*Topics in Theoretical Economics*

*Volume* 6, *Issue* 1 2006 *Article* 11

# Age Bias in Fiscal Policy: Why Does the Political Process Favor the Elderly?

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## Abstract

Across countries, government expenditures tend to favor the elderly. This paper provides a political economy explanation for this phenomenon. I consider the classic problem of dividing a fixed payoff in an overlapping generations setting. Any share of the payoff can be given to any generation. Using a new solution concept for majority rule in dynamic settings (Bernheim and Slavov, 2006), I demonstrate that policies favoring the old are easier to sustain politically than other policies. This result appears across a broad class of majoritarian institutions and thus reflects general forces at work in the political process. Age bias arises because it is easy to induce the young to support policies favoring the elderly by promising them large rewards later in their lives. On the other hand, older generations cannot be rewarded in a similar manner. This asymmetry helps to generate broad political support for large transfers to older individuals.


**KEYWORDS:** Political Economy, Intergenerational Transfers, Overlapping Generations, Majority Rule

*Department of Economics, Occidental College, Los Angeles, CA 90041. For helpful discussion and comments, I would like to thank Bevin Ashenmiller; Doug Bernheim; Antonio Rangel; Luis Rayo; John Shoven; Steve Tadelis; members of the Public Economics Group at Stanford; seminar participants at Chicago, Minnesota, Georgetown, Brown, Connecticut, and Texas; and an anonymous referee. This paper is based on my doctoral dissertation at Stanford, where I was supported by a National Science Foundation Graduate Research Fellowship.

# 1 Introduction

Governments frequently make transfers that target particular age groups. For example, public retirement programs provide income for the elderly, while education expenditures primarily benefit the young. The empirical evidence suggests that, on balance, across industrialized countries, the elderly tend to receive large net transfers. In the United States, individuals over the age of 65 receive more than seven times as much public spending per capita as those aged 20-35 (Rogers et. al., 2000).[1] A similar pattern holds in other OECD countries as well (see Poterba, 1997). Not only are expenditures on the elderly large, but they are also immensely popular, even among the young. When given a hypothetical choice between raising payroll taxes and cutting retirees' benefits to address Social Security's long-term funding shortfall, 57 percent of Americans preferred raising taxes, while only 21 percent preferred cutting benefits (Gallup Poll, July 13-14, 1998). A poll taken in 2003 indicated that 76 percent of adults overall – and 80 percent of 18-29-year-olds – supported the creation of a Medicare prescription drug program (Gallup Poll, June 27-29, 2003).

A variety of efficiency considerations may explain the existence of particular age-targeted public expenditures. Education has positive externalities, which may lead to underinvestment by the young; imperfections in the market for health insurance may justify some degree of public provision, and the elderly have more health needs than the young. However, while efficiency may partly explain the large net transfers toward the elderly, it is unlikely to provide the whole story. As discussed by Mulligan and Sala-i-Martin (1999), transfers to the elderly must be financed by distortionary taxation, which has an increasing marginal cost. Thus, one would expect an increase in the population share of the elderly to cause the GDP share of public expenditures on the elderly to rise by a smaller factor. Yet empirically, the GDP share of public expenditures on the elderly tends to increase more than proportionally with relative increases in the elderly population. The political sustainability of Social Security, and the resulting implications for its reform, are the subject of a number of papers (e.g., Mulligan and Sala-i-Martin, 2004; Galasso and Profeta, 2004; Conesa and Krueger, 1999).

Age bias results are not uncommon in political economy analyses of intergenerational transfers. Some authors have demonstrated that particular programs designed to favor the elderly (e.g., social security or public debt) are politically sustainable. These include Browning (1975), Cukierman and Meltzer (1989), Tabellini (1990,1991), Azariadis and Galasso (1995, 1998), Cooley and Soares (1999), and Boldrin and Rustichini (2000). Other authors consider a more general set of transfers in which programs that favor the elderly are decided on jointly with programs that favor the young; they find, explicitly or implicitly, that programs benefiting the elderly are easier to sustain than programs benefiting the young. For example, Mulligan and Sala-i-Martin (1999) argue that the elderly are more "single-minded" since they are retired and not divided into occupational interest groups. This characteristic increases the benefits from forming a political interest group. Rangel (2003) shows that middle-aged individuals can be induced to make transfers to the elderly

[1] This analysis includes federal, state, and local transfer payments, as well as expenditures on education

if they expect to receive similar transfers in return when they are old. On the other hand, middle-aged individuals have already received any transfers intended for the young, leaving no instrument to punish them for failing to make similar transfers to future generations. The key insight is that transfers to future generations are only sustainable if they can be linked with transfers to older generations (a similar result appears in Boldrin and Montes, 2005).[2]

One could use a number of political economy solution concepts to study the sustainability of age-specific transfers. Many of the papers cited above use a game-theoretic approach, which consists of modeling the political process as a game and characterizing its equilibria (e.g., Mulligan and Sala-i-Martin's, 1999, interest-group based model). Others (e.g., Browning, 1975; Renstrom, 1996; Cukierman and Meltzer, 1989; Poutvaara, 1996) use a non-game-theoretic solution concept: they specify a policy set and look for a Condorcet winner, or a policy that majority-defeats all other policies in pairwise comparison, in each period.[3] The Condorcet winner concept is appealing in part because of its robustness. When a Condorcet winner exists, a number of majoritarian institutions – including two-party competition (Downs, 1957) and various forms of sequential pairwise voting (Shepsle and Weingast, 1984; Bernheim et. al., 2002) – will deliver it in equilibrium. Since the existence of a Condorcet winner is not guaranteed in multidimensional policy spaces, some type of restriction on the policy space must be imposed. For example, if the policy space is restricted to a single dimension over which individuals have single-peaked preferences, then a median voter exists whose preferred policy is a Condorcet winner (Black, 1958).

The contribution of this paper is to demonstrate that age bias is a robust finding that is inherent to any democratic system. I abstract from particular policies, like social security or education, and allow any payment to be made to any age group. In particular, I consider the classic "divide-the-dollar" problem, in which a fixed surplus must be split among a group of individuals. This problem is faced each period by an ongoing community consisting of overlapping generations of individuals. Each period, any share of the surplus can be given to any age group. This setup provides a very general policy space by allowing transfer payments to all age group to be chosen simultaneously; it also abstracts from any efficiency considerations.

Of course, the multidimensional policy set makes the application of the traditional Condorcet winner concept impossible. Instead, I use a new solution concept, developed in Bernheim and Slavov (2006) for infinite-lived agents, that is a natural analog of the Condorcet winner notion in a dynamic setting. In a dynamic setting, policy choices can be history dependent, allowing for the imposition of "punishments" for collective deviations from a particular path. A *Dynamic Condorcet Winner* (DCW) assigns a policy outcome to every history in such a way that the specified outcome for each history is a Condorcet winner given the resulting continuation paths. I extend the DCW concept to overlapping generations settings. It turns out that DCWs exist in the divide-the-dollar model. Moreover, like their static counterparts, DCWs also summarize the equilibria of a number of voting in-

[2]A number of other papers also examine the political economy of intergenerational transfers without focusing explicitly on age bias, including Lambertini and Azariadis (2003) Bassetto (1999), Poutvaara (1996), Renstrom (1996), and Grossman and Helpman (1998).

[3]Bassetto (1999) uses Nash bargaining, a different non-game theoretic alternative.

stitutions. Thus, the DCW solution concept allows me to explore general forces at work in the political process.

I characterize the set of divide-the-dollar outcomes that are sustainable in a DCW and demonstrate that the DCW set exhibits clear bias towards the elderly. Since the DCW set encompasses all policy outcomes that can be sustained under repeated majority rule, one can interpret this set as a political feasibility constraint on policy design (see Bernheim and Slavov, 2006). Given this interpretation, one can use the DCW set to study how policy makers with various objectives are affected by a political feasibility constraint requiring that the selected outcome be implementable by majority rule. I demonstrate that the maximum politically feasible average discounted payoff for a cohort is increasing in the age of the cohort; that is, a policy maker representing the old can obtain a higher average discounted payoff for the favored cohort than a policy maker representing the young. Moreover, if one restricts attention to steady state DCWs – i.e., DCWs in which the same policy occurs every period on the equilibrium path – then the share of the dollar that can be given to any age group tends to increase with age. In other words, when faced with a majoritarian feasibility constraint, a policy maker who favors the young in every period – and seeks to implement a steady state DCW – will tend to do worse than a policy maker who favors the old in every period.

Under the majoritarian feasibility interpretation, the multiplicity of DCWs becomes a positive, rather than a negative, result. The notion of choosing a policy out of a politically feasible set makes the results of this paper applicable to a variety of political systems, not only those that approximate direct democracy. It is reasonable to think that interest groups, legislators, bureaucrats, and other decision makers face majoritarian feasibility constraints in a broad sense: a policy is unsustainable if it cannot be made acceptable to a majority even by constructing a punishment for opposing it.

The intuition for the existence of age bias is that, since the young outlive the old, one has more flexibility to reward the young for supporting any given policy early in their lives. In particular, the young can be induced to support large transfers to the elderly if they expect to be rewarded for their support after older generations die. On the other hand, the old cannot be induced to support large transfers to the young using this reward system, as they will not outlive the young to collect any rewards. This intuition is more robust than the popular argument that the young support social security "so it will be there for them" because it allows for the current social security regime to be scrapped before the current young retire. However, the analysis of steady state DCWs does encompass this traditional explanation as a special case: if the young expect a policy path in which the current social security system will be in place when they are old, this in itself can serve as a reward. On the other hand, giving large transfers to the young each period does not hold out a similar reward for older agents.

A similar intuition appears in game theoretic analyses of organizations consisting of overlapping generations of players (e.g., Crémer, 1986; Dickson and Shepsle, 2001; Shepsle, 1999). In these models, the elderly never cooperate (e.g., by voluntarily contributing to a public good) because they cannot be punished. The intuition in this paper, however, is more subtle. In a game, the elderly can deviate unilaterally; thus, non-cooperation is not surprising. In contrast, since the DCW solution concept has no notion of unilateral deviations, younger generations can form a coalition to expropriate the elderly. However, the young have little desire to do so if they are sufficiently patient and expect to be rewarded later in their lives for "good behavior." On the other hand, older generations have a strong desire to expropriate the young, as there is less flexibility to reward them in the future. Therefore, policies that transfer resources to the elderly receive broader political support.

This paper is organized as follows. Section 2 introduces the model and the solution concept. Section 3 presents the main results. Section 4 explores some extensions to the basic model, including the introduction of economic growth and the ability to borrow and save. Section 5 concludes.

