general standards of living. Consumption by the elderly depends less on what they earned during their working years and more on what their children and current generations of taxpayers are earning.

The structure of our paper is as follows. First, we describe the key data on which our calculations are based, including estimated age profiles. Second, we present the theoretical model. Third, we use simulation analysis to track consumption, assets, and other macroeconomic variables over a demographic transition. We then consider the sensitivity of the results to alternative parameters, discuss limitations, and conclude.

Economic Lifecycles: A Comparative Perspective

Two fundamental aspects of the economic lifecycle are crucial to our purpose. First, how does economic dependency vary with age? The broad answer to this question is well-known – children and the elderly produce much less than they consume while prime age adults produce much more. Little, however, is known about details, which are essential to understanding more about the timing and magnitude of macroeconomic effects of changes in age structure. Second, how do societies shift resources from the working ages to the dependent ages? One possibility is that prime-age adults transfer resources to children and to the elderly through public programs or through private institutions – the family, in particular. The second possibility, relevant mostly to old-age support, is lifecycle saving. Prime-age adults accumulate assets. In old-age they dis-accumulate the assets and rely on the income generated by those assets. The mechanism by which assets are shifted across age groups is important because it determines whether population aging leads to the accumulation of assets or to the expansion of public and private transfer programs.

Estimates of the economic lifecycle for the United States in 2000 and Taiwan in 1977 are presented in Figure 1A and 1B. The labor income profiles incorporate and summarize, for men and women combined, labor force participation, hours worked, wages, and all of the factors that influence these variables. They are cross-sectional profiles and, hence, reflect the varied experiences of each of the age groups represented in the respective profiles. Despite the many ways in which Taiwan in 1977 differed from the United States in 2000, the labor income profiles are strikingly similar. There are some discernible differences, however. The US labor income profile rises somewhat more slowly with age and begins to decline at a somewhat later age than in Taiwan.

The consumption profiles shown in Figure 1 consist of both public and private consumption. Public consumption in the United States favors children, via spending on education, and the elderly, via spending on health care. Private consumption in the US rises steadily with age until around age 60 and then declines. Public and private consumption combined are highly favorable to the elderly. We estimate that average consumption by a 90-year-old was over $40,000 in 2000 as compared with only $25,000 by a young adult. The difference between them is essentially a consequence of health care spending.

The situation in Taiwan 1977 was very different. Consumption clearly favored young adults with total consumption declining from about NT$40,000 per year for young adults to around NT$30,000 per year for those who were 90 (or older). Public education programs were important in Taiwan in 1977, but public spending on health care was unimportant.

The key difference between the two cases, then, occurs at the older ages. In the US, per capita consumption of those 58 and older exceeded per capita labor income. In Taiwan the cross-over age was 62. In relative terms the gap between consumption and production at older ages is much larger in the US than in Taiwan. In contrast, the dependency profiles at young ages appear to be quite similar in the two countries. The cross-over ages are the same – 26 years of age in both countries – and the magnitudes of the gap between consumption and production relative to labor income appear to be similar.

The United States and Taiwan also differ in the way resources are shifted to the elderly. In the US, about 60% of consumption of the elderly is funded by income from the ownership of assets. Public transfer accounts for 35% and *inter vivos* familial transfers only 4% of consumption of the elderly (Figure 2). In Taiwan, asset-based reallocations are less important – about 40% of consumption of the elderly. Transfers are more important, with public transfers equal to 23% and familial transfers equal to 33% of consumption of the elderly. In both economies, public and familial transfers combined rival or exceed the importance of assets (Mason, Lee, et al. forthcoming). Clearly, a focus on life cycle saving and asset accumulation alone, or on transfers alone, would miss much of the story.

Theory

Recent studies of the economic growth effects of population have focused on per capita income. Our own work has emphasized two effects of age structure on per capita income, for which we have coined the terms “first and second dividend”. The first dividend arises because the effective number of producers is growing faster than the effective number of consumers. Other things equal, income per equivalent consumer rises. The underlying demographic cause is a decline in fertility and the share of children in the population. The second dividend arises to the extent that longer life and changes in age structure lead to more rapid accumulation of assets and, in a closed economy, to capital deepening. The second dividend is not a free lunch, however, because current generations must reduce their consumption in order to increase their wealth and achieve higher consumption in future periods. These ideas can be framed using the following identity for consumption per effective consumer:

$$\frac{C(t)}{N(t)} = c(t)\frac{Y(t)}{L(t)}\frac{L(t)}{N(t)} \tag{1}$$

C(t) and Y(t) are total consumption and labor income respectively, and c(t) is the ratio of consumption to labor income. N(t) and L(t) are the effective number of consumer and producers, respectively:

$$\begin{aligned} N(t) &= \sum_{a=0}^{v} f(a)P(a,t) \\ L(t) &= \sum_{a=0}^{w} g(a)P(a,t), \end{aligned} \tag{2}$$

P(a,t) is the population aged a at time t, $f(a)$ and $g(a)$ are age-specific, time-invariant vectors of coefficients measuring age variation in consumption and productivity, respectively.

The effect of current age structure on current consumption, the first dividend, is captured in equation (1) by the economic support ratio (L/N), the number of effective producers per effective consumer. Suppose the consumption ratio c(t) and labor productivity Y(t)/L(t) were unaffected by demography.[1] Per capita consumption would vary directly with the support ratio. An increase in the support ratio, which occurs during the demographic transitions, would lead to higher consumption. Later when population aging depresses the support ratio, consumption would decline. That would be the end of the story.

The story is quite different and much more complex if the consumption ratio c(t) declines, as we will argue here, as the support ratio rises. Current per capita consumption would rise by less than the support ratio, but saving rates would increase, assets held by future generations would rise, and, if assets were domestically invested, labor productivity and wages would increase. If assets were invested abroad, then national income would rise. Domestic labor productivity and wages would be unaffected, but foreign labor productivity and wages would rise. But whether assets were invested domestically or abroad higher levels of consumption could be sustained in the future. In this response rests the possibility for a second demographic dividend that we explore in this paper.

Assets, Lifecycle Wealth and Child Costs

Wealth is defined broadly as a net claim on future income. Individuals can consume more in the future than they produce only if they hold wealth. Wealth can take two broad forms, however. One form is *assets*, e.g., private savings, funded pensions, or a home. The second form is *transfer wealth* consisting of the present value of net transfers received through familial support systems and through public programs, e.g., PAYGO pensions and publicly provided health care. Either assets or transfer wealth can be used to fund future expenditures in excess of future labor income. From the perspective of the individual, they are equivalent.[2] From the perspective of the macroeconomy, however, transfer wealth and assets are not equivalent. By accumulating more assets, higher levels of aggregate consumption can be sustained in the future. Thus, the effect on growth of changes in the demand for wealth is closely tied to whether that wealth is created by expanding transfer programs or accumulating assets. In this analysis we assume this to be a matter of exogenously determined policy.

The aggregate demand for wealth depends on the future trajectories of consumption and production. Our strategy is to assume that the cross-sectional age profiles of consumption and earnings estimated for a particular country in a particular year retain their shapes in the future, while their levels shift upwards over time. The labor income profile is assumed to shift at some exogenously given rate due to technological progress.[3] The rate at which the consumption profile shifts is endogenous and depends on technological progress, population age structure, the shapes of the consumption and production profiles, and public policy.

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[1] In a closed economy, a decline in labor force growth would lead to capital deepening and an increase in output per worker. This would not be the case, however, in a small open economy.

[2] We abstract from uncertainty about whether future income streams are realized.

[3] This analysis is confined to a small, open economy. Hence, capital per worker is exogenously determined by international capital markets.

Our assumption that the cross-sectional shape of the consumption age profile is fundamental and unchanging is novel, and this requires some interpretation. In a strict life cycle savings model, the age profile of consumption would not be constant. Instead it would depend on the relative economic fortunes of each generation. For example, the young in Taiwan who may earn six times as much as their parents did at a comparable age (6 = exp(30*.06)) would consume correspondingly more at each age over their life cycle. But this is not what we see, and not what would emerge under a system of familial co-residence and income sharing. In fact, the cross sectional age profiles of consumption in the US and Taiwan have been fairly stable in shape over the period from 1980 to 2000 for which we have calculated them. This is what we would expect if individuals in families are altruistically linked. Differences emerging from different earnings histories would be offset by both familial and public sector transfers. This is our working hypothesis for the calculations reported below in this paper.

The key idea is that variation in consumption across generations at any point in time is a product of preferences or altruism that expresses itself through the host of transfer programs – both public and private – that permeate all modern societies. In the lifecycle model, the consumption of the elderly depends on their tastes and their lifetime earnings. In this model, the consumption of the elderly depends on general standards of living, the needs of the elderly, and social and familial preferences about the consumption of the elderly as compared with that of prime-age adults and children. In a life cycle saving model, individuals save at the cost of their current consumption in order to consume more themselves in the future. In our approach, individuals save to raise the future consumption of all those to whom they are altruistically linked.

Given the trajectory of labor income, the lifecycle wealth of adults can be readily calculated for any consumption trajectory. The lifecycle wealth of adults can be decomposed into two components. One component is called "child wealth", which is the present value of the net costs of supporting children in the future and will be negative. The second component, "pension wealth", is the wealth used to fund consumption at older ages. Pension wealth consists of assets and pension transfer wealth.

We do not know to what extent the future consumption needs of an aging population will be met by unfunded transfer systems versus funded systems or private saving. This will depend on how policies and institutions develop over coming decades. For our simulations, we will assume that the ratio of assets to pension transfer wealth will remain constant.[4] Thus, for any exogenous transfer policy and for any endogenous consumption trajectory, we can compute a trajectory of assets. Which consumption trajectory is feasible depends on characteristics of the macroeconomy, e.g., the rate of return to assets.

Before turning more formally to the methods we use, a few comments are in order. First, the age boundaries assumed for dependency ratios, such as 20 and 65, are obviously arbitrary. Our calculations are based on the actual age profiles of labor earnings by age. Second, in any society, it is the elderly who have the highest ownership of assets, following a life time of accumulation. Holding the age profile of capital, or equivalently saving rates, constant, and multiplying it times the changes in age distribution over the demographic transition, would clearly imply rising capital to income ratios. We might call this a pure compositional effect. In our analysis, however, we do

[4] In 2000 approximately 35% of US pension wealth was transfer wealth and about 65% was assets.

not hold the age profile of wealth constant. Rather, the demand for wealth by age will depend on fertility and mortality. Couples with fewer children assign a greater share of their life cycle earnings to their own consumption, and therefore have a greater demand for wealth to provide for higher consumption in retirement. People who expect to live longer have a greater demand for wealth to finance their longer period of post-work consumption. These changes associated with the demographic transition and changes in age structure are also reflected in our analysis.

Assets $A(t)$

The approach to modeling assets can be motivated by considering a simpler case in which the lives of prime-age adults were divided into two distinct phases. During the first phase adults would raise their children. All labor income in excess of personal consumption would be devoted to childrearing. During the second phase adults would accumulate pension wealth – assets and transfer wealth needed to fund their retirement. Aggregate pension wealth at any point in time would be equal to the wealth held by adults, including the elderly, who had completed their childrearing (Mason 2005).

A more realistic approach recognizes that supporting children and providing for retirement overlap. Adults at all ages hold at least some pension wealth and adults at all ages make transfers to children. Aggregate pension wealth depends on the pension wealth held by all age groups. This, in turn, depends on the total lifecycle wealth held by all adults and the capitalized obligation to support children.

Aggregate lifecycle wealth is the wealth that adults must hold, as a group, in year t in order to achieve a given path of consumption and labor income over the remainder of their collective existence. *W(t)*, the lifecycle wealth of all adults in year *t*, is equal to the present value of the consumption less the present value of the labor income of those adults over the remainder of their lives. Let PV[] be the present value operator. Then,

$$W(a,t) = PV\left[C(a,t)\right] - PV\left[Y(a,t)\right] \tag{3}$$

where *C(a,t)* and *Y(a,t)* are vectors of current and future consumption and current and future labor income, respectively, for the cohort of age *a* in year *t*. Summing *W(a,t)* across all adult ages in year t yields aggregate lifecycle wealth, *W(t)*.

Lifecycle wealth comes in three forms: assets (A), transfer wealth associated with childrearing (T_K) and pension transfer wealth (T_P), i.e.,

$$W(t) = A(t) + T_k(t) + T_P(t). \tag{4}$$

Transfer wealth associated with childrearing is the present value of the current and future costs of childrearing and, hence, is negative. Pension wealth is defined as $W_P(t) = A(t) + T_P(t)$, i.e., assets plus pension transfer wealth. Assets by assumption can only be held by adults.

The relative size of pension transfer wealth is captured by $t(t) = T_P(t)/W_p(t)$ and the relative size of child transfer wealth by $t_k(t) = T_k(t)/W(t)$. Substituting into equation (4) and rearranging terms gives the total assets of adults and, because only adults hold assets, aggregate assets in year t:

$$A(t) = (1 - t(t))(1 - t_k(t))W(t). \tag{5}$$

Total assets depend on three factors: pension transfer policy (both public and private), the cost of children both public and private, and the lifecycle wealth required to support adult consumption. Transfer policy is exogenous, whereas the cost of children and lifecycle wealth are both endogenous.

Labor Income and Consumption: $Y(t)$ and $C(t)$

Lifecycle wealth and child wealth depend on the trajectory of labor income and consumption. Labor income at each age depends on the effective number of producers belonging to that age group and the general level of wages which shifts over time due to technological change. We assume that the rate of technological change is constant and exogenous.[5] Thus, labor income of year t adults in period t+x is:

$$\begin{aligned} Y(a,t+x) &= \bar{y}(t)G_y(x)L(a,t+x) \\ G_y(x) &= (1+g_y)^x \end{aligned} \tag{6}$$

where $\bar{y}(t)$ is the labor income index for the current year t, g_y is the annual rate of growth in that index due to technological change, $G_y(x)$ measures the effect of technological change over an interval of x years, and $L(a,t+x)$ is the number of year t adults alive in year $t+x$ measured as equivalent producers.

By assumption the shape of the per capita cross-sectional profile of consumption is fixed and incorporated into the equivalent number of consumers. Consumption at each age grows at the same rate g_c, but that rate varies from year to year. Hence,

$$\begin{aligned} C(a,t+x) &= \bar{c}(t)G_c(t,x)N(a,t+x) \\ G_c(t,x) &= \prod_{z=0}^{x-1}\left(1+g_c(t+z)\right) \end{aligned} \tag{7}$$

where $C(a,t+x)$ is the total consumption of year t adults in year $t+x$, $\bar{c}(t)$ is the consumption index that determines the level of the consumption profile in year t, $G_c(t,x)$ is the proportional increase in the consumption index between years t and $t+x$, $N(a,t+x)$ is the number of year t adults alive in year $t+x$ measures as equivalent consumer, and $g_c(t+z)$ is the rate of growth of the consumption index between period $t+z$ and $t+z+1$. The consumption trajectory, defined by the initial consumption index and the vector of consumption growth rates, is endogenous.

Lifecycle Wealth: $W(t)$

These general rules can be applied to year t adults to determine their labor income income and consumption over their remaining adult years and, hence, their lifecycle wealth in year t. Let *NTOT(t,x)* denote the number of effective consumers in year t+x who were adults in year t. Similarly, *LTOT(t,x)* denotes the number of effective producers in year t+x who were adults in year t:

$$\begin{aligned} NTOT(t,x) &= \sum_{a=a_0+x}^{w} N(a,t+x) \\ LTOT(t,x) &= \sum_{a=a_0+x}^{w} L(a,t+x). \end{aligned} \tag{8}$$

[5] In the current analysis, we do not consider any feedbacks from capital deepening to wages. Hence, the model considered here is appropriate for a small open economy in which the rate of return on investment is determined by international capital markets and the shift in the wage profile is determined by exogenous technological change.

In a closed population NTOT and LTOT would depend only on survival rates, but in an open population they will include migrants who were adults in year t.

The labor income of year t adults at age $a = a_0 + t$ in year $t+x$ is:

$$Y(a,t+x) = \bar{y}(t+x)L(a,t+x) \tag{9}$$

and consumption by year t adults in year $t+x$ is:

$$C(a,t+x) = \bar{c}(t+x)N(a,t+x). \tag{10}$$

The present value in year t of the current and future lifetime consumption of all adults is given by:[6]

$$PVC(t) = \bar{c}(t)\sum_{x=0}^{w-a_0} D(x)G_c(x,t)NTOT(t,x), \tag{11}$$

and the present value in year t of the current and future lifetime production of all adults is given by:

$$PVY(t) = \bar{y}(t)\sum_{x=0}^{w-a_0} D(x)G_y(x)LTOT(t,x), \tag{12}$$

where $D(x)$ is the discount factor $(1+d)^{-x-1}$. The lifecycle wealth of all adults in year t is:

$$\begin{aligned} W(t) = {} & \bar{c}(t)\sum_{x=0}^{w-a_0} D(x)G_c(x,t)NTOT(t,x) \\ & -\bar{y}(t)\sum_{x=0}^{w-a_0} D(x)G_y(x)LTOT(t,x). \end{aligned} \tag{13}$$

Child Transfer Wealth: $T^k(t)$

The final variable that determines assets in equation (5) is child transfer wealth which measures the costs to year t adults of providing resources consumed by children. If adults spend more on children in the current and future periods, then child transfer wealth is a larger negative value. Or as represented in equation (5), the ratio of child transfer wealth to adult transfer wealth is a larger negative value.

What determines child transfer wealth? In part, it depends on the difference between what children consume and what children produce in the current and in future periods. Production and consumption are determined in the same manner for children as for adults. The shape of the age profiles of production and consumption ($g(a)$ and $f(a)$) are held constant for all ages including children. The shifts of the profiles over time are governed by the shifts in the production and consumption indexes discussed above.

The cost of children to year t adults also depends on their share of the costs of children in future periods. By assumption all of the *current* costs of children are born exclusively by year t adults, i.e., children do not support children. Year t adults are responsible only for a portion of the cost of children in subsequent years, because some portion of the costs of children is shifted to persons who become adults after year t.

The share of child costs borne by year t adults depends on a host of factors, including the extent to which child costs are born by families as opposed to taxpayers, the system of taxation that is used to finance public transfers to children, and the extent to which parents, grandparents, and other family members finance familial transfers to

[6] We assume that income and consumption accrue at the end of the period.

children. The model distinguishes two ways in which child costs are financed: familial transfers and public transfers. Adult parents are assumed to bear the cost of familial transfers. Public transfers are financed through a proportional tax on labor income. The relative mix of these two mechanisms is an exogenously determined policy variable.

Child transfer wealth is equal to:

$$T_k(t) = \bar{y}(t)\sum_{x=0}^{w-a_0} D(x)G_y(x)KLTOT(t,x) - \bar{c}(t)\sum_{x=0}^{w-a_0} D(x)G_c(t,x)KNTOT(t,x) \qquad (14)$$

where $KLTOT(t,x)$ and $KNTOT(t,x)$ are the effective numbers of child producers and consumers, respectively, in year $t+x$ for which year t adults are financially responsible. A detailed description of the methods involved in calculating these variables is provided in the appendix.

Lifecycle Pension Wealth: $W_p(t)$

Pension wealth is equal to lifecycle wealth less child transfer wealth. Combining the results from equations (13) and (14) and rearranging terms yields:

$$\begin{aligned} W_p(t) &= \bar{c}(t)\sum_{x=0}^{w-a_0} D(x)G_c(t,x)\left(NTOT(t,x) + KNTOT(t,x)\right) \\ &\quad - \bar{y}(t)\sum_{x=0}^{w-a_0} D(x)G_y(x)\left(LTOT(t,x) + KLTOT(t,x)\right). \end{aligned} \qquad (15)$$

Lifecycle pension wealth is the discounted present value of current and future consumption by year t adults and their dependent children less the present value of current and future production by year t adults and their dependent children. Recall that assets in year t are equal to $(1-t(t))W_p(t)$. Thus, equation (15) gives pension wealth and assets in each year conditional on the consumption trajectory as determined by $\bar{c}(t)$ and $G_c(t,x)$.

Solving for the Consumption Trajectory

The relationship between the trajectory of consumption and lifecycle pension wealth and assets is apparent from inspecting equation (15). If the consumption trajectory is higher, either because $\bar{c}(t)$ or an element in the vector of growth rates $G_c(t,x)$ is higher, lifecycle pension wealth in the current period and current assets must be higher. However, if aggregate consumption is higher, saving must be lower, and the trajectory of assets must be lower. The feasible consumption trajectory is the one for which lifecycle accounting and the macroeconomic accounting lead to the same assets in all periods.

The solution will depend on whether the economy is open or closed. Here we assume that the economy is open and, hence, the rate of return to capital, r, is exogenous. Thus, the aggregate flow constraint is governed by:[7]

$$A(t+1) = (1+r)A(t) + Y(t) - C(t). \qquad (16)$$

Two methods are available for solving the model. We have used both methods and they produce results that are equivalent in all important respects. One approach uses numerical search to find a solution using forward recursion. A second approach, which we will use for the results in this paper, finds an exact closed form solution based on

[7] We assume that assets are measured at the beginning of the period and that consumption and income accrue at the end of the period.

backward recursion. In it, we assume that the population achieves stability and the model reaches steady state at some point in the distant future, t^*.[8] Under those conditions, the consumption index will grow at the same rate as labor productivity. We can solve directly for the consumption index and assets in year t^* and all years thereafter. Next we solve for consumption in year t^*-1 given consumption in all subsequent periods, equation (16), and the lifecycle model that governs the demand for assets by consumers. We work backwards to the present period or historically.