# 2 Model

Consider an overlapping generations (OLG) model in which three generations are alive at any given time $t = 0, 1, \ldots$. Every period, a new generation of young is born, and each generation lives for three periods. There is no population growth, and the generations are of equal size (normalized to 1); thus, the total population at any given time consists of three individuals. At time $t$, an individual is identified by her age $i = 1, 2, 3$, where 1 is the youngest individual and 3 is the oldest individual. Let $I \equiv \{1, 2, 3\}$ be the set of age groups/individuals alive in any period.[4]

## 2.1 "Divide-the-Dollar"

Every period, the community of living individuals is faced with the collective choice problem of splitting a fixed surplus (a "dollar") among themselves. Any share of the dollar can be given to any age group. Disposal is not allowed, so the entire dollar must be paid out each period. The policy space in any period is the unit simplex

$$U = \left\{ u \in \mathbb{R}^3_+ \,\middle|\, \sum_{i=1}^{3} u_i = 1 \right\}, \tag{1}$$

where $u_i$ is the share of the dollar received by age group $i$.

This simple "divide-the-dollar" problem is a canonical one in political science (see, e.g., Baron and Ferejohn, 1987, 1989; Epple and Riordan, 1987), and it isolates the issue of pure age-based transfers. Analytically, the problem turns out to be quite complex because not only does a Condorcet winner fail to exist, but every division of the dollar is part of a Condorcet cycle. That is, for any division of the dollar, we can find another division that is strictly preferred by a majority.

I assume that individuals must consume any part of the dollar they receive in the current period; borrowing and saving are not allowed. I also assume that an individual's lifetime utility is equal to the present value of her future consumption. Section 4 considers the implications of relaxing these two assumptions.

[4]An earlier version of this paper considered the general case with $N$ generations. This adds a significant amount of computational complexity without providing much additional insight; therefore, it has been omitted here. The $N$-generation version is available from the author upon request.

## 2.2 Solution Concept

Bernheim and Slavov (2006) define and explore a dynamic analog of the Condorcet concept for infinite-lived agents. This concept and some of its properties are summarized here, with appropriate modifications for the OLG setting.

The static Condorcet winner concept is defined as follows:

**Definition 1** *A* ***Condorcet winner*** *is a policy $u \in U$ such that $|\{i \in I | u_i \geq u'_i\}| \geq 2$ for all $u' \in U$.*[5]

In words, a policy $u$ is a Condorcet winner if and only if, for any alternative policy $u'$, a majority of individuals (i.e., at least two) prefer $u$ to $u'$. This solution concept is widely used in political economy problems, including some that deal with the issue of age bias (e.g., Tabellini, 1991; Browning, 1975). It is appealing because, if a Condorcet winner exists, it emerges in equilibrium across a number of majoritarian institutions. For example, a Condorcet winner is the unique pure strategy equilibrium of the Downsian two-party competition model (Downs, 1957). Condorcet winners are also selected by various institutions based on sequential pairwise voting (e.g., Shepsle and Weingast, 1984; Bernheim et. al., 2002) and citizen-candidate models of representative democracy (Besley and Coate, 1997).

However, a Condorcet winner is not guaranteed to exist in a multidimensional policy space.[6] As a result, restrictions are necessary. For example, the policy space may be restricted to a single dimension over which individuals have single-peaked preferences; this restriction ensures the existence of a median voter whose most preferred policy is a Condorcet winner. In the "divide-the-dollar" problem in (1), there is no Condorcet winner. In fact, for any $u \in U$, there exists $u' \in U$ such that $|\{i \in I | u'_i > u_i\}| \geq 2$.

Moving to a dynamic setting, we can define individuals' preferences over sequences of policies, rather than single policies. Furthermore, in a dynamic setting, collective choices can be history dependent. For any infinite policy sequence $(u^1, u^2, ...) \in U^\infty$ the continuation payoffs for the three age groups alive in the current period are:

$$\begin{aligned} V_1(u^t, u^{t+1}, ...) &= u_1^t + \delta u_2^{t+1} + \delta^2 u_3^{t+2} \\ V_2(u^t, u^{t+1}, ...) &= u_2^t + \delta u_3^{t+1} \\ V_3(u^t, u^{t+1}, ...) &= u_3^t \end{aligned}$$

where $0 \leq \delta \leq 1$ is the discount factor for future consumption. Define

$$V\left(u^t, u^{t+1}, ...\right) \equiv \left[V_1\left(u^t, u^{t+1}, ...\right), V_2\left(u^t, u^{t+1}, ...\right), V_3\left(u^t, u^{t+1}, ...\right)\right].$$

In any period $t$, let $h^t = (u^1, \ldots, u^{t-1})$ be the history of policies implemented through the previous period. Let $H^t$ represent the set of all possible histories at time $t$, and let $H = \cup_{t=0}^{\infty} H^t$. Collective choice takes the form of a mapping from histories to outcomes, as defined below:

[5] I use the notation $|S|$ to denote the cardinality of a finite set $S$.

[6] In fact, the *nonexistence* of a Condorcet winner is practically guaranteed in arbitrary multidimensional settings (see, e.g., McKelvey, 1976; Schofield, 1978).

**Definition 2** *A **policy program** is a mapping $P : H \to U$ that specifies a policy outcome in period t for any history $h^t \in H$.*

Given a policy program, $P$, we can iteratively find the entire continuation path for any history $h^t$. Denote this as $C^P(h^t) = (P(h^t), P(h^t, P(h^t)), \ldots)$. The payoffs associated with this continuation path are denoted $V^P(h^t) = V\left(C^P(h^t)\right)$.

The following definition provides a natural generalization of the Condorcet concept in a dynamic setting:

**Definition 3** *A **Dynamic Condorcet Winner (DCW)** is a policy program $P$ such that for any $t$, $h^t \in H$, and $u \in U$, $\left|\left\{i \in I \mid V_i^P(h^t) \geq u_i + \delta V_{i+1}^P(h^t, u)\right\}\right| \geq 2.$*[7]

Intuitively, individuals anticipate that if a particular policy $u$ is implemented today, the continuation path $C^P(h^t, u)$ will occur beginning next period. A policy program is a DCW if the outcome it prescribes, $P(h^t)$, is majority-preferred to any other policy, $u$, taking into account the consequent continuation paths. That is, a collective deviation to $u$ today is not profitable for a majority of individuals given the resulting "punishment," $C^P(h^t, u)$.[8]

The DCW solution concept has some properties that make it appropriate for studying the divide-the-dollar problem. First, if individuals are sufficiently patient DCWs exist. Second, like their static counterparts, DCWs also summarize the equilibria of a broad class of institutions. In particular, consider the set of one-shot games whose equilibrium outcomes correspond exactly to the set of Condorcet winners whenever that set is nonempty (e.g., Downsian competition). Now consider any dynamic game formed by the infinite repetition of such a static game. For every DCW, it is possible to construct a subgame perfect equilibrium (SPE) of the dynamic game with the following property: given any history of outcomes, the continuation path for payoffs is the same in both the DCW and the SPE. Furthermore, DCWs correspond to the only equilibria that are robust across all such dynamic games.[9] Of course, it is true that any particular game may have non-DCW equilibria. Knowing about non-DCW equilibria may be useful if one is interested in studying a particular institution. However, the DCW concept is appealing for this analysis because it allows me to explore the general forces at work in the political process without taking a stand on a particular institutional arrangement.

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[7] Since subscripts refer to age, $V_4\left(u^1, u^2, ...\right) \equiv 0$.

[8] Krusell and Ríos Rull's (1999) "recursive political equilibrium" is a related solution concept in which the current policy choice can depend only on a state variable, rather than the entire history of outcomes. In the absence of a state variable (as in this paper), their solution concept collapses to finding a static Condorcet winner each period. The recursive political equilibrium notion is applied in an overlapping generations setting by Krusell and Ríos Rull (1996), although the policy set in their paper is binary.

[9] The argument for the OLG setting is a straightforward adaptation of the one in Bernheim and Slavov (2006) for infinite-lived agents.

# 3 Results

This section characterizes the set of DCWs and explores its implications for age bias. Define

$$V_\delta^0 = \left\{ w \in \mathbb{R}_+^3 \;\middle|\; w = V\left(u^1, u^2, ...\right) \text{ where } u^t \in U \text{ for all } t \right\}.$$

That is, $V_\delta^0$ is the set of age-dependent continuation payoffs that are feasible for the three currently living generations at any point in time. Let $V_\delta^* \subseteq V_\delta^0$ denote the set of all such continuation payoffs that are sustainable in a DCW. The characterization of $V_\delta^*$ turns out to be technically complex. Thus, it is illustrative to consider first an intuitive characterization of the DCW set. The second part of the section presents the formal argument. The final part of the section investigates the implications of the results for age bias.

## 3.1 An Intuitive Argument

Consider a possible policy sequence, $(u^1, u^2, ...)$. Under what conditions can this path be sustained in a DCW? Let us begin by establishing a necessary condition for sustainability. Suppose that at some point, $t$, in the sequence, $V_i(u^t, u^{t+1}, ...) + V_j(u^t, u^{t+1}, ...) < 1$ for two distinct age groups, $i$ and $j$; that is, the aggregate continuation value of two individuals is less than 1. Consider a collective deviation to a policy $u \in U$ with $u_i = V_i(u^t, u^{t+1}, ...) + \varepsilon$ and $u_j = V_j(u^t, u^{t+1}, ...) + \varepsilon$ for $\varepsilon > 0$. Such a policy exists for sufficiently small $\varepsilon$ since $V_i(u^t, u^{t+1}, ...) + V_j(u^t, u^{t+1}, ...) < 1$. Note that

$$\begin{aligned} V_i\left(u^t, u^{t+1}, ...\right) &< u_i + \delta V_{i+1}\left(v^{t+1}, v^{t+2}, ...\right) \\ V_j\left(u^t, u^{t+1}, ...\right) &< u_j + \delta V_{j+1}\left(v^{t+1}, v^{t+2}, ...\right) \end{aligned}$$

for any policy sequence $(v^{t+1}, v^{t+2}, ...)$, since $V(v^{t+1}, v^{t+2}, ...) \geq 0$. That is, the deviation to $u$ makes both $i$ and $j$ strictly better off regardless of the resulting continuation path. This suggests that if $P$ is a policy program in which $C^P(h^t) = (u^t, u^{t+1}, ...)$ for some $h^t$, then $P$ cannot possibly be a DCW since a deviation to $u$ would make a majority better off regardless of $C^P(h^t, u)$. Thus we find a property that must be shared by all DCWs: if $P$ is a DCW, then for all $h^t$, $V_i^P(h^t) + V_j^P(h^t) \geq 1$ for all $i \neq j$. Alternatively, if the path $(u^1, u^2, ...)$ can be sustained in a DCW, then $V_i(u^t, u^{t+1}, ...) + V_j(u^t, u^{t+1}, ...) \geq 1$ for all $t$ and $i \neq j$. Intuitively, if the aggregate continuation payoff for some majority coalition is less than 1, then it is possible to divide the dollar in such a way that every individual in this majority coalition is strictly better off even if they will subsequently receive nothing for the rest of their lives.

It turns out that the condition described above is also sufficient for sustainability if individuals are sufficiently patient (i.e., $\delta$ is sufficiently close to 1). To see this, consider any sequence $(u^1, u^2, ...)$ satisfying

$$V_i\left(u^t, u^{t+1}, ...\right) + V_j\left(u^t, u^{t+1}, ...\right) \geq 1 \text{ for all } t \text{ and } i \neq j \tag{2}$$

Consider a collective deviation to any $u \in U$ in period $t$. We need to find a suitable punishment $(v^{t+1}, v^{t+2}, ...)$ such that a majority of individuals prefer to stick with the current path than to receive $u$ followed by the punishment. Moreover, the punishment itself must be a sustainable path.