Simulating the Demographic Transition in a Small Open Economy

To fully appreciate the influences of population on the economy one must take a long view. The demographic transition and its accompanying changes in age structure occur over a period of several centuries. No country has yet completed the demographic transition and historical records that predate the transition are available only in a few instances. The strategy we employ here is to use population estimates from 1950 to 2000 and UN projections to 2300 to capture close to the entire demographic transition for a single country, Niger, selected because in 2000 it had the highest total fertility rate (TFR) of any country in the world, 7.9 births per woman.[9]

Niger's demographic transition follows the classic pattern. In 1950-55, life expectancy at birth for both sexes combined was only 36.2 years, increasing gradually to 44.3 years for 2000-05. More rapid increases are anticipated in the future with life expectancy reaching 61.4 years in 2045-50 and eventually reaching almost 90 in 2300. The TFR rose slightly from 7.7 births per woman in 1950-55 to peak at 8.2 births per woman from 1975-1990. The medium scenario employed here assumes that the TFR will decline gradually to reach 3.6 births per woman in 2045-50 and replacement level, just over two births per woman, in 2080 and thereafter.

Age structure is fundamental to our analysis and the trends in three broad groups, the working ages (20-64), children (0-19), and the elderly (65+), are charted in Figure 3. The early part of the demographic transition in Niger is dominated by the large and rising share of the child population. The percentage under the age of 20 reached 60 percent of the population in 2000 rising from 54 percent in 1950. The increase in the share of children was a consequence, in part, of the rise in the TFR, but mostly a consequence of improvements in child survival. The infant mortality rate dropped from 198 deaths per 1000 births in 1950-55 to 153 deaths per 1000 births in 2000-05. The percentage of the population in the working ages, 20-64 declined from 43.4 percent in 1950 to only 38.4 percent in 2000.

The 21st Century is dominated by the rise in the working age population and the decline in the child population. Between 2000 and 2090, the percentage of the population in the working ages increases from 38.4 percent to 61.6 percent, while the percentage in

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[8] We assume that steady-state is achieved in 2300 the last year for which long-range population projections are available. Simulated values for 2000 to 2150 are nearly identical when we assume steady-state is reached in 2200.

[9] Estimates and projections covering the 1950-2050 period are based on UN Population Division (2005). Projections from 2050 to 2300 are based on UN Population Division (2004). Long-range projections for most individual countries are unpublished but were provided to the authors by the Population Division. The long-range projections have been adjusted to eliminate a discontinuity in 2050 between the short-term and long-range projections.

the child ages drops from 60 percent in 2000 to 28 percent in 2090. The 22nd Century is dominated by the rise in the elderly population and the decline in the working age population. From the peak reached in 2090, the percentage of the population in the working ages drops to 50.0 percent in 2200. Over the same period, the percentage of the population 65 and older rises from 10.3 percent to 27.9 percent. Changes after 2200 (not shown) are very slow with the percentage in the old ages rising to about 30 percent. An important point to note is that this is a classic demographic transition with fertility declining to replacement level. Although many countries have fertility rates well-below replacement, the TFR is not projected to drop below two births per woman in Niger, even temporarily.

Figure 3 about here.

The simulation results rely on Niger's demography but other important model parameters are not based, even loosely, on Niger. For the baseline simulation, we use the age-profiles of production and consumption for Taiwan 1977 (Figure 1). The scaling of the profiles is arbitrary because there is no natural unit for measuring equivalent consumers and producers. We have scaled the production profile so that when applied to the global population for 2000, it produces the world GDP in US$2000 (World Bank 2005). We have scaled the consumption ratio so that the support ratio for the world in 2000 was 1 equivalent producer per equivalent consumer.

We assume that two-thirds of child costs are met through familial transfers and one-third through public transfers, figures consistent with the experience both in Taiwan and the US. Assets are held constant at sixty-five percent and pension transfer wealth at thirty-five percent of pension wealth, a figure similar to the US in 2000. For the discount rate we use a risk-free rate of return of 3%. For the depreciation rate we use 3% (Mankiw, Romer et al. 1992) and for the international real rate of return on assets we use 6% (Barro and Sala-i-Martin 1995) declining linearly to a steady-state rate of interest of 4.42% in 2300.[10] We assume that productivity growth is 1.5% per year. The sensitivity of the results to variation of key parameters is discussed below.

An important assumption is that the economy is small and open. Hence, the accumulation of assets at the country level has no effect on wages or interest rates. These are determined in international markets. Global aging will influence interest rates and wage rates in ways that we intend to explore more in future research. The effects on consumption will be substantially greater when the influences of capital on labor income are incorporated into the model.

Baseline Simulation

Key features of the demographic transition and its implications for consumption are captured in Figure 4. Early in the demographic transition the support ratio declined to a low level in 2000 because high fertility and declining infant mortality produced a population with many young dependents. The support ratio does not begin to rise until around 2020, but then it rises steeply, by about 50%, during the next 70 years. During

[10] The steady-state international rate of return is calculated using the same set of assumptions for the global economy and the global population distribution in 2300 weighted to reflect current differences in per capita income between the developing and developed countries. In the global calculations, however, the rate of return is endogenous. Details are available from the authors.

this phase, the first dividend phase, the direct effect of an increase in the support ratio (equation (1)), is an increase in consumption per equivalent consumer of 50%.

Figure 4 about here.

The transitory nature of the first dividend is quite clear. The support ratio drops from its peak in 2090, at first rapidly and then more gradually. At the end of the transition the support ratio is well below the peak, about 10 percent above the 1950 support ratio. The first dividend has turned negative and is adversely influencing per capita consumption during this phase of the demographic transition.

In the absence of the second demographic dividend, the consumption index ($\bar{c}/\bar{y}$) would track the support ratio exactly. The index measures the extent to which consumption per equivalent consumer rises relative to productivity changes induced by technological innovation. Consumption deviates from the support ratio because consumers vary the shares of their income that they devote to consumption and saving, and, as a result, assets and asset income vary.

Early in the transition, the consumption index fluctuates, but between 1950 and 2000 it declines by about the same amount as the support ratio. From 2010 to 2070, the consumption index is lower than the support ratio by roughly 5 percentage points. After 2010 the consumption index is growing at more or less the same rate as the support ratio. Just as the first dividend is coming to an end, the effects of the second dividend become apparent. Even though the support ratio begins to decline, consumption continues to increase relative to labor productivity. Consumption does begin to fall early in the 22nd Century, but it remains above the support ratio peak for over 150 years. In steady-state the consumption peak is about 18% above the level produced by the first dividend alone. The reason higher consumption can be sustained is that consumers have accumulated more assets. By doing so, they have converted a transitory dividend into a permanent one.

The annual growth effects of the dividends are modest, but important. The peak rate of growth for the support ratio was 0.9% per year in 2045-50. Between 2030 and 2065, the support ratio increase by 0.5% per year or more. The consumption index grew at an annual rate peaking at about 1.1% per year for 2065-70 and exceeding 0.5% per year between 2030 and 2090. During this sixty year period, the only other source of economic growth is technological progress of 1.5% per year. Hence, the two demographic dividends combined contribute 25% or more of the economic growth experienced.

The second demographic dividend in a small open economy is the rate of growth of C/Y, the rate of growth of consumption relative to labor income. This is the difference between the rate of growth of the consumption index and the rate of growth of the support ratio. The second dividend is usually a small negative value between 2000 and 2055. Consumption is depressed by roughly 0.1% per year during the 55-year period. The first dividend turns positive in 2055 and remains positive until 2155. Between 2060 and 2105, the second dividend averages between 0.4% and 0.5% per annum.

Figure 5 about here.

Next we turn to the wealth effects of the demographic transition (Figure 6). A key point is that the demand for pension wealth is very weak in the early stages of the demographic transition. First, most adults are young and they have not yet accumulated much pension wealth irrespective of their expectations about retirement. Second, those who are currently working, particularly those who have been working sufficiently long to have accumulated pension wealth, have little incentive to do so, because they anticipate a relatively short duration of retirement. Third, the costs of childrearing are very high. Because Niger's population is dominated by such a large number of children, and its support ratio is correspondingly low, consumption during retirement is low. Moreover, the support ratio will continue to be low for decades. Hence, anticipated consumption during retirement is low. This leads to an even lower demand for retirement wealth. Pension wealth is barely discernible only in 2020 and later.[11]

Figure 6 about here.

Of particular importance in the early part of the demographic transition is the decline in fertility and its implications for child costs. The wealth calculations emphasized here are the present values of future childrearing costs. Between 1950 and 2025, the childrearing debt was very high – equal to six to seven times annual labor income. The childrearing debt declines for two reasons. First, fertility drops more rapidly than child mortality and, hence, the surviving number of children declined. That this is occurring is evident in the age distribution graph shown above (Figure 3). Second, the age distribution of adults begins to shift towards older ages so that adults, as a group, have completed a larger portion of their childrearing. There is a countervailing effect, however, because consumption per child rises relative to labor income as the number of children declines. The process is largely completed after 100 years. At that point childrearing debt stabilizes at about 3 times annual labor income.

Pension wealth and assets rise very substantially over the transition. In 2000 there is a negligible demand for pension wealth as explained above. By 2150, pension wealth has reached nearly 8 times annual labor income and assets have reached near 5 times annual labor income, as compared with a value of about 4 for current-day US.

The impact of this enormous increase in assets is muted in the simulation results presented here because the economy is small and open. The effect of a four-fold increase in the ratio of capital to labor income, experienced between 2050 and 2100 for example, would be to double output per worker assuming a standard Cobb-Douglas production function. This would produce an effect on consumption substantially higher in a closed economy. Of course, even if the world consists entirely of open economies, the rise of assets will produce increases in wages and output.

The final baseline results we present are for the aggregate consumption rate. Consumption as a fraction of labor income and consumption as a fraction of national income (Y+rA) are charted in Figure 7. Consumption as a fraction of labor income is of interest because of the key role it plays in equation 1. Consumption rises about 30

[11] The small negative values for pension wealth in the earliest years are an artifact of the simulation methodology. The capital value of transfers to older children, those 25-29, are included in pension wealth rather than in child wealth.

percent relative to labor income between 2055-60 and 2150-55. This is the cumulative effect of the second dividend.

The ratio of consumption to national income is of interest because its complement is saving as a fraction of national income. We see clear evidence of the saving boom often attributed to demographic effects in the literature – the saving rate rises from close to zero at the beginning of the simulation to peak at about 15 percent of national income. Thereafter, the saving rate declines to a steady state rate of about 6% of national income.

Figure 7 about here.

Is it Mortality or Fertility Decline?

The demographic transition is driven by changes both in fertility and mortality. Which plays the more important role in the demographic dividends? We explore this issue by considering demographic transitions that vary substantially from those projected for Niger. Two alternative scenarios are considered. In the first, the declines in the age-specific fertility rates are delayed for 50 years and, then, follow the path of the baseline scenario shifted rightward by 50 years. All other parameters are held equal to their baseline values. The second scenario is similar in construction, except the declines in age-specific death rates are delayed for 50 years rather than the declines in fertility rates.[12]

The effects of a delay in fertility decline are substantial, whereas the effects of a delay in mortality decline are modest. Figure 8 present the support ratio and the consumption index for the three scenarios. The decline in fertility decline leads to further deterioration in the support ratio because constant fertility combined with a decline in infant and child mortality is leading to an increase in the number of surviving children and a substantially higher childrearing burden. The support ratio eventually recovers and reaches nearly the same height as the baseline scenario, but decades later. The delayed fertility decline depresses consumption over an extended period. It does not recover to the value for 2000 until near the end of the century. Eventually consumption rises and stabilizes at the same steady state value as in the baseline scenario.

Figure 8 about here.

The effects of delayed mortality decline are very different in part because the effects of a decline in death rates at young ages are so different from the effects of a decline in death rates at older ages. When the decline in infant and child mortality is delayed, the number of surviving children declines relative to the baseline and, hence, the support ratio is higher and the costs of childrearing are lower. This leads to an immediate boost in consumption as compared with the baseline. Later in the simulation the support ratio is higher in the delayed mortality scenario because higher death rates at older ages reduce the relative size of the old-age population. As compared with the baseline scenario, then, the cumulative effects of the first dividend on consumption are consistently greater when mortality decline is delayed. This does not translate, however, into permanently higher consumption. The reason, as shown below, is that that higher

[12] These calculations were carried out by constructing a population projection model that closely mimics UN projections.

adult death rates under the delayed mortality decline scenario lead to lower capital accumulation and a smaller second dividend.

As discussed above an immediate and important impact of delayed fertility decline is to increase the number and cost of children, whereas the impact of delayed mortality decline is to reduce the number and cost of children. The capitalized value of the lifetime costs of childrearing declines from about 7 times annual labor income in 2000 to about 3 times annual labor income in 2100 for both the baseline and delayed mortality decline scenarios. Child costs are reduced when mortality decline is delayed, but the differences are small. For the delayed fertility decline scenario, child costs rise gradually during the first half of the 21st Century and begin to decline only at mid-Century. They remain above the other scenarios until well into the 22nd Century.

Figure 9 about here.

As compared with the baseline scenario, assets are reduced by a small amount in the delayed mortality decline scenario. The costs of children are lower in the delayed mortality scenario, which would lead to higher consumption at older ages and greater accumulation of assets. This is more than offset, however, by the reduced number of years lived at older ages. Under the delayed fertility decline scenario, asset accumulation is very substantially depressed.

Figure 10 about here.

Sensitivity to Other Model Parameters

A series of additional sensitivity tests have been conducted and the results are presented in Tables 1 and 2. Three alternatives simulations are carried out: the first uses the US age profiles of consumption and production (Figure 1) rather than the Taiwan profiles. Second, we consider the implications of relying more heavily on transfer wealth rather than assets to support old age consumption by increasing the share of transfer wealth in pension wealth to 0.65 from 0.35. In the third simulation we consider the implications of shifting the costs of childrearing from parents to taxpayers by reducing the share of familial transfers to children from 0.67 to 0.33.

The US lifecycle differs from the Taiwan lifecycle in that consumption by children is lower and consumption by the elderly is higher relative to consumption by prime age adults. Part of the high consumption at older ages is offset by higher production at older ages in the US. The differences in the economic lifecycle are summarized by the support ratios compared in Table 1. When the population age structure is young, in 1950 or 1990 for example, the support ratio based on the US age profiles of consumption and production are slightly more favorable (2.5% higher in 1950). When the population age structure is old – in 2090 or 2300, for example, the US support ratio based on the US age profiles is unfavorable. This is particularly the case towards the end of the demographic transition. In 2300, the US-based support ratio is 11% below the Taiwan-based support ratio.

Table 1 about here.

During the dividend phases the US age profiles are advantageous, the consumption index is higher from the beginning of the simulation through the first dividend period. The US-based consumption index reaches a higher maximum. As the dividend phases end, however, the relatively high level of old-age consumption in the US turns into a disadvantage. US-based assets are always higher than Taiwan-based assets, but part of that advantage is lost because interest rates are substantially lower (Table 2).[13] Greater assets are insufficient to overcome the adverse effect of a low US-based support ratio and lower rates of return to capital. Whether or not this would be true in a closed economy is a question that remains to be answered.

Table 2 about here.

The implications of an increased reliance on transfers to support old-age consumption are substantial. During the first dividend period, growth of the consumption index in the baseline economy and the high transfer economy are similar. The cost of relying more heavily on transfers comes at sustaining the second dividend and it is substantial. Consumption at the peak (the end of the second dividend period) is almost 10% lower in the high transfer economy a disadvantage that persists in perpetuity. The reason, of course, is that the asset accumulation in the high transfer economy has been much lower (Table 2). Note that the effect is more than proportional to the increase in the share of transfer wealth. Because consumption is reduced at older ages, the amount of pension wealth that is accumulated is also reduced and, hence, the adverse effect on assets is more than proportional.

The final sensitivity test considers the effect of an increase in the taxpayer's share of child costs. Instead of one-third, taxpayers are paying two-thirds of the cost of children. A rise in the burden on taxpayers increases the child costs for year t adults because taxpayers are on average older than parents. Thus, year t adults will pay a higher portion of the future costs of children. The effect is quite modest even though the assumed change is quite extreme. Because we assume that taxes are levied on wages only, it may be that the average age of taxpayers is not much greater than the average age of parents. Given a different tax system, the effects could be larger.

Qualifications and Further Research

The results presented here are promising, but much remains to be done. First, there are features of the theoretical model that require further development. The most obvious and important is to relax the small, open economy feature of the model. Another is to relax the assumption that the cross-sectional consumption profile is fixed. We could, for example, explore the implications of a quantity-quality tradeoff for child expenditures or the implications of political economy models that might influence the consumption of the elderly. A second area requiring more work is empirical. Comparing results from the US and Taiwan shows that variation in the economic lifecycle across countries is important and, hence, the need for more estimates of the economic lifecycle and more analysis of

[13] The global economy and thus international interest rates are determined using the same economic lifecycle as used for the Niger simulation. The results would be different if, for example, the Taiwan lifecycle were used to represent the global economic lifecycle.

how it varies and why. Of equal importance is improving estimates and analysis of transfers. Again, we have estimates for Taiwan and the US that can be employed here, but we know very little about transfer policy in the comprehensive sense of the term used here. In particular, estimates of familial transfers are not widely available. Finally, the model does not incorporate the effects of capital accumulation on wages. Aging is a global phenomenon and, hence, to the extent capital accumulation increases world-wide, as implied by our model, wages will increase on a global scale. This may prove to be the most important implication of population aging for economic growth.

CONCLUSIONS

Over the coming decades we will find ourselves in uncharted waters. The share of the elderly population will reach unprecedented levels and not just in the industrialized world. Many low- and middle-income countries are also far along in their demographic transitions. Even if adults begin to delay retirement, it is virtually certain that the number of retirees will rise relative to the number of workers – in most countries and in the world as a whole.

If labor were the only factor of production, the first order effects of population aging would be easily assessed. Per capita income and per capita consumption would vary directly with the economic support ratio. An increase in the number of retirees would add to the number to be supported but not to the number producing nor the amount produced. The economy would be fixed pie divided among more consumers, and thus, per capita consumption would decline in direct proportion to the decline in the support ratio. The favorable effects of changing age structure that occur early in the demographic transition would be undone as young dependents are replaced by old ones.

Retirees do not, however, rely exclusively on the labor of others (through public and familial transfer systems). Retirees depend on pension funds, personal savings, homes acquired during their working years, and other assets to finance some part of their retirement. How much is a matter of some dispute and varies widely from place to place, but estimates we present show that assets are an important source of support for the elderly in Taiwan and especially in the United States. Thus, the lifecycle demand for assets, the size of the capital stock, and total production increase as populations age. The size of the pie increases with aging, but the important question is by how much.

This paper answers the question using a new conceptual approach. This model acknowledges the close ties and pervasive links across generations. Consumption at each age is not governed by an individualistic lifetime budget constraint as in the lifecycle model. Rather, consumption is governed by altruism and constrained by total production.

The simulation results indicate that fertility decline, increased longevity, and the accompanying changes in age structure have potentially large effects on the demand for assets. Early in the demographic transition, the demand for assets is near zero. By the time that the fertility transition has been completed in the baseline simulation, assets relative to labor income exceeds 3. For this to happen, however, requires a commitment to asset accumulation over transfer systems in the provision of old-age support. Perhaps the most important feature of the simulations to note is that the foundation for the second demographic dividend occurs early in the demographic transition. Long before fertility rates are low and life expectancy is high, a larger share of output is being committed to the accumulating assets. The first dividend eases the pain of foregoing consumption.

Some of the resources previously devoted to supporting large numbers of children can be devoted to capital accumulation. In this way the demographic dividends provide an opportunity for sustaining permanently higher standards of living into the future.

The opportunities are significant, but they are not certain. They depend on public and familial transfer policy. If the response to population aging were exclusively to expand public transfer programs and to increase the burden on adult children of providing support to their parents, then we will merely be dividing a fixed pie among varying number of consumers.

APPENDIX

Child Transfer Costs

The cost of all children age z in year t+x is:

$$\begin{aligned} COST(z,t+x) &= Y(z,t+x) - C(z,t+x) \\ &= \bar{y}(t)G_y(x)L(z,t+x) - \bar{c}(t)G_c(t,x)N(z,t+x) \qquad z < a_0 \end{aligned} \tag{17}$$

A fraction of the cost of children of age *z* in year *t+x* is financed through transfers by year *t* adults; the remainder is financed through transfers by persons who became adults between year *t* and *t+x*. Let $TAX_k(z,t,x)$ be the share of child costs paid by year t adults. Then, child transfer wealth in year t for year t adults is:

$$T_k(t) = \sum_{x=0}^{w-a_0} D(x) \sum_{z=0}^{a_0-1} \mathrm{TAX_k}(z,\mathrm{t,x})\mathrm{COST}(z,\mathrm{t+x}) \tag{18}$$

Substituting for COST from equation (17) yields:

$$T_k(t) = \bar{y}(t)\sum_{x=0}^{w-a_0} D(x)G_y(x)KLTOT(t,x) - \bar{c}(t)\sum_{x=0}^{w-a_0} D(x)G_c(t,x)KNTOT(t,x)$$

$$KLTOT(t,x) = \sum_{z=0}^{a_0-1} \mathrm{TAX_k}(z,\mathrm{t,x})L(z,t+x) \tag{19}$$

$$KNTOT(t,x) = \sum_{z=0}^{a_0-1} \mathrm{TAX_k}(z,\mathrm{t,x})N(z,t+x)$$

where $KLTOT(t,x)$ is the total number of children in year t+x dependent on year t adults measured in equivalent production units and in year t+x and $KNTOT(t,x)$ is the total number of children in year t+x dependent on year t adults measured in equivalent consumption units.