Define

$$z_i \equiv \frac{V_i(u^t, u^{t+1}, ...) - u_i}{\delta}.$$

For each individual $i$, $z_i$ represents the maximum continuation payoff the punishment can provide if that individual is to oppose the deviation to $u$. That is, any individual $i$ prefers to continue with the current path if and only if $V_{i+1}\left(v^{t+1}, v^{t+2}, ...\right) \leq z_i$. Of course, if $z_i < 0$ then $i$ prefers the deviation regardless of the consequences.

Note the following two properties of the $z_i$'s:

1. At least two individuals have $z_i \geq 0$. If $z_i < 0$ for two individuals, say $i$ and $j$, then it must be the case that $V_i(u^t, u^{t+1}, ...) + V_j(u^t, u^{t+1}, ...) < u_i + u_j \leq 1$, which violates (2).

2. The $z_i$'s sum to at least 1 ($z_1 + z_2 + z_3 \geq 1$). This follows from

$$\begin{aligned} & V_1(u^t, u^{t+1}, ...) + V_2(u^t, u^{t+1}, ...) + V_3(u^t, u^{t+1}, ...) \\ = \ & u_1^t + u_2^t + u_3^t + \delta\left[V_2\left(u^{t+1}, u^{t+2}, ...\right) + V_3\left(u^{t+1}, u^{t+2}, ...\right)\right] \\ \geq \ & 1 + \delta \end{aligned}$$

   where the inequality follow from (2).

Now, consider two cases:

*Case 1*: Suppose that $u_3 > u_3^t$. Since $V_3(u^t, u^{t+1}, ...) = u_3$, we know $z_3 < 0$. The oldest cohort prefers to deviate and cannot be punished; thus, we must design a punishment that will induce 1 and 2 to oppose the deviation. Since $z_3 < 0$, we must have $z_1 \geq 0$ and $z_2 \geq 0$ (follows from property 1) and $z_1 + z_2 \geq 1$ (follows from property 2). Consider the following punishment:

$$\begin{aligned} v^{t+1} &= \left(0, \frac{z_1}{z_1 + z_2}, \frac{z_2}{z_1 + z_2}\right) \\ v^{t+2} &= (1 - \delta^2, \delta^2, 0) \\ v^s &= (0, 0, 1) \text{ for all } s \geq t + 2 \end{aligned}$$

This continuation path is summarized in table 1.

Note that $v^s \in U$ for all $s$ because $z_i \geq 0$ for $i = 1, 2$, and $z_1 + z_2 > 0$. Moreover, for $i = 1, 2$,

$$V_{i+1}(v^{t+1}, v^{t+2}, ...) = \frac{z_i}{z_1 + z_2} \leq z_i$$

where the inequality follows from $z_1 + z_2 \geq 1$. Hence 1 and 2 prefer to continue with the original sequence $(u^t, u^{t+1}, ...)$ than to receive $u$ followed by the punishment

| period/ period of birth | $t+1$ | $t+2$ | $t+3$ | $t+4$ | $t+5$ |
|---|---|---|---|---|---|
| $t-2$ | - | - | - | - | - |
| $t-1$ | $\frac{z_2}{z_1+z_2}$ | - | - | - | - |
| $t$ | $\frac{z_1}{z_1+z_2}$ | 0 | - | - | - |
| $t+1$ | 0 | $\delta^2$ | 1 | - | - |
| $t+2$ | - | $1-\delta^2$ | 0 | 1 | - |
| $t+3$ | - | - | 0 | 0 | 1 |

Table 1: Punishment if $z_3 < 0$

| period/ period of birth | $t+1$ | $t+2$ | $t+3$ | $t+4$ | $t+5$ |
|---|---|---|---|---|---|
| $t-2$ | - | - | - | - | - |
| $t-1$ | 1 | - | - | - | - |
| $t$ | 0 | 0 | - | - | - |
| $t+1$ | 0 | $\delta^2$ | 1 | - | - |
| $t+2$ | - | $1-\delta^2$ | 0 | 1 | - |
| $t+3$ | - | - | 0 | 0 | 1 |

Table 2: Punishment if $z_3 \geq 0$, $z_1 \geq 0$

$(v^{t+1}, v^{t+2}, ...)$. It remains to show that the new sequence $(v^{t+1}, v^{t+2}, ...)$ is itself sustainable. I defer this discussion until the end of the second case.

*Case 2*: Suppose that $u_3 \leq u_3^t$. The oldest cohort, which does not care about next period's policy, weakly prefers not to deviate. Thus, we only need to design a punishment such that one of the younger cohorts also prefers not to deviate. Since $z_3 \geq 0$, we have either $z_1 \geq 0$ or $z_2 \geq 0$ (follows from property 1). If $z_1 \geq 0$, let

$$\begin{aligned} v^{t+1} &= (0,0,1) \\ v^{t+1} &= (1-\delta^2, \delta^2, 0) \\ v^s &= (0,0,1) \text{ for all } s \geq t+2. \end{aligned}$$

This continuation path is summarized in table 2. Since $V_2(v^{t+1}, v^{t+2}, ...) = 0$ and $z_1 \geq 0$, clearly individual 1 prefers not to deviate. If instead $z_3 \geq 0$, $z_1 < 0$, and $z_2 \geq 0$, let

$$\begin{aligned} v^{t+1} &= (1-\delta^2, \delta^2, 0) \\ v^s &= (0,0,1) \text{ for all } s \geq t+1. \end{aligned}$$

This continuation path is shown in table 3. Since $V_3(v^{t+1}, v^{t+2}, ...) = 0$ and $z_2 \geq 0$, clearly individual 2 prefers not to deviate.

| period/ period of birth | $t+1$ | $t+2$ | $t+3$ | $t+4$ | $t+5$ |
|---|---|---|---|---|---|
| $t-2$ | - | - | - | - | - |
| $t-1$ | 0 | - | - | - | - |
| $t$ | $\delta^2$ | 1 | - | - | - |
| $t+1$ | $1-\delta^2$ | 0 | 1 | - | - |
| $t+2$ | - | 0 | 0 | 1 | - |
| $t+3$ | - | - | 0 | 0 | 1 |

Table 3: Punishment if $z_3 \geq 0$, $z_1 < 0$, and $z_2 \geq 0$

It remains to be seen that the punishment itself is sustainable. It is straightforward to confirm from the tables above that, provided $\delta^3 + \delta^2 \geq 1$, the punishment sequences $(v^{t+1}, v^{t+2}, ...)$ satisfy equation (2), just like the original sequence $(u^1, u^2, ...)$. That is, for all $s$, $V_i(v^s, v^{s+1}, ...) + V_j(v^s, v^{s+1}, ...) \geq 1$ for all $i \neq j$. This implies that any collective deviation from the new path can be punished with another continuation path also satisfying the same property, and so on. In other words, (2) has a self-sustaining property: if a path $C^P(h^t)$ satisfies (2), then for any $u$, there exists a punishment $C^P(h^t, u)$ also satisfying (2) such that a majority prefers not to deviate. The continuation path $C^P(h^t, u)$ can be sustained because it satisfies (2) as well.

Intuitively, punishments are constructed by finding the majority coalition that is "easiest" to punish. The oldest individual cannot be punished and will therefore never oppose a deviation that provides a higher payoff in the current period. However, if the old favor the deviation, then $z_1 + z_2 \geq 1$; that is, the maximum continuation payoffs that can be given to 1 and 2 sum to at least 1. Hence, we can construct a suitable punishment that gives the two younger cohorts exactly 1 (which enables the continuation to satisfy (2)). On the other hand, if the deviation makes the old worse off, then this cohort will always oppose it. We can find another opponent because the maximum continuation payoff, $z_i$, for either 1 or 2 is nonnegative; hence, giving this cohort a continuation payoff of zero ensures opposition. The continuation payoff for the remaining generation can then be chosen to satisfy (2).

As noted earlier, there is a similarity to game theoretic models of "ongoing" organizations with overlapping generations of players, such as Crémer (1986), Dickson and Shepsle (2001), and Shepsle (1999). In these models, players choose how much work effort to contribute towards a public good, and the static Nash equilibrium results in underprovision. Cooperation can be sustained, however, using Nash reversion. An age bias result also emerges: older individuals cannot be induced to cooperate because of the difficulty in punishing them.

The model in this section is different in two important ways. First, unilateral deviations are not allowed. The young and middle-aged constitute a majority and can in principle force the elderly to accept a low payoff. Thus, the finding of age bias is more surprising. Second, in designing punishments, there is no equivalent of Nash reversion. To begin with, there is no static equilibrium to which one can revert. Moreover, a stationary policy path (i.e., reverting to a single policy to be repeated forever) cannot serve as a punishment. This is because any punishment must give either the current young or current middle-aged a very low continuation payoff. To make the punishment acceptable to a majority over the next two periods, future young/middle-aged individuals must receive large lifetime payoffs – so large that they are unsustainable in the long-run. For example, table 1 shows that the generations born in periods $t+1$ and $t+2$ receive lifetime payoffs of $\delta^2+\delta^3$ and 1 respectively. These are necessary because the older generations in periods $t+1$ and $t+2$ receive very little (they are being punished). If the $t+1$ and $t+2$ birth cohorts received any less, then they might prefer to form a majority coalition with one of these older generations and bring about a collective deviation. However, such high lifetime payoffs are unsustainable in a steady state – in the first case, $\delta^2+\delta^3>1$ is technically infeasible, and in the second, 1 is politically infeasible (it requires the young to receive the entire dollar every period and does not satisfy (2)). Thus, the policy path must eventually revert to one that is feasible – in this case, $(0,0,1)$.

Here, the existence of DCWs is demonstrated for $\delta^2+\delta^3\geq 1$ (approximately $\delta\geq .7549$). Intuitively, one might expect that if $\delta$ is sufficiently *low*, there are no DCWs: as $\delta$ decreases, the dynamic problem begins to resemble the static divide-the-dollar problem, which has no Condorcet winner. Indeed, this is the case. Equation (2) requires the following for all $t$:

$$\begin{aligned} V_1\left(u^t,u^{t+1},...\right)+V_2\left(u^t,u^{t+1},...\right) &= u_1^t+\delta u_2^{t+1}+\delta^2 u_3^{t+2}+u_2^t+\delta u_3^{t+1}\geq 1 \quad (3)\\ V_1\left(u^t,u^{t+1},...\right)+V_3\left(u^t,u^{t+1},...\right) &= u_1^t+\delta u_2^{t+1}+\delta^2 u_3^{t+2}+u_3^t\geq 1 \quad (4)\\ V_2\left(u^t,u^{t+1},...\right)+V_3\left(u^t,u^{t+1},...\right) &= u_2^t+\delta u_3^{t+1}+u_3^t\geq 1. \quad (5)\end{aligned}$$

Using the fact that $u_1^t+u_2^t+u_3^t=1$, these three conditions can be rewritten as follows:

$$\begin{aligned} \delta u_2^{t+1}+\delta^2 u_3^{t+2}+\delta u_3^{t+1}-u_3^t &\geq 0 \quad (6)\\ \delta u_2^{t+1}+\delta^2 u_3^{t+2}-u_2^t &\geq 0 \quad (7)\\ u_2^t+\delta u_3^{t+1}+u_3^t &\geq 1. \quad (8)\end{aligned}$$

The first inequality (6) implies that $u_3^t\leq\delta+\delta^2$. The second inequality (7) implies that $u_2^t\leq\delta+\delta^2$ as well. Substituting these into (8), we get $2\delta+3\delta^2+\delta^3\geq 1$. But if $\delta<.3247$, then this inequality does not hold, which demonstrates that no sequences satisfy (6-8).