Tax burden of year t adults

The share of year t adults depends on whether child costs are financed through public or private (familial) transfer programs. We assume that the shares of public and private transfers are constant and exogenous, i.e., they are a matter of public policy. Let the familial share be t^f and the public share be $1-t^f$. Then the share of cost paid by year t adults is a weighted sum of the taxes paid through a familial transfer system and the taxes paid through a public transfer system, i.e.,

$$TAX_k(z,t,x) = t^f TAX_k^f(z,t,x) + (1-t^f)TAX_k^g(z,t,x) \tag{20}$$

where $TAX_k^f(z,t,x)$ is the share of child costs paid by year t adults under a familial transfer systems and $TAX_k^g(z,t,x)$ is the share of child costs paid by year t adults under a public transfer system.

We assume that all public transfers to children are financed by a proportional tax on labor income. Thus,

$$TAX_k^g(z,t,x) = \sum_{a=a_0+x}^{w} Y(a,t+x)/Y_A(t+x) \tag{21}$$

where $Y_A(t+x) = \sum_{a=a_o}^{w} Y(a,t+x)$ is the total labor income of all in year t+x.

The tax share of year t adults is in year t+x is their share of labor income in year $t+x$. Note that the public tax share is independent of the age of the child, z. Henceforth, we drop the z argument.

We assume that familial transfers are determined by parentage. If we let F(z, t, x) equal the proportion of those aged z with parents (mothers) age $a_0 + x$ or older in year $t+x$, then

$$TAX_k^f(z,t,x) = F(z,t,x) \tag{22}$$

where F is calculated using the distribution of births to women:

$$F(z,t,x) = \frac{\sum_{a=a_0+x-z}^{AGEM} B(a,t+x-z)}{\sum_{a=a_0}^{AGEM} B(a,t+x-z)} \quad \text{for } x > z \tag{23}$$

$$= 1 \quad \text{for } x \leq z.$$

and $B(a, t+x-z)$ is births to women aged a in year t+x-z. Children who are x years or older are all the offspring of year t adults (mothers) and hence F has a value of 1. The value of F declines to zero as x increases. (Note that F can be represented as a function of t and x-z. It isn't really three dimensional.)

We can substitute into equation (20) and the share of year t adults is:

$$TAX_k(z,t,x) = t^f F(z,t,x) + (1-t^f) \sum_{a=a_0+x}^{w} Y(a,t+x)/Y_A(t+x) \tag{24}$$

Substituting into equation (19) yields child transfer wealth for year t adults. Note that the tax shares devoted to childrearing are determined exogenously by population age structure, fertility, the age profile of earnings – all exogenous factors. Thus, in the determination of child transfer costs, the only endogenous variable is the vector of the consumption index.

Steady-state Results

The trajectory of assets must satisfy the macroeconomic flow constraint:

$$(1+r)A(t) + Y(t) - C(t) = A(t+1). \tag{25}$$

In steady-state, assets grow at the same rate as total labor income, g_Y. Substituting $(1+g_Y)A(t)$ for $A(t+1)$, substituting for income and consumption, and rearranging terms, assets in steady state must satisfy:

$$A(t^*) = \frac{1}{r-g_Y}\left[\bar{c}(t^*)N(t^*) - \bar{y}(t^*)L(t^*)\right]. \tag{26}$$

From the analysis of the lifecycle the relationship between assets and lifecycle pension wealth is governed by exogenously specified pension transfer policy:

$$A(t^*) = (1-t(t^*))W_P(t^*), \tag{27}$$

where $W_P(t)$ is given in equation (15). Combining the macro and lifecycle conditions, and noting that the growth rate of the consumption index must equal the growth rate of the production index in steady-state, the consumption index in steady-state must satisfy:

$$\frac{1}{r-g_Y}\left[\bar{c}(t^*)N(t^*)-\bar{y}(t^*)L(t^*)\right]=(1-t(t^*))W_p(t^*). \quad (28)$$

Rearranging terms yields:

$$\frac{\bar{c}(t^*)}{\bar{y}(t^*)}=\frac{L(t^*)}{N(t^*)}\left[1+(r-g_Y)(1-t(t^*))w_p(t^*)\right], \quad (29)$$

where $w_p(t^*)$ is the ratio of lifecycle pension wealth to current labor income.

Equation (29) tells us the level of consumption that can be sustained in steady-state given any level of labor productivity. Age-structure determines the steady-state consumption ratio through two multiplicative factors – the economic support ratio and a second factor that captures the influence of age structure on lifecycle pension wealth and, hence, assets.

Backward Recursion

The backward recursion solution computes the consumption index and, hence, all other variables in period t-1 conditional on the values in period t. The steady-state values are known. Hence, we can begin in period t*, solve for period t*-1, and recursively solve for all periods t.

From lifecycle accounting, assets in period t-1 depend on pension policy and lifecycle wealth in year t-1. From equations (15) and (27):

$$\begin{aligned}A(t-1)&=\bar{c}(t-1)(1-t)\sum_{x=0}^{w-a_0}D(x)G_c(t-1,x)\left(NTOT(t-1,x)+KNTOT(t-1,x)\right)\\&-\bar{y}(t-1)(1-t)\sum_{x=0}^{w-a_0}D(x)G_y(x)\left(LTOT(t-1,x)+KLTOT(t-1,x)\right).\end{aligned}$$

(30)

Pension policy may vary with year, but here we drop t to simplify notation. The right-hand-side variables include consumption in year t-1, consumption in year t and subsequent years, and labor income terms in year t-1 and later. Only the consumption terms in year t-1 are unknown and must be solved for. These are distinguished in:

$$\begin{aligned}A(t-1)&=\bar{c}(t-1)(1-t)N(t-1)D(0)\\&+(1-t)\sum_{x=1}^{w-a_0}D(x)\bar{c}(t-1+x)\left(NTOT(t-1,x)+KNTOT(t-1,x)\right) \quad (31)\\&-\bar{y}(t-1)(1-t)\sum_{x=0}^{w-a_0}D(x)G_y(x)\left(LTOT(t-1,x)+KLTOT(t-1,x)\right).\end{aligned}$$

From macro accounting, we know that:

$$A(t-1)=\frac{A(t)+\bar{c}(t-1)N(t-1)-\bar{y}(t-1)L(t-1)}{1+r}. \quad (32)$$

This gives us two equations in two unknowns, assets and the consumption index in period t-1. Substituting for *A(t-1)* yields:

$$\bar{c}(t-1)(1-t)N(t-1)D(0)+(1-t)\sum_{x=1}^{w-a_0} D(x)\bar{c}(t-1+x)\left(NTOT(t-1,x)+KNTOT(t-1,x)\right)$$

$$-\bar{y}(t-1)(1-t)\sum_{x=0}^{w-a_0} D(x)G_y(x)\left(LTOT(t-1,x)+KLTOT(t-1,x)\right)$$

$$=\frac{A(t)+\bar{c}(t-1)N(t-1)-\bar{y}(t-1)L(t-1)}{1+r} \quad (33)$$

Multiplying both sides by 1+r and rearranging terms yields:

$$\bar{c}(t-1)N(t-1)\left((1-t)(1+r)D(0)-1\right)$$

$$=A(t)-(1+r)(1-t)\sum_{x=1}^{w-a_0} D(x)\bar{c}(t-1+x)\left(NTOT(t-1,x)+KNTOT(t-1,x)\right)$$

$$+\bar{y}(t-1)(1+r)(1-t)\sum_{x=0}^{w-a_0} D(x)G_y(x)\left(LTOT(t-1,x)+KLTOT(t-1,x)\right)-\bar{y}(t-1)L(t-1) \quad (34)$$

Further algebra gives the consumption index for t-1:

$$\bar{c}(t-1)=\frac{\left\{\begin{array}{l} A(t)-(1+r)(1-t)\sum_{x=1}^{w-a_0} D(x)\bar{c}(t-1+x)\left(NTOT(t-1,x)+KNTOT(t-1,x)\right) \\ +\bar{y}(t-1)\left\{(1+r)(1-t)\sum_{x=0}^{w-a_0} D(x)G_y(x)\left(LTOT(t-1,x)+KLTOT(t-1,x)-L(t-1)\right)\right\} \end{array}\right\}}{N(t-1)\left((1-t)(1+r)D(0)-1\right)}. \quad (35)$$

Assets in period t-1 can be calculated using either equation (31) or equation (32).

References

Barro, R. J. and X. Sala-i-Martin (1995). Economic growth. New York; London and Montreal, McGraw-Hill.

Bloom, D. E. and D. Canning (2001). Cumulative Causality, Economic Growth, and the Demographic Transition. Population Matters: Demographic Change, Economic Growth, and Poverty in the Developing World. N. Birdsall, A. C. Kelley and S. W. Sinding. Oxford, Oxford University Press**:** 165-200.

Bloom, D. E., D. Canning, et al. (2002). The Demographic Dividend: A New Perspective on the Economic Consequences of Population Change. Santa Monica, CA, RAND.

Bloom, D. E. and J. G. Williamson (1998). "Demographic Transitions and Economic Miracles in Emerging Asia." World Bank Economic Review **12**(3): 419-56.

Lee, R., A. Mason, et al. (2000). "Life Cycle Saving and the Demographic Transition in East Asia." Population and Development Review **26**(Supplement).

Lee, R., A. Mason, et al. (2003). "From Transfers to Individual Responsibility: Implications for Savings and Capital Accumulation in Taiwan and the United States." Scandinavian Journal of Economics **105**(3): 339-357.

Lee, R. D., S.-H. Lee, et al. (2005). "Charting the Economic Lifecycle." mimeo.

Lee, R. D., A. Mason, et al. (2001). Saving, Wealth, and Population. Population Does Matter: Demography, Poverty, and Economic Growth. N. Birdsall, A. C. Kelley and S. W. Sinding. Oxford, Oxford University Press**:** 137-164.

Mankiw, G., D. Romer, et al. (1992). "A Contribution to the Empirics of Economic Growth." Quarterly Journal of Economics **107**(2): 407-37.

Mason, A. (2001). Population Change and Economic Development in East Asia: Challenges Met, Opportunities Seized. Stanford, Stanford University Press.

Mason, A. (2005). Demographic Transition and Demographic Dividends in Developed and Developing Countries. United Nations Expert Group Meeting on Social and Economic Implications of Changing Population Age Structures, Mexico City.