## 3.2 A Formal Argument

The formal proof of the result stated above uses an analog of Abreu et. al.'s (1990) self-generation mapping (SGM), which is a tool used to characterize the subgame perfect equilibria of infinitely repeated games. Intuitively, the SGM takes any set of feasible continuation payoffs, $W\subseteq V_\delta^0$, and determines the payoffs for each age group that are politically sustainable given that continuation values must be chosen from this set. The DCW set is then identified as the largest set of continuation payoffs that can be sustained using other continuation payoffs within the same set. This section is a technical one that demonstrates how to apply the SGM to the model at hand, and to the general problem of finding DCWs in OLG models. The reader may wish to skip to the next section for the discussion of age bias.

For any $W \subseteq V_\delta^0$, the SGM is defined as follows:[10]

$$\Psi_\delta(W) = \left\{ w \in R^N \;\middle|\; \begin{array}{c} \text{(i) } \exists\,(u'', w'') \in U \times W \text{ such that } w_i = u_i'' + \delta w_{i+1}'' \\ \text{(ii) } \forall u \in U, \exists w' \in W \text{ such that} \\ \left| \left\{ i \in I \,\middle|\, w_i \geq u_i + \delta w_{i+1}^{'} \right\} \right| \geq 2 \end{array} \right\}. \quad (9)$$

Condition (i) requires that all payoffs in $\Psi_\delta(W)$ can be generated by a current policy, $u''$, and a continuation payoff, $w''$, in the set $W$. Condition (ii) requires that for any collective deviation, $u \in U$, there exists some "punishment" $w' \in W$ such that $u''$ is majority preferred to $u$, given the respective continuation values of $w''$ and $w'$. Intuitively, for any subset of feasible continuation payoffs, the SGM tells us the payoffs that can be supported using only these continuation payoffs. Note that since $w'' \in W \subseteq V_\delta^0$, for any $w \in \Psi_\delta(W)$, $w_i = u_i'' + \delta V_{i+1}(u^1, u^2, \ldots) = V_i(u'', u^1, u^2, \ldots)$ for some infinite policy sequence $(u^1, u^2, \ldots)$; thus $\Psi_\delta(W) \subseteq V_\delta^0$.

By starting with $V_\delta^0$ and iteratively applying the mapping $\Psi_\delta(\cdot)$, we can find the set of all payoffs that are sustainable in a DCW. In particular, for $n = 2, 3, \ldots,$ define $\Psi_\delta^n(W) = \Psi_\delta\left(\Psi_\delta^{n-1}(W)\right)$, and set $\Psi_\delta^1(W) = \Psi_\delta(W)$. Let

$$\Psi_\delta^\infty(W) = \cap_{n=1}^\infty \Psi_\delta^n(W).$$

If one takes $W = V_\delta^0$, then $\Psi_\delta^1(V_\delta^0)$ generates payoffs that can be supported given that continuations must be chosen in $V_\delta^0$. The next set $\Psi_\delta^2(V_\delta^0)$ generates payoffs that can be supported given that continuations must be chosen in $\Psi_\delta^1(V_\delta^0)$, and so on. As I will demonstrate below, an infinite iteration of this process converges to the fixed point $\Psi_\delta^\infty(V_\delta^0)$, a compact set such that any value in this set can be supported using continuation values in the same set. Moreover, $\Psi_\delta^\infty(V_\delta^0)$ is the set of payoffs that are sustainable in a DCW.

In applying the SGM to the problem at hand, it is useful to note the following result.

**Lemma 1** $V_\delta^0 = \left\{ w \in \mathbb{R}_+^3 \;\middle|\; \begin{array}{c} \textit{(i) } \sum_{j=i}^3 w_j \leq \sum_{j=i}^3 \delta^{3-j} \textit{ for all } i = 1,2,3 \\ \textit{(ii) } \sum_{i=1}^3 w_i \geq 1 \end{array} \right\}$

The proof of this result and all others appear in the appendix. The main purpose of this result is to allow us to avoid using infinite sequences in characterizing $V_\delta^0$. It also follows easily from this characterization that $V_\delta^0$ is a compact set.

[10] A similar version of the SGM is developed in Bernheim and Slavov (2006) for infinite-lived agents. Lemmas 2-7 represent an adaptation – appropriate for OLG models – of the proof of theorem 1 presented in that paper.

### 3.2.1 Properties of the Self-Generation Mapping

The first step in applying the SGM to the problem at hand is to demonstrate the properties of the sequence $\Psi_\delta^n(V_\delta^0)$ claimed above.

**Lemma 2** *If $W \subseteq W' \subseteq V_\delta^0$, then $\Psi_\delta(W) \subseteq \Psi_\delta(W')$.*

The above result states that if a set of continuation values, $W$, is contained in another set, $W'$, then the set of payoffs that are sustainable based on $W$ is contained in the set of payoffs that are sustainable based on $W'$. Intuitively, any continuation values in $W$ that are used to sustain a payoff can also be found in $W'$ and used to sustain the same payoff.

**Lemma 3** *Suppose $W \subseteq V_\delta^0$ is compact. If $\Psi_\delta(W)$ is nonempty, then it is compact.*

In words, this lemma shows that if one applies the SGM to a compact set, the resulting set (if nonempty) is also compact.

**Lemma 4** *$\Psi_\delta^n(V_\delta^0)$ is a nested sequence of compact sets.*

This result follows easily from the first three lemmas. Starting with the compact set $V_\delta^0$, one applies the SGM to obtain $\Psi_\delta^1(V_\delta^0)$, which is also a compact set. Applying the SGM again produces $\Psi_\delta^2(V_\delta^0)$ – again, a compact set. Moreover, as argued above, $\Psi_\delta^1(V_\delta^0) \subseteq V_\delta^0$. Applying lemma 2 by taking $W = \Psi_\delta^1(V_\delta^0)$ and $W = V_\delta^0$, we find that $\Psi_\delta^2(V_\delta^0) \subseteq \Psi_\delta^1(V_\delta^0)$. The same argument applies for all $n = 1, 2, ...$ An immediate corollary of lemma 4 is that, if the infinite intersection $\Psi_\delta^\infty(V_\delta^0)$ is nonempty, then it is compact.

**Lemma 5** *If $W \subseteq \Psi_\delta(W)$, then $\Psi_\delta(W) \subseteq \Psi_\delta^\infty(V_\delta^0)$.*

The above result shows that if every payoff in a set $W$ can be sustained using continuation payoffs in the same set, then $\Psi_\delta(W)$ is contained in the infinite intersection of $\Psi_\delta^n(V_\delta^0)$. This lemma is useful in characterizing the DCW set because if this property holds for some set $W$, then one can establish that every payoff in $W$ is sustainable in a DCW.

**Lemma 6** *$\Psi_\delta(\Psi_\delta^\infty(V_\delta^0)) = \Psi_\delta^\infty(V_\delta^0)$.*

This lemma shows that any payoff in the infinite intersection $\Psi_\delta^\infty(V_\delta^0)$ can be sustained using continuation values in the same set.

**Lemma 7** *$V_\delta^* = \Psi_\delta^\infty(V_\delta^0)$.*

Finally, the above result establishes that the infinite intersection of $\Psi_\delta^n(V_\delta^0)$ coincides with the set of payoffs that are sustainable in a DCW.

### 3.2.2 Applying the Self-generation Mapping

Given the above results, we can now apply the SGM to find $V_\delta^*$. This procedure consists of characterizing the sets $\Psi_\delta^1(V_\delta^0)$ and $\Psi_\delta^2(V_\delta^0)$. I then demonstrate that $\Psi_\delta\left(\Psi_\delta^2(V_\delta^0)\right) = \Psi_\delta^2(V_\delta^0)$. Since any payoff in $\Psi_\delta^2(V_\delta^0)$ can be sustained using continuation values in the same set, lemmas 4 and 5 establish that $V_\delta^* = \Psi_\delta^\infty(V_\delta^0) = \Psi_\delta^2(V_\delta^0)$.

**Lemma 8** $\Psi_\delta^1(V_\delta^0) = \{w \in V_\delta^0 \,|w_i + w_j \geq 1 \textit{ for all } i \neq j\}$.

The above lemma requires that all DCW payoffs give each majority coalition a continuation payoff of at least 1. The intuition for this result is explained in the previous section: if any majority coalition receives less than 1, then there is a way to divide the dollar among the members of this coalition in such a way that each individual prefers to deviate even if they will subsequently receive nothing for the rest of their lives. Of course, lemma 8 requires only that one be able to find a policy sequence $(u^1, u^2, ...)$ such that $V_i\left(u^1, u^2, ...\right) + V_j\left(u^1, u^2, ...\right) \geq 1$ for all $i \neq j$. That is, the condition that each majority coalition receive at least 1 is only enforced in the initial period; it need not hold in subsequent periods. However, it was argued in the previous section that, in a DCW, this condition must hold *at every point in time* to ensure that no *future* community prefers to deviate from the policy path. Thus, $\Psi_\delta^1(V_\delta^0)$ imposes conditions on the initial cohorts' payoffs that are necessary but not sufficient for a DCW payoff; further conditions are necessary to enforce the requirements of (6-8).

**Lemma 9** *If* $\delta^3 + \delta^2 \geq 1$, *then,*

$$\Psi_\delta^2(V_\delta^0) = \left\{ w \in V_\delta^0 \;\middle|\; \begin{array}{c} w_i + w_j \geq 1 \textit{ for all } i \neq j \\ w_1 + w_2 + w_3 \geq 1 + \delta \\ w_1 + w_3 \leq 1 + \delta^2 + \delta^3 \end{array} \right\}.$$

The first condition defining $\Psi_\delta^2(V_\delta^0)$ is the same as that defining $\Psi_\delta^1(V_\delta^0)$. This is natural since $\Psi_\delta^2(V_\delta^0) \subseteq \Psi_\delta^1(V_\delta^0)$. The second condition requires a minimum aggregate payoff for all three individuals alive in the initial period. The purpose of this requirement is to guarantee that $w_2''$ and $w_3''$ (the continuation payoffs received next period by the initial young and middle aged) will sum to at least 1, since

$$w_2'' + w_3'' = \frac{w_1 - u_1}{\delta} + \frac{w_1 - u_1}{\delta} \leq \frac{w_1 + w_2 - (1 - w_3)}{\delta}.$$

More intuitively, it prevents the stream of payoffs to today's young and middle aged from being too front loaded: if $w_1$ and $w_2$ are small, and if a large share of the dollar is given to the young and middle aged today (because $w_3$ is also small), then this will leave next period's middle-aged and old with an aggregate continuation payoff that is too small.

The third condition essentially protects next period's young against extravagant future promises made to today's young. This condition is violated if the initial young are promised a sufficiently high lifetime payoff, and a large fraction of this payoff must be received in the future because the initial old (due to a high $w_3$) must receive a large share of today's resources. In this situation, fulfilling the promise made to the initial young requires a large continuation payoff for next period's middle aged. A sufficiently large payoff to next period's middle aged implies that next period's young and old cannot receive the minimum aggregate payoff of 1 no matter how high a lifetime payoff next period's young are promised. (A symmetric condition protecting the future young against promises made to today's middle aged also applies. However, this condition is not binding because technical feasibility already requires $w_2 + w_3 \leq 1 + \delta$, which is less than $1 + \delta^2 + \delta^3$ for $\delta^2 + \delta \geq 1$. In other words, making overly extravagant future promises to the current middle aged is impossible because they do not live long enough.)

**Proposition 1** *If $\delta^3 + \delta^2 \geq 1$, then,*

$$V_\delta^* = \left\{ w \in V_\delta^0 \;\middle|\; \begin{array}{c} w_i + w_j \geq 1 \textit{ for all } i \neq j \\ w_1 + w_2 + w_3 \geq 1 + \delta \\ w_1 + w_3 \leq 1 + \delta^2 + \delta^3 \end{array} \right\}.$$

The above result fully characterizes the set of DCW payoffs for sufficiently large $\delta$. This set can be interpreted in light of the intuitive minimum payoff requirement presented in the previous section: once the conditions defined in the proposition are met, there is enough flexibility to construct a policy sequence generating these payoffs that also satisfies (6-8) at every point in time. One arrives at the result by demonstrating that any payoff in $\Psi_\delta^2(V_\delta^0)$ can be sustained using continuation payoffs in the same set – that is, $\Psi_\delta^2(V_\delta^0) \subseteq \Psi_\delta\left(\Psi_\delta^2(V_\delta^0)\right)$. This implies that $\Psi_\delta^2(V_\delta^0) \subseteq \Psi_\delta^\infty(V_\delta^0)$. Moreover, by definition, any $w \in \Psi_\delta^\infty(V_\delta^0)$ is also in $\Psi_\delta^2(V_\delta^0)$, and therefore $\Psi_\delta^2(V_\delta^0) = \Psi_\delta^\infty(V_\delta^0)$.

Figure 1 depicts the convergence of the SGM for $\delta = .8$. Each graph represents a "slice" of the continuation payoff set for a fixed value of $w_3$ – that is, it shows the values of $w_1$ and $w_2$ that are attainable in a DCW for a given value of $w_3$. The outermost areas show the set of mechanically feasible payoffs, $V_\delta^0$, and the two smaller areas represent $\Psi_{.8}^1(V_{.8}^0)$ and $\Psi_{.8}^2(V_{.8}^0)$. Figures 2 and 3 show slices of the DCW set for $\delta = .76$ (approximately the value for which $\delta^3 + \delta^2 = 1$) and $\delta = 1$ respectively. For comparison, the light outer areas show the sets of mechanically feasible payoffs, and the dark inner areas show the DCW payoff sets.

## 3.3 Age Bias

The DCW set encompasses all the possible payoffs that can be implemented under repeated majority rule. The previous section shows that there is a multiplicity of DCWs whenever they exist. While this might be viewed as a negative result, an appealing way to deal with the multiplicity issue is to interpret the DCW set as a political feasibility constraint on policy design (see Bernheim and Slavov, 2006). Given this interpretation, one can think of a policy maker or interest group choosing

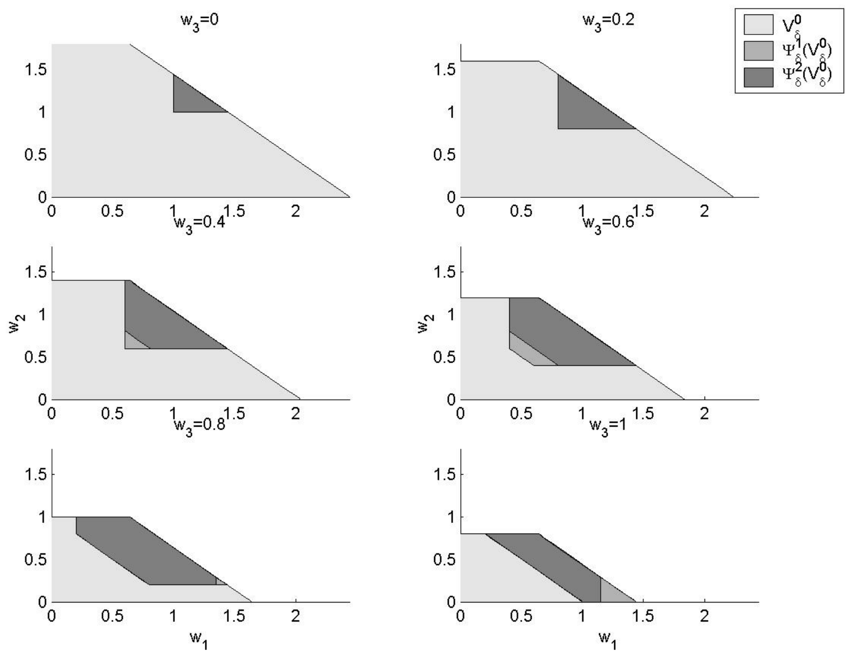


Figure 1: Convergence of the Self-Generation Mapping

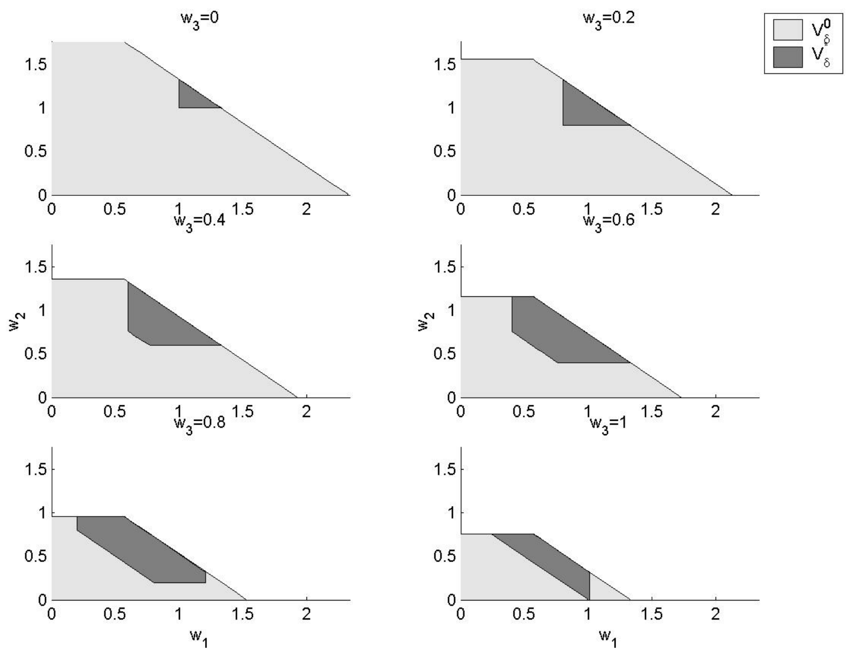


Figure 2: DCW Payoff set for $\delta = .76$

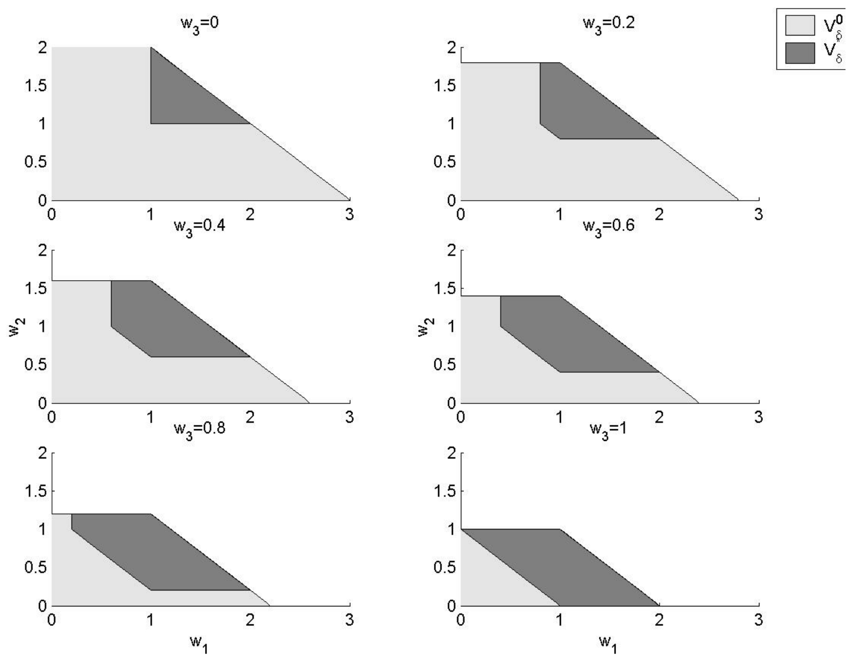


Figure 3: DCW Payoff set for $\delta = 1$

its favorite outcome subject to the constraint that the chosen payoffs lie in the DCW set. The DCW set then enables us to explore the ways in which a majoritarian feasibility requirement constrains policy makers with various objectives.

In this section, I use this political feasibility interpretation of the DCW set to explore the issue of age bias. According to the previous section's results, in a DCW, the continuation values for any majority must always sum to 1 or more, regardless of the number of periods left in the members' lives. Loosely, one could say that the continuation values for all individuals alive today receive equal weight, even though some individuals may have already received high payoffs earlier in their lives. This observation hints at the existence of age bias. A more formal analysis follows.

### 3.3.1 Average Discounted Payoffs

To begin with, I consider policy makers whose interests are aligned with each of the three cohorts currently alive. The thought experiment is as follows: in the current period only, each policy maker is allowed to select a set of payoffs that will maximize the lifetime payoff of the preferred cohort. The policy maker's choice is made once and for all with the knowledge that the chosen payoffs must subsequently be implemented by majority rule.

For any continuation value $w \in V_\delta^0$, define the average discounted continuation payoff received by age group $i$ as

$$\pi_i(w;\delta) = \frac{w_i}{\sum_{j=i}^{3} \delta^{3-j}}.$$

Note that for all $i$, $0 \leq \pi_i(w;\delta) \leq 1$; thus, the normalization allows one to compare continuation payoffs across age groups. Let $\pi_i^{\max}(\delta)$ be the maximum sustainable normalized payoff for age group $i$. That is, $\pi_i^{\max}(\delta) = \max_{w \in V_\delta^*} \pi_i(w;\delta)$. Clearly, the policy maker whose interests are aligned with the cohort currently aged $i$ will prefer that the payoff $\pi_i^{\max}(\delta)$ be implemented.