Mason, A. and R. Lee (2004). Reform and Support Systems for the Elderly in Developing Countries: Capturing the Second Demographic Dividend. International Seminar on the Demographic Window and Healthy Aging: Socioeconomic Challenges and Opportunities, China Centre for Economic Research, Peking University, Beijing.

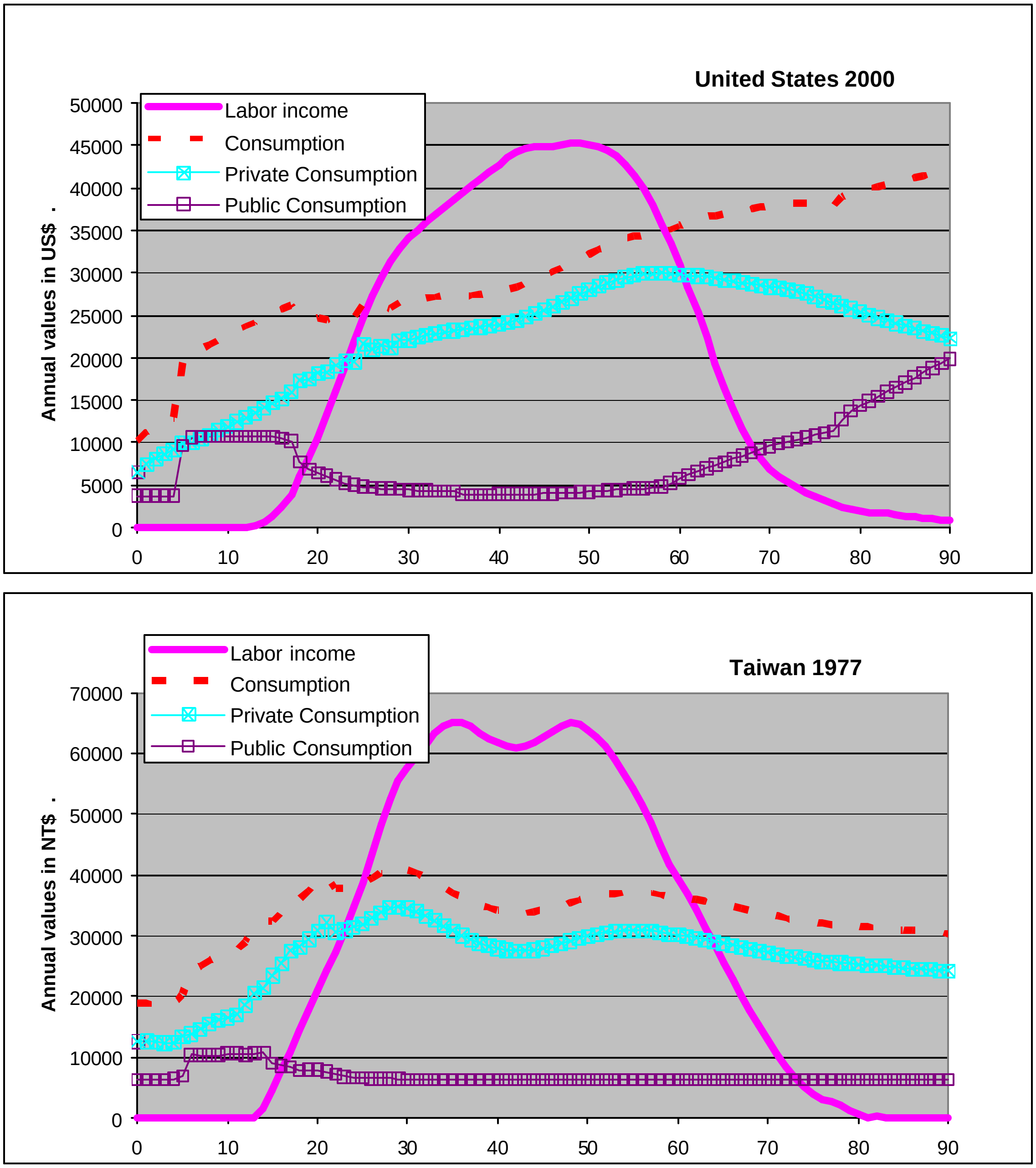
Mason, A., R. Lee, et al. (forthcoming). Population Aging and Intergenerational Transfers: Introducing Age into National Accounts. Economics of Aging Series. D. Wise. Chicago, NBER and University of Chicago Press.

United Nations Population Division (2004). World Population to 2300. New York, United Nations.

United Nations Population Division (2005). World Population Prospects: The 2004 Revision. New York, United Nations.

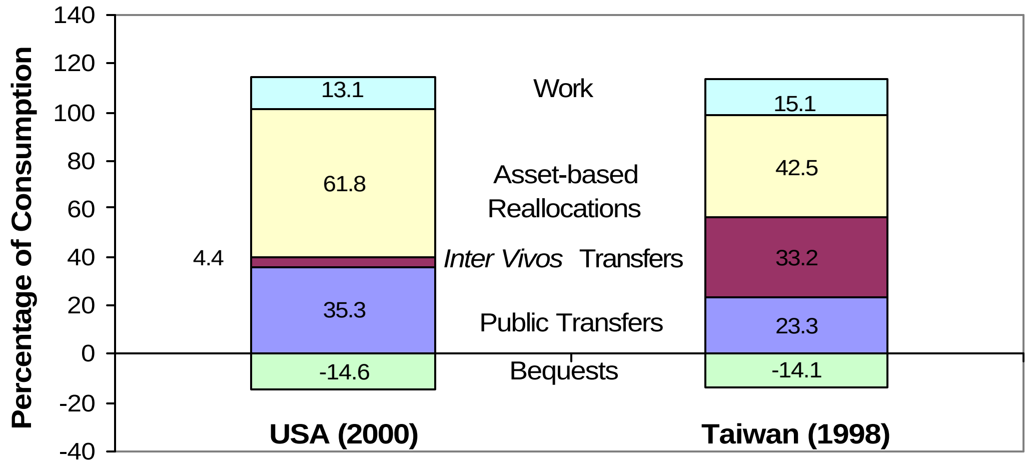
World Bank (2005). World Development Indicators 2005. New York, World Bank.

Figure 1. Consumption and Labor Income Age Profiles for the United States, 2000 and Taiwan, 1977.



Source: Lee, Lee, and Mason (2005)

Figure 2. How the Elderly Finance Consumption in the US and Taiwan (Age 65+)



Source: Mason, Lee, et al. (forthcoming)

Figure 3. Age Distribution of Niger's Population, 1950-2200

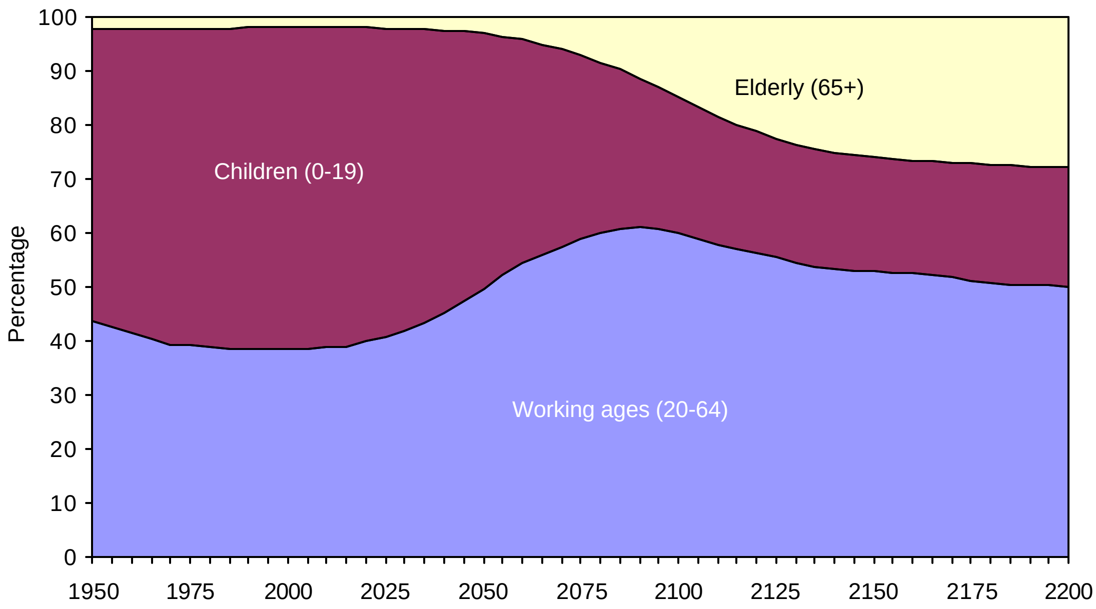


Figure 4. **The Support Ratio (L/N) and the Index of Consumption (c_{bar}/y_{bar}), Niger Population, 1950-2300, Taiwan Economic Lifecycle**

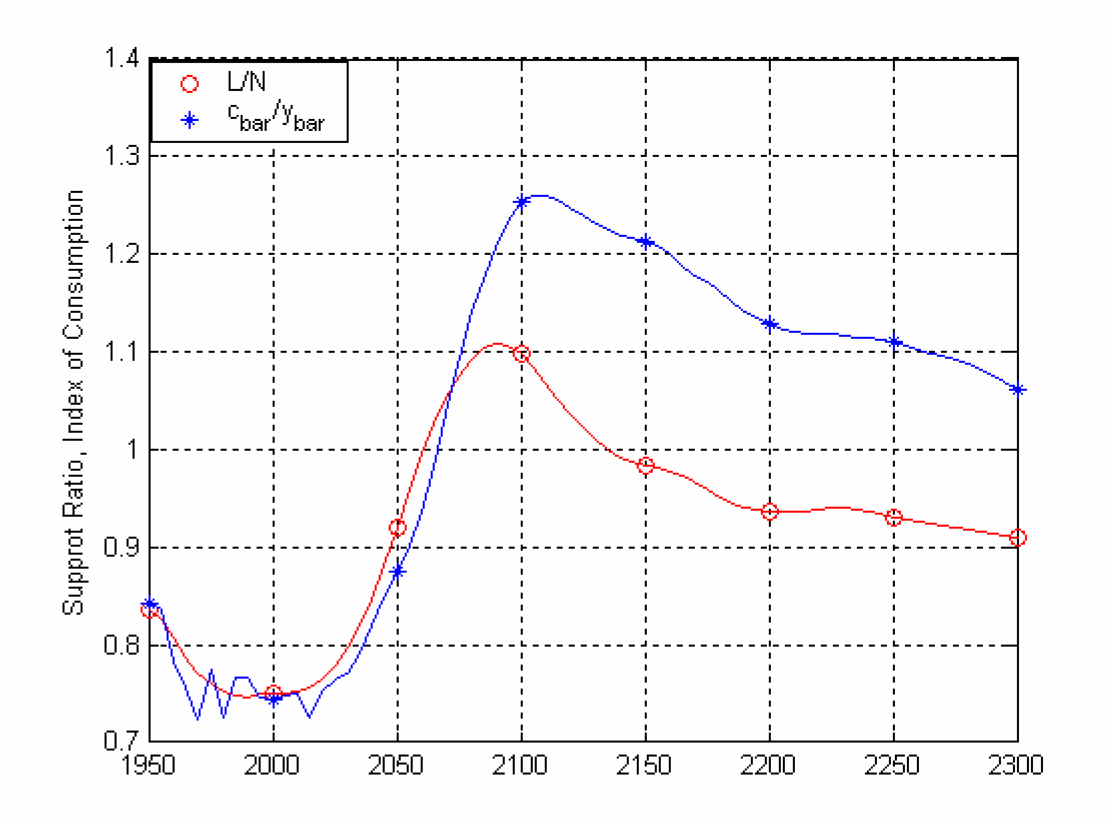


Figure 5. Dividend Variables (Annual rates of growth), Niger Population, 2000-2150, Taiwan Economic Lifecycle.