**Proposition 2** *Assume $\delta^3 + \delta^2 \geq 1$. Then*

*(i)* $\pi_1^{\max}(\delta) = \frac{\delta+\delta^2}{1+\delta+\delta^2}$

*(ii)* $\pi_2^{\max}(\delta) = \delta$

*(iii)* $\pi_3^{\max}(\delta) = 1$

The logic behind the proposition is straightforward. It is always possible to give the elderly the entire dollar; in particular, as was demonstrated in the previous section, a policy sequence in which $u = (0,0,1)$ is repeated every period is sustainable in a DCW. Since the elderly can always receive the entire dollar, then clearly the middle-aged must be able to attain a continuation payoff of no less than $\delta$; this payoff is achieved by giving the middle aged cohort the entire dollar in period $t+1$ (when they are old). It is also possible to give the middle aged part – but not all – of the dollar in the current period. Hence the result in (ii): the maximum possible average discounted payoff for the middle aged is less than 1 (unless $\delta = 1$). A similar logic extends to the young cohort in the current period. Clearly, if a continuation value of $\delta + \delta^2$ (corresponding to a normalized payoff of $\delta$) is achievable for the middle-aged, then the young must be able to attain a payoff of at least $\delta^2 + \delta^3$ (by receiving nothing now and a continuation value of $\delta+\delta^2$ in period $t+1$). The young can also receive a portion (but not all) of the dollar today, bringing the maximum sustainable normalized payoff to $\left(\delta + \delta^2\right) / \left(1 + \delta + \delta^2\right)$.

More intuitively, giving a normalized payoff of 1 to the young requires all older cohorts to receive nothing for the rest of their lives. These older cohorts constitute a majority coalition that opposes the policy sequence regardless of the consequences. The older majority must receive transfers during the lifetime of the young cohort, thereby diminishing the fraction of the dollar available for the young cohort towards the beginning of its life. Giving a normalized payoff of 1 to the middle-aged cohort requires the elderly to receive nothing in the current period, although the young can receive positive transfers after the current middle-aged cohort dies. It turns out that for $\delta < 1$, it is not possible to reward the younger cohort sufficiently to prevent a majority coalition of the young and the elderly from opposing the policy sequence. The oldest cohort, however, will be outlived by a majority of individuals. By promising this majority sufficiently large transfers after the oldest cohort dies, any payoff for the oldest cohort is sustainable. Thus, a policy maker representing the young cohort can obtain, at best, a policy path that promises this group large rewards when they are older. On the other hand, an interest group representing the old cohort can obtain large immediate rewards for this group.

### 3.3.2 Steady State Payoffs

A commonly-cited explanation for why the young support social security is the following: the young expect to be old someday, and if they support social security now, then they can receive its benefits in the future. This explanation assumes stationarity along the equilibrium path – i.e., that in the absence of a collective deviation, the same policy will occur every period. Stationarity is an artificial restriction, however; as a result, this intuition is fragile. In the previous section, a stationary equilibrium path was not assumed. The "social security system" under which today's young will "retire" is not required to be the same as the social security system under which the current old receive benefits. The finding of age bias without a stationarity restriction is in many ways surprising, and the intuition for its existence is more subtle.

However, the notion that the young support social security "so it will be there for them" is encompassed by the model in this paper, and it turns out to be a special case of the intuition presented above. This section formalizes this idea by limiting attention to "steady state DCWs" – i.e., DCWs that satisfy the property $C^P(h^1) = (u, u, u, \dots)$. This property requires stationarity on the equilibrium path; however, continuation paths for other histories need not be stationary. A policy $u \in U$ is said to be sustainable if there exists a steady state DCW $P$ with $C^P(h^1) = (u, u, \dots)$. Let $U_\delta^{SS}$ denote the set of policies that are sustainable in a steady state DCW.

Imposing the stationarity requirement ($C^P(h^1) = (u, u, \dots)$) on (6-8) gives us

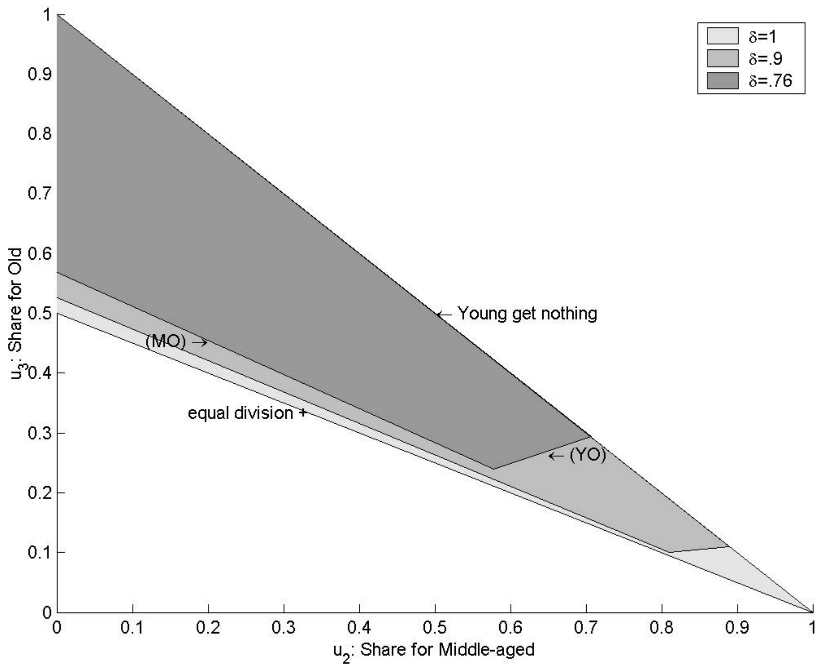


Figure 4: Steady State Sustainable Policies

the following characterization of any $u \in U_\delta^{SS}$:

$$\delta u_2 + (\delta^2 + \delta - 1)u_3 \geq 0 \quad (10)$$

$$(\delta - 1)u_2 + \delta^2 u_3 \geq 0 \quad (11)$$

$$u_2 + (1 + \delta)u_3 \geq 1. \quad (12)$$

These inequalities are illustrated in figure 4 for various values of $\delta$. The share received by the middle aged ($u_2$) is on the horizontal axis, and the share received by the old ($u_3$) is on the vertical axis. The residual ($1 - u_2 - u_3$) is the share received by the young ($u_1$). Along the diagonal line from the upper left to the lower right, the young receive nothing. The innermost area represents the set of sustainable policies when $\delta = .76$. This set is contained in the set of sustainable policies for $\delta = .9$, which is in turn contained in the set of sustainable policies for $\delta = 1$. For the $\delta = .76$ area, equation (11) is labeled in figure 4 as (YO) (since it requires that the young and the old must get a continuation value of at least 1), and equation (12) is labeled as (MO) (since it requires that the middle-aged and old must get a continuation value of at least 1). Note that (10), the constraint requiring individuals aged 1 and 2 to receive a continuation value of at least 1, is never binding.

The figure indicates that is impossible to give the entire dollar to the young and middle aged unless $\delta = 1$, while there are many sustainable policies along the diagonal that give nothing to the young. Equal division – under which each age group gets $1/3$ – is only sustainable if $\delta = 1$; when $\delta = 1$, the equal division point lies on the constraint (12).[11]

Figure 4 also allows us to apply the majoritarian feasibility interpretation of the DCW set. Figure 5 depicts the maximum politically feasible payoff for each age group. Thus, a policy maker representing the old age group can implement a steady state DCW in which this age group receives the entire dollar every period. However, a policy maker representing the middle-aged an only obtain a per-period payment of $\delta^2/\left(1-\delta+\delta^2\right)$ for the favored age group. An advocate for the young age group does even worse, achieving, at best, a per-period payment of only $\delta/(1+\delta)$.

The intuition for these results is straightforward, and it can be thought of as a special case of the intuition from the non-steady state case. The share of the dollar received by the oldest age group, $u_3$, enters into the continuation payoff of every individual. If today's young expect to receive $u_3 = 1$ when they are old, this serves as their reward for supporting the status quo. A similar argument applies for the share received by the middle-aged. However, the share for the young does not enter a majority of individuals' payoffs. If the entire dollar is paid to the young, all older individuals prefer to deviate to any other policy that gives them a positive share even if they expect to get nothing for the rest of their lives. It is not possible to construct a stationary policy sequence that rewards older individuals for supporting large payments to the young.

# 4 Extensions

In the model thusfar, the payoff to be divided remains constant over time. Moreover, individuals must immediately consume any resources they receive. In this section, I explore the consequences of relaxing these assumptions.

[11] Proposition 4 also suggests an interesting contrast between overlapping generations and infinite-lived agents. In an OLG setting, even when $\delta = 1$, not all policies are sustainable; for example, it is not possible to give the entire dollar to individuals below the median age. With infinite-lived agents, on the other hand, any payoff that is part of a Condorcet cycle is sustainable for sufficiently high $\delta$. Thus, in the divide-the-dollar problem, all divisions are sustainable if $\delta = 1$ (Bernheim and Slavov, 2006). This "folk theorem" result relies on the existence of Condorcet cycles: when cycles exist, a deviation from any element of a cycle may be punished by a permanent switch to the next element (which a majority views as strictly inferior). In an OLG setting, however, this mechanism fails. It may be true that a given policy $A$ is majority preferred to another policy $B$ in the current period. However, an individual's preferences over policies depend on age. As a result, $A$ being majority preferred to $B$ does not necessarily imply that a majority of individuals prefer the path in which $A$ occurs every period to the path in which $B$ occurs every period. If $A$ is extremely favorable to the young, for example, then young people prefer $A$ today followed by a policy that favors older individuals.

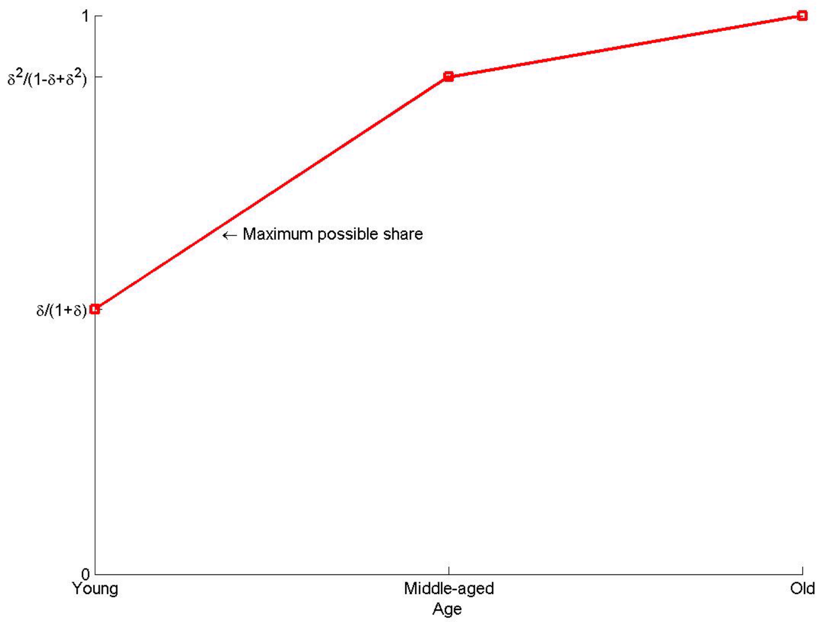


Figure 5: Maximum Sustainable Share by Age

## 4.1 The Role of Growth

Suppose that the fixed surplus grows (or shrinks) over time at a constant rate $g$. That is, $(1+g)^{t-1}$ dollars are available to divide in period $t$. Let $\mathcal{P}^*(\delta, g)$ denote the set of DCWs given discount factor $\delta$ and growth rate $g$. A policy, $u$, specifies the share of the current surplus payoff to be given to each individual. For any infinite policy sequence $(u^1, u^2, ...)$, the continuation payoff of individual $i$ at time $t$ depends on both $\delta$ and $g$. I write this as $V_i(u^t, u^{t+1}, ...; \delta, g) = \sum_{j=0}^{3-i} \delta^j u_{i+j}^{t+j} (1+g)^{t+j-1}$. For any policy program $P$ and history $h^t$, let $V_i^P(h^t; \delta, g) = V_i(C^P(h^t); \delta, g)$.

**Proposition 3** *If* $(1+\widehat{g})\widehat{\delta} = (1+g)\delta$, *then* $\mathcal{P}^*(\delta, g) = \mathcal{P}^*(\widehat{\delta}, \widehat{g})$.

A corollary of this result is that $P$ is a DCW given $g$ and $\delta$ if and only if it is a DCW in a setting with growth rate $\hat{g} = 0$ and discount factor $\hat{\delta} = (1+g)\delta$. Intuitively, the growth rate simply changes the effective discount factor in the problem. Being able to promise a larger reward tomorrow is equivalent to having more patient individuals.

## 4.2 Borrowing and Saving

Allowing borrowing and saving is not such a straightforward extension. Suppose that individuals have standard intertemporally separable preferences: age group $i$'s utility in period $t$ is given by $\zeta(c_i^t)$, where $\zeta'(\cdot) > 0$, $\zeta''(\cdot) < 0$, and $c_i^t$ is the consumption of age group $i$ in period $t$ . Thus, lifetime utility at birth for an individual born at time $t$ is $\sum_{j=0}^{3-1} \zeta(c_{j+1}^{t+j})\bar{\delta}^j$, where $\bar{\delta}$ is the discount factor for future utility. Individuals can borrow and save freely at interest rate $r$. Hence, an individual born at time $t$ faces the lifetime budget constraint

$$\sum_{j=0}^{2} \left(\frac{1}{1+r}\right)^j c_{j+1}^{t+j} \leq \sum_{j=0}^{2} \left(\frac{1}{1+r}\right)^j u_{j+1}^{t+j},$$

where $u^t$ is the policy in period $t$.

If the sequence of policies is fixed, an individual's utility in period $t$ can be written as a function of the present value of her income alone. Given a policy sequence $(u^1, u^2, \ldots)$, the present value of income for cohort $i$ at time $t$ is

$$\sum_{j=0}^{3-i} \left(\frac{1}{1+r}\right)^j u_{i+j}^{j+1},$$

which is identical to $V_i(u^1, u^2, ...)$, as defined in section 2.2, for $\delta = 1/(1+r)$. Thus, for fixed $(u^1, u^2, \ldots)$, there is no loss of generality in assuming that individuals have linear utility and cannot borrow and save. However, in the context of policy programs, there is a complication: one has to allow for the possibility that the policy sequence can change, and that the budget constraint may be violated as a result of such a change. For example, consider a policy program, $P$, and suppose that in period $t$, individual $i$ has already borrowed a given amount, $B > 0$. When considering an alternative policy, $u \in U$, followed by the resulting continuation path, $C^P(h_t, u)$, one has to specify what happens if the continuation path does not generate enough income for $i$ to pay back the loan of $B$. One way to address this problem is to allow for negative consumption. With this flexibility, the budget constraint can always be enforced.

An alternative approach is to drop the linear utility assumption while continuing to assume that individuals cannot borrow or save. This implies that the payoff to age group $i$ from policy sequence $(u^1, u^1, \dots)$ is $\sum_{j=0}^{3-i} = \bar{\delta}^j \zeta\left(u_{i+j}^{j+1}\right)$. In this case, the simple condition for sustainable policies in proposition 1 no longer holds. Nonlinear utility complicates the process of finding DCWs analytically; however, it may be possible to obtain results computationally.

# 5 Conclusion

This paper has provided a robust explanation for the political power of the elderly. In particular, I have shown that the set of DCWs exhibits bias towards the elderly in the sense that there are DCWs in which the old receive large transfers, but no symmetric DCWs in which the young receive transfers of the same magnitude. This implies that a policy maker representing the elderly and facing a majoritarian feasibility constraint can do much better for the favored group than a similarly constrained policy maker favoring the young. This age-bias result holds in a multidimensional policy space, in which any transfer can be made to any age group. It also does not rely on a stationary policy sequence to generate support for public retirement programs among the young. Moreover, the set of DCWs summarizes the equilibria of a broad class of voting institutions, suggesting that my results capture general forces at work in the political process. Intuitively, age bias exists because the young outlive the old. As a result, one has more flexibility to reward the young in the future for supporting any particular policy (including large transfers to the elderly) early in their lives. Older generations lack such a future and must therefore receive rewards now.

One immediate extension of this work is to allow the government to borrow and save. This may provide an explanation for the findings of the generational accounting literature: fiscal policy favors the living at the expense of the unborn (e.g., Auerbach et. al., 1991).

# Appendix

**Proof of Lemma 1.**

**Step 1** $V_\delta^0 \subseteq \left\{ w \in \mathbb{R}_+^3 \middle| \begin{array}{c} \text{(i)} \quad \sum_{j=i}^3 w_j \leq \sum_{j=i}^3 \delta^{3-j} \text{ for all } i = 1,2,3 \\ \text{(ii)} \ \sum_{i=1}^3 w_i \geq 1 \end{array} \right\}$.

Consider any $w \in V_\delta^0$. By definition, $w_1 = u_1^1 + \delta u_2^2 + \delta^2 u_3^3$, $w_2 = u_2^1 + \delta u_3^2$, and $w_3 = u_3^1$ for some $u^1, u^2, u^3 \in U$. Thus, $w_3 \leq 1$. Also, $w_3 + w_3 = (u_2^1 + u_3^1) + \delta u_3^2 \leq 1 + \delta$, and $w_1 + w_3 + w_3 = (u_1^1 + u_2^1 + u_3^1) + \delta (u_3^2 + u_2^2) + \delta^2 u_3^3 \leq 1 + \delta + \delta^2$. Finally, $w_1 + w_2 + w_3 = (u_1^1 + u_2^1 + u_3^1) + \delta (u_3^2 + u_2^2) + \delta^2 u_3^3 = 1 + \delta (u_3^2 + u_2^2) + \delta^2 u_3^3 \geq 1$.

**Step 2** $\left\{ w \in \mathbb{R}_+^3 \middle| \begin{array}{c} \text{(i)} \quad \sum_{j=i}^3 w_j \leq \sum_{j=i}^3 \delta^{3-j} \text{ for all } i = 1,2,3 \\ \text{(ii)} \ \sum_{i=1}^3 w_i \geq 1 \end{array} \right\} \subseteq V_\delta^0$.

Consider any $w \in \mathbb{R}_+^3$ satisfying $w_3 \leq 1$, $w_3 + w_2 \leq 1 + \delta$, and $1 \leq w_3 + w_2 + w_1 \leq 1 + \delta + \delta^2$. I will show that $w \in V_\delta^0$ by constructing an infinite sequence $(u^1, u^2, ...)$ such that $w = V(u^1, u^2, ...)$. Define

$$\begin{aligned} u^1 &= [1 - \min(w_2, 1 - w_3) - w_3, \min(w_2, 1 - w_3), w_3] \\ u^2 &= \begin{bmatrix} 1 - \min\left(\frac{w_1 - u_1^1}{\delta}, 1 - \frac{w_2 - u_2^1}{\delta}\right) - \frac{w_2 - u_2^1}{\delta} \\ \min\left(\frac{w_1 - u_1^1}{\delta}, 1 - \frac{w_2 - u_2^1}{\delta}\right) \\ \frac{w_2 - u_2^1}{\delta} \end{bmatrix}' \\ u^3 &= \left[0, 1 - \frac{w_1 - u_1^1 - \delta u_2^2}{\delta^2}, \frac{w_1 - u_1^1 - \delta u_2^2}{\delta^2}\right] \\ u^t &= (0,0,1) \text{ for all } t > 3 \end{aligned}$$

It is straightforward to see that $u_3^1 = w_3$, $u_2^1 + \delta u_3^2 = w_2$, and $u_1^1 + \delta u_2^2 + \delta^2 u_3^3 = w_1$. It remains to be see that $u^t \in U$ for all $t$. First note that $u^1 \in U$ because $\sum_{i=1}^3 u_i^1 = 1$ and $u^1 \geq 0$ (where the latter claim follows from $w_3 + \min(w_2, 1 - w_3) \leq 1$). Second, I will show that $u^2 \in U$. Clearly $\sum_{i=1}^3 u_i^2 = 1$. Since $u_2^1 \leq w_2$, it must be the case that $u_3^2 \geq 0$. Now consider two possibilities for $u_2^2$. We could have $u_2^2 = (w_1 - u_1^1)/\delta$, which is nonnegative because $u_1^1 = \max(1 - w_2 - w_3, 0) \leq w_1$. Alternatively, we could have $u_2^2 = 1 - (w_2 - u_2^1)/\delta$. In this case as well, $u_2^2$ is nonnegative because $u_2^1 = \min(w_2, 1 - w_3) \geq w_2 - \delta$, implying that $(w_2 - u_2^1)/\delta \leq 1$. Now note that $u_1^2 \geq 0$ since $u_3^2 + u_2^2 \leq 1$. Thus, $u^2 \geq 0$ and $u^2 \in U$. Third, I will show that $u^3 \in U$. To see this, note that $\sum_{i=1}^3 u_i^3 = 1$. Moreover, $u_1^1 + \delta u_2^2 = \min(w_1, 1 + \delta - w_2 - w_3)$, which implies that $w_1 \geq u_1^1 + \delta u_2^2 \geq w_1 - \delta^2$. It follows from this result that $1 \geq u_3^3 \geq 0$, and thus $u^3 \geq 0$. Therefore $u^3 \in U$ as well. Finally, it is obvious that $u^t \in U$ for $t > 3$. Thus, $u^t \in U$ for all $t$, as required.