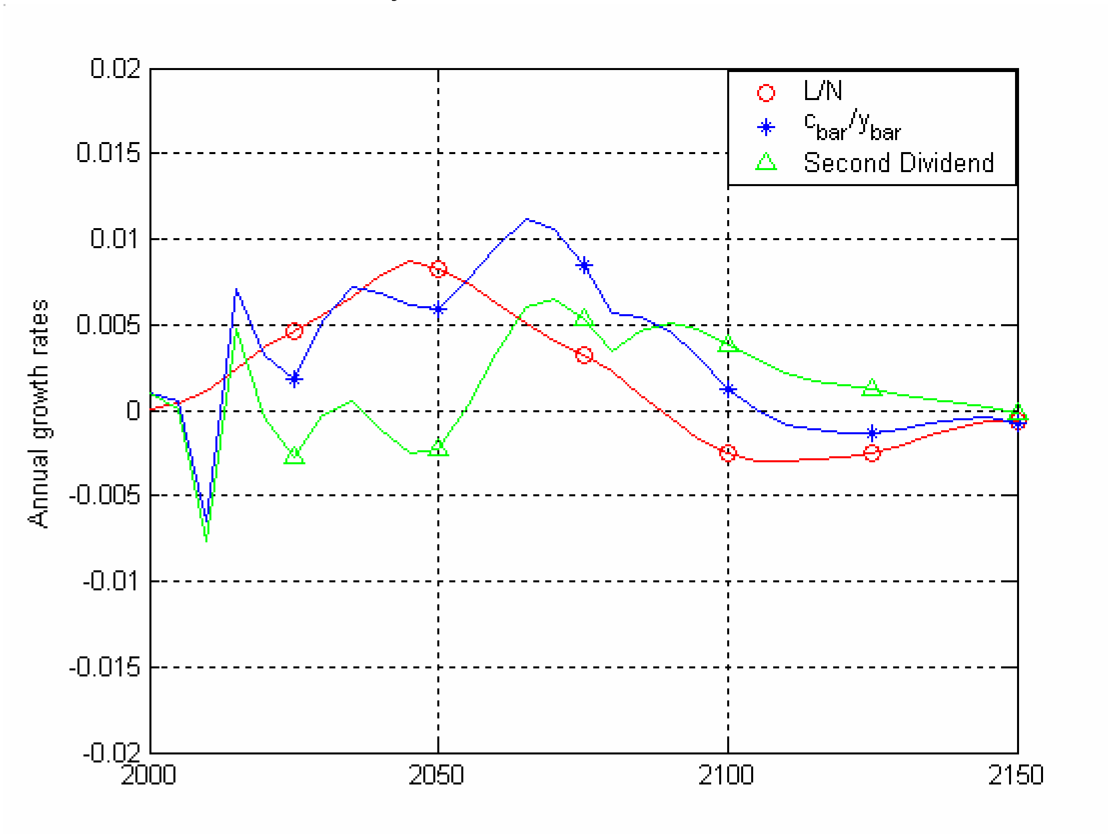


Figure 6. **Components of Wealth,**
Niger Population, 1950-2300, Taiwan Economic Lifecycle

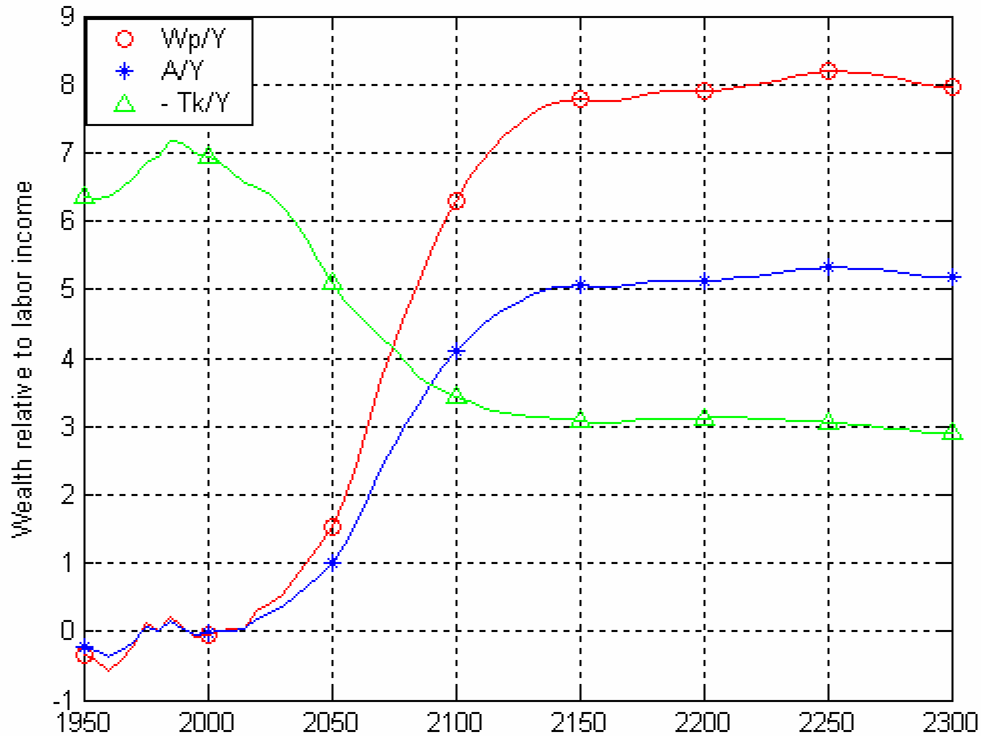


Figure 7. **Consumption Ratio,**
Niger Population, 1950-2300, Taiwan Economic Lifecycle

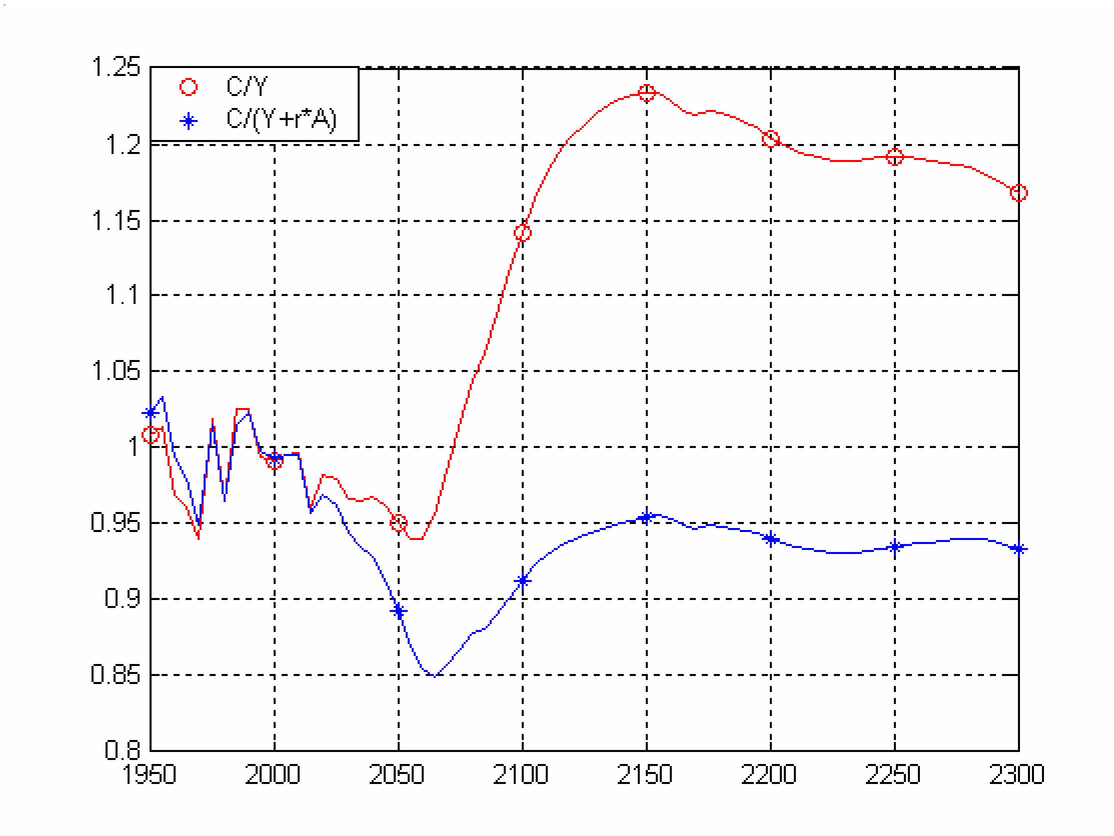


Figure 8. The Support Ratio (L/N) and the Index of Consumption (cbar/ybar), Alternative Population Scenarios, 2000-2300, Taiwan Economic Lifecycle

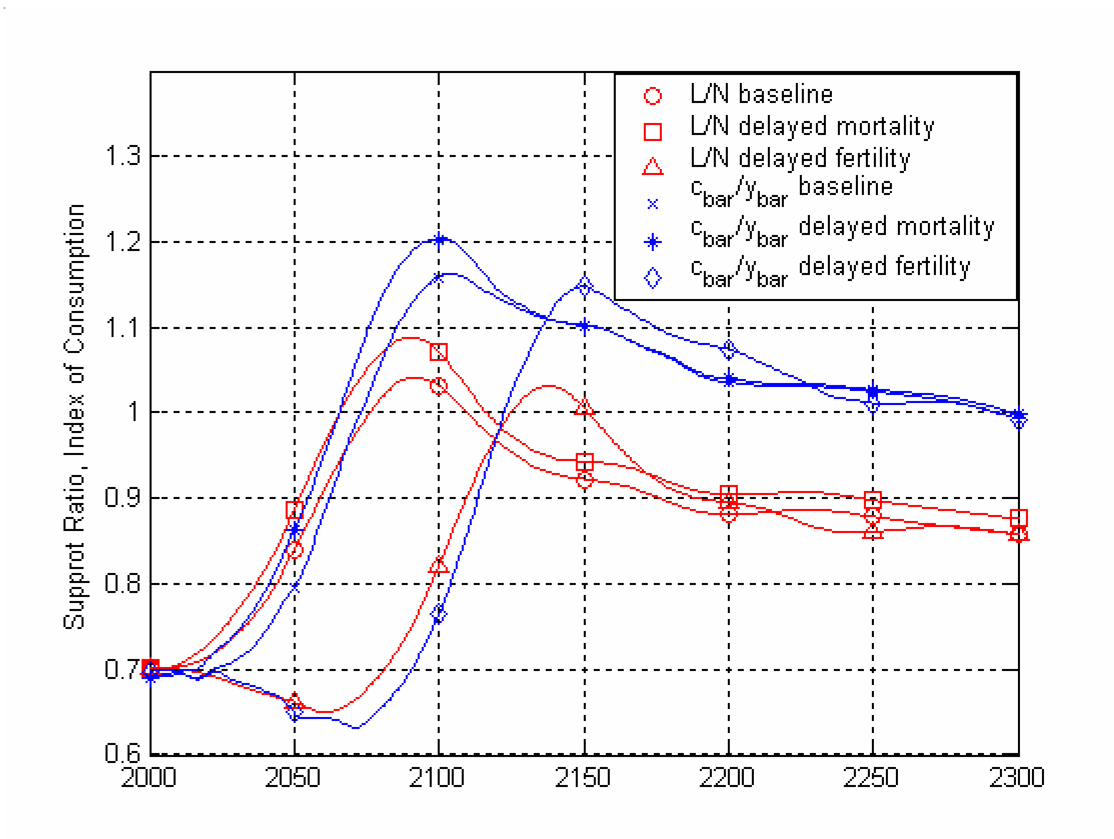


Figure 9. Child Transfer Wealth Relative to Labor Income (-Tk/Y), Alternative Population Scenarios, 2000-2300, Taiwan Economic Lifecycle

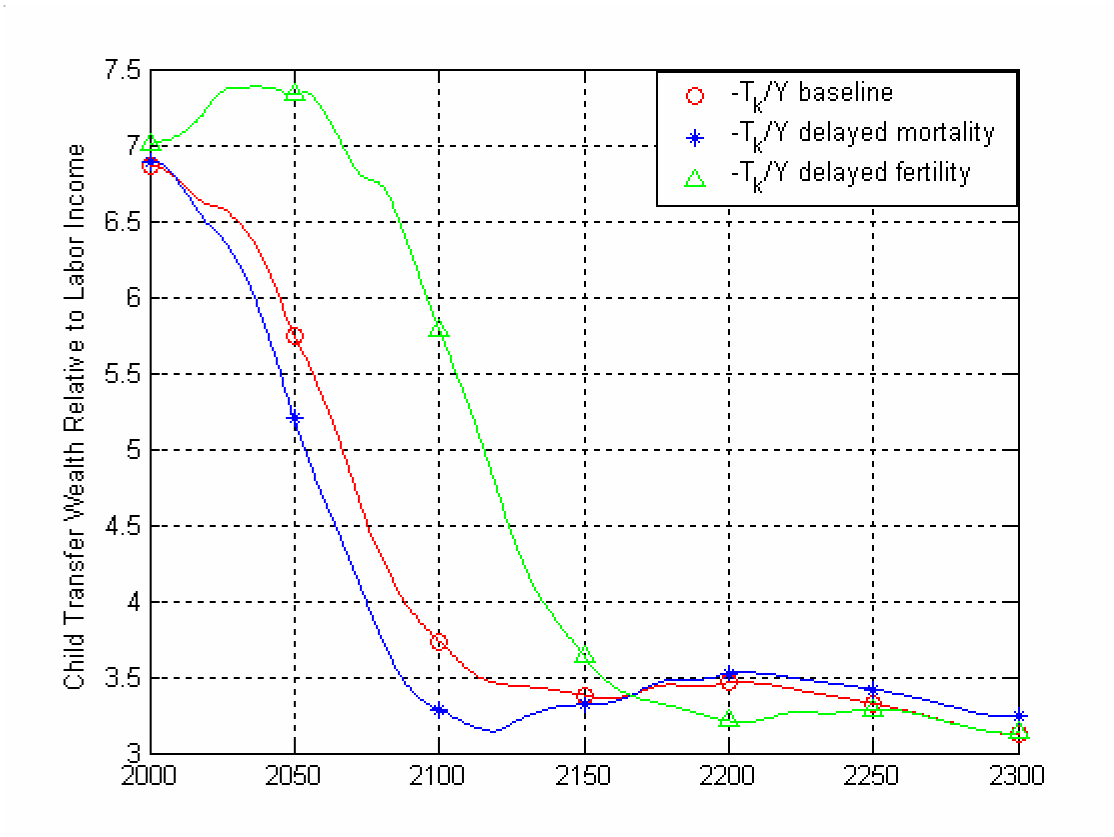


Figure 10. Assets Relative to Labor Income (A/Y), Alternative Population Scenarios, 2000-2300, Taiwan Economic Lifecycle

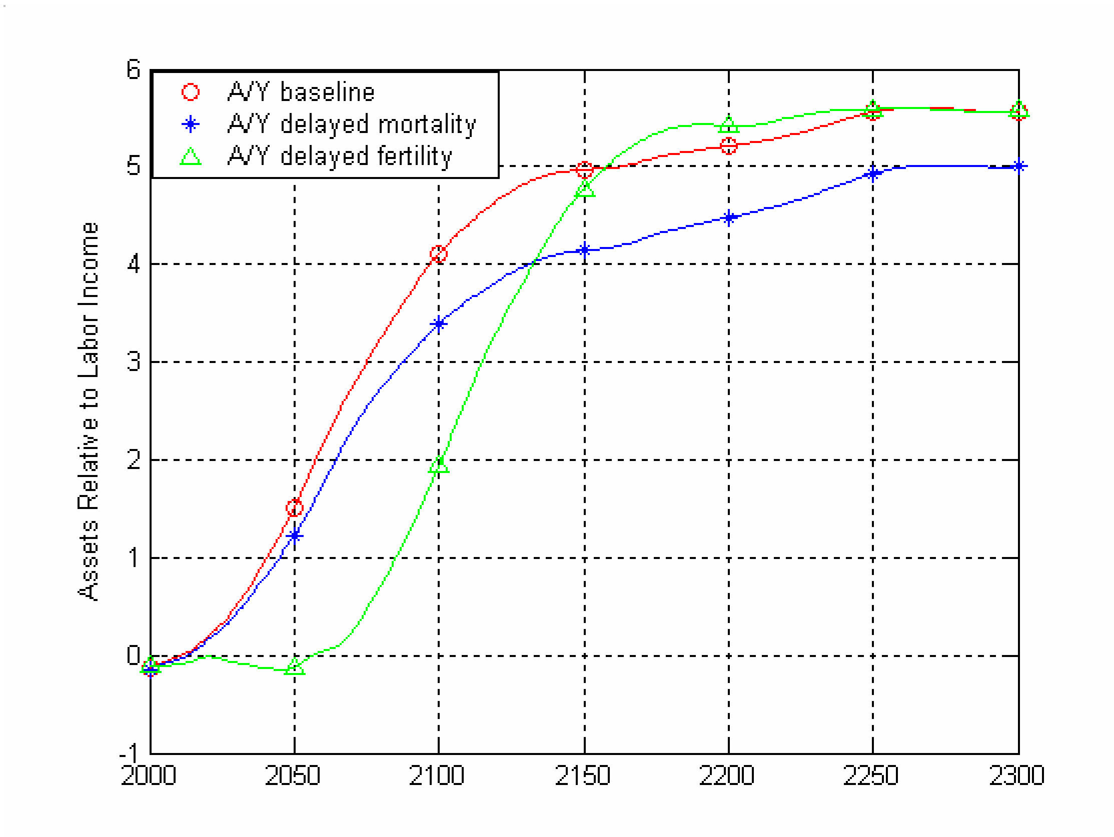


Table 1. Effects of Parameters on Simulated Values.

Variable	Baseline	US lifecycle	$t = 0.65$	$t_f = 0.33$
First dividend period	1990-2090	1990-2085	1990-2090	1990-2090
Year second dividend ends	2110	2100	2095	2105
L/N, 1950	0.8352	0.8553	0.8352	0.8352
L/N, 1990	0.7466	0.7606	0.7466	0.7466
L/N, 2090 (2085 for US lifecycle)	1.1091	1.0905	1.1091	1.1091
L/N, 2300	0.9099	0.8130	0.9099	0.9099
$\bar{c}/\bar{y}$, 1950	0.8419	0.8745	0.8337	0.8409
$\bar{c}/\bar{y}$, 1990	0.7659	0.7895	0.7477	0.7576
$\bar{c}/\bar{y}$, maximum	1.2606	1.2997	1.1603	1.2891
$\bar{c}/\bar{y}$, 2300	1.0621	0.9206	0.9693	1.0715
L/N, annual growth (%), 1st div period	0.40%	0.36%	0.40%	0.40%
$\bar{c}/\bar{y}$, annual growth (%),1st div period	0.46%	0.46%	0.44%	0.48%

Notes: First dividend period begins when L/N reaches a minimum and ends when L/N reaches a maximum. Second dividend ends when $\bar{c}/\bar{y}$ reaches its maximum value. Unless otherwise indicated simulation is based on the following assumptions: Taiwan 1977 lifecycle profiles; annual labor productivity growth of 1.5% per year; family share of transfers to children of 0.67; a discount rate of 3% per year; an interest declining from 6% for 1950-2000 to a steady-state value of 4.22% for the Taiwan lifecycle and 3.52% for the US lifecycle; transfer wealth as a share of total pension wealth of 0.35; medium scenario of the UN Population Projections.

Table 2. Effects of Parameters on Simulated Values of Wealth Relative to Labor Income and the Consumption Ratio.

Variable	Baseline	US lifecycle	$t = 0.65$	$t_f = 0.33$
Child wealth, max	-7.1794	-7.7468	-7.0728	-7.5037
Child wealth, min	-2.8965	-2.2420	-2.5138	-3.2051
Pension wealth, max	8.1881	10.8842	5.7727	8.7330
Assets, max	5.3222	7.0747	2.0205	5.6765
C/(Y+rA) max	1.0331	1.0115	1.0048	0.9984
C/(Y+rA) min	0.8494	0.7531	0.9416	0.8408

Note. All assets values expressed relative to total labor income. See notes for Table 2.