■

**Proof of Lemma 2.** Consider any $w \in \Psi_\delta(W)$. From the definition of $\Psi_\delta(W)$, $w_i = u''_i + \delta w''_{i+1}$for some $u'' \in U$ and $w'' \in W$; and for any $u \in U$,

$$\left|\left\{i \in I \left| w_i \geq u_i + \delta w'_{i+1}\right.\right\}\right| \geq 2$$

for some $w' \in W$. But $w''$ and $w'$ are also in $W'$. Thus, $w \in \Psi_\delta(W')$. ■

**Proof of Lemma 3.** Consider any compact set $W \subseteq V_\delta^0$ and suppose $\Psi_\delta(W)$ is nonempty. First note that $\Psi_\delta(W) \subseteq V_\delta^0$ is bounded. Second, to see that $\Psi_\delta(W)$ is closed, consider any convergent sequence $\{w_s\}_{s=1}^\infty$ such that $w_s \in \Psi_\delta(W)$ for all $s$. Let $w_\infty = \lim_{s\to\infty} w_s$. To demonstrate that part (i) of (9) holds for $w_\infty$, observe that, for all $s$, there exist $u''_s \in U$ and $w''_s \in W$ such that $w_{s,i} = u''_{s,i} + \delta w''_{s,i+1}$ for all $i \in I$. Let $(u''_\infty, w''_\infty)$ be a limit point of the sequence $\{(u''_s, w''_s)\}_{s=1}^\infty$. By continuity, $w_{\infty,i} = u''_{\infty,i} + \delta w''_{\infty,i+1}$. It suffices to show that $u''_\infty \in U$ and $w''_\infty \in W$. These results follow from the compactness of $U$ and $W$ respectively. To demonstrate that part (ii) of (9) holds for $w_\infty$, consider any $u \in U$. We know that for all $s$, there exists some $w' \in W$ such that

$$\left|\left\{i \in I \left| w_{s,i} \geq u_i + \delta w'_{s,i+1}\right.\right\}\right| \geq 2 \tag{13}$$

Choose an infinite subsequence $\{w'_{s_k}\}_{k=1}^\infty$ such that for some for some $S \subseteq I$ with $|S| \geq 2$, $w_{s_k,i} \geq u_i + \delta w'_{s_k,i+1}$ for all $i \in S$ and $k \geq 0$. In words, $\{w'_{s_k}\}_{k=1}^\infty$ is a subsequence such that, for every element of the subsequence, condition (13) holds for *the same* majority coalition $S \subseteq I$. Such a subsequence exists because $I$ gives rise to a finite number of majority coalitions. Let $w'_\infty$ be a limit point of the subsequence $\{w'_{s_k}\}_{k=0}^\infty$. By continuity, $w_{\infty,i} \geq u_i + \delta w'_{\infty,i+1}$ for all $i \in S$. It suffices to show that $w'_\infty \in W$. This result follows from the compactness of $W$. ■

**Proof of Lemma 4.** First of all, note that $\Psi_\delta^1(V_\delta^0) \subseteq V_\delta^0$; hence $\Psi_\delta^2(V_\delta^0) \subseteq \Psi_\delta^1(V_\delta^0)$. An inductive argument using lemma 2 establishes that for any $n \geq 1$, $\Psi_\delta^n(V_\delta^0) \subseteq \Psi_\delta^{n-1}(V_\delta^0)$. Second, since $V_\delta^0$ is compact, an inductive argument using lemma 3 establishes the compactness of each set $\Psi_\delta^n(V_\delta^0)$. ■

**Proof of Lemma 5.** Suppose $W \subseteq \Psi_\delta(W)$. We know that $W \subseteq V_\delta^0$. Thus, lemma 2 implies that $\Psi_\delta(W) \subseteq \Psi_\delta(V_\delta^0) = \Psi_\delta^1(V_\delta^0)$. Now suppose $\Psi_\delta(W) \subseteq \Psi_\delta^n(V_\delta^0)$ for some $n \geq 1$. Then, by lemma 2, $\Psi_\delta(\Psi_\delta(W)) \subseteq \Psi_\delta(\Psi_\delta^n(V_\delta^0)) = \Psi_\delta^{n+1}(V_\delta^0)$. Lemma 2 also implies that, since $W \subseteq \Psi_\delta(W)$, $\Psi_\delta(W) \subseteq \Psi_\delta(\Psi_\delta(W))$. Hence $\Psi_\delta(W) \subseteq \Psi_\delta^{n+1}(V_\delta^0)$. By induction, $\Psi_\delta(W) \subseteq \Psi_\delta^n(V_\delta^0)$ for all $n$, as required. ■

**Proof of Lemma 6.** If $\Psi_\delta^\infty(V_\delta^0)$ is empty, then the proof is trivial. So suppose $\Psi_\delta^\infty(V_\delta^0)$ is nonempty. The proof has two steps.

**Step 1** $\Psi_\delta^\infty(V_\delta^0) \subseteq \Psi_\delta(\Psi_\delta^\infty(V_\delta^0))$. Consider any $w \in \Psi_\delta^\infty(V_\delta^0)$. By definition, $w \in \Psi_\delta^n(V_\delta^0)$ for all $n$. To see that part (i) of (9) holds, note that for any $n \geq 1$, there exist $u''_n \in U$ and $w''_n \in \Psi_\delta^{n-1}(V_\delta^0)$ such that $w_i = u''_{n,i} + \delta w''_{n,i+1}$ for all $i \in I$. Let $(u''_\infty, w''_\infty)$ be a limit point of the sequence $\{(u''_n, w''_n)\}_{n=1}^\infty$. By continuity, $w_i = u''_{\infty,i} + \delta w''_{\infty,i+1}$ for all $i \in I$. Since $U$ is compact, it is clear that $u''_\infty \in U$. To demonstrate that $w''_\infty \in \Psi_\delta^\infty(V_\delta^0)$, it is sufficient to show that $w''_\infty \in \Psi_\delta^n(V_\delta^0)$ for all $n$. Consider any $n$. We know that for all $m \geq n+1$, $w''_m \in \Psi_\delta^{m-1}(V_\delta^0) \subseteq \Psi_\delta^n(V_\delta^0)$ (where the last relation follows from lemma 4). Since $\Psi_\delta^n(V_\delta^0)$ is compact, $w''_\infty \in \Psi_\delta^n(V_\delta^0)$, as required. To see that part (ii) of (9) holds, consider any $u \in U$. For any $n \geq 1$, there exists $w'_n \in \Psi_\delta^{n-1}(V_\delta^0)$ such that $\left|\left\{i \in I \,\middle|\, w_i \geq u_i + \delta w'_{n,i+1}\right\}\right| \geq 2$. Let $\{w'_{n_k}\}_{k=0}^\infty$ be a subsequence such that for some $S \subseteq I$ with $|S| \geq 2$, $w_i \geq u_i + \delta w'_{n_k,i+1}$ for all $i \in S$ and for all $k$. Let $w'_\infty$ be a limit point of this subsequence. By continuity, $w_i \geq u_i + \delta w'_{\infty,i+1}$ for all $i \in S$. To demonstrate that $w'_\infty \in \Psi_\delta^\infty(V_\delta^0)$, it is sufficient to show that $w'_\infty \in \Psi_\delta^n(V_\delta^0)$ for all $n$. Consider any $n$. We know that for all $m \geq n+1$, $w'_m \in \Psi_\delta^{m-1}(V_\delta^0) \subseteq \Psi_\delta^n(V_\delta^0)$ (where the last relation follows from lemma 4). Since $\Psi_\delta^n(V_\delta^0)$ is compact, $w'_\infty \in \Psi_\delta^n(V_\delta^0)$, as required.

**Step 2** $\Psi_\delta(\Psi_\delta^\infty(V_\delta^0)) \subseteq \Psi_\delta^\infty(V_\delta^0)$. This result follows immediately from lemma 5 and step 1.

■

**Proof of Lemma 7.** This proof has two steps.

**Step 1** $V_\delta^* \subseteq \Psi_\delta^\infty(V_\delta^0)$. Consider any $w \in V_\delta^*$. We know there is a DCW $P$ such that $V^P(h^t) = w$ for some $h^t$. It must be the case that $w_i = P_i(h^t) + \delta V_{i+1}^P(h^t, P(h^t))$ for all $i \in I$. Letting $u'' = P(h^t) \in U$ and $w'' = V^P(h^t, P(h^t)) \in V_\delta^*$ establishes that $w$ satisfies property (i) of (9). Now consider any $u \in U$. By definition of a DCW,

$$\left|\left\{i \in I \,\middle|\, V_i^P\left(h^t\right) \geq u_i + \delta V_{i+1}^P\left(h^t, u\right)\right\}\right| \geq 2$$

Letting $w' = V^P(h^t, u) \in V_\delta^*$, we can see that $w$ satisfies property (ii) of (9). This argument establishes that $V_\delta^* \subseteq \Psi_\delta(V_\delta^*) \subseteq \Psi_\delta^\infty(V_\delta^0)$, where the last relation follows from lemma 5.

**Step 2** $\Psi_\delta^\infty(V_\delta^0) \subseteq V_\delta^*$. Consider any $w \in \Psi_\delta^\infty(V_\delta^0)$. Define a function $x : H \to W$ recursively as follows. First, let $x(h^1) = w$. Note that $x(h^1) \in \Psi_\delta^\infty(V_\delta^0)$. Now assume that $x(h^{t-1}) \in \Psi_\delta^\infty(V_\delta^0)$ for all $h^{t-1}$. By lemma 6, there exist $u''(h^{t-1}) \in U$ and $w''(h^{t-1}) \in \Psi_\delta^\infty(V_\delta^0)$ such that $x_i(h^{t-1}) = u''_i(h^{t-1}) + \delta w''_{i+1}(h^{t-1})$ for all $i \in I$. Moreover, for any $u \in U$, there exists $w'(h^{t-1}, u) \in W$ such that

$$\left|\left\{i \in I \,\middle|\, x_i\left(h^{t-1}\right) \geq u_i + \delta w'_{i+1}\left(h^{t-1}, u\right)\right\}\right| \geq 2$$

For any $h^t$, construct $x(h^t)$ as follows:

$$\begin{aligned} x\left(h^{t-1}, u''\left(h^{t-1}\right)\right) &= w''\left(h^{t-1}\right) \\ x\left(h^{t-1}, u\right) &= w'\left(h^{t-1}, u\right) \text{ for any } u \neq u''\left(h^{t-1}\right) \end{aligned}$$

By construction, $x(h^t) \in \Psi_\delta^\infty(V_\delta^0)$, allowing the recursion to continue. Note that for any $h^t$, $x(h^t) = V(u''(h^t), u''(h^t, u''(h^t)), ...)$. Now construct a policy program $P$ as follows: for all $h^t$, $P(h^t) = u''(h^t)$ (where $u''(h^t)$ is as defined above). It remains to show that $P$ is a DCW that generates $w$ as a continuation payoff for some history. Consider any $h^t$. Note that $C(h^t) = (u''(h^t), u''(h^t, u''(h^t)), ...)$, which in turn implies that $V^P(h^t) = x(h^t)$. I have shown that for any $u \in U$,

$$\begin{aligned} \left|\left\{i \in I \left| x_i\left(h^t\right) \geq u_i + \delta x_{i+1}\left(h^t, u\right)\right.\right\}\right| &\geq 2 \\ \Rightarrow \left|\left\{i \in I \left| V_i^P\left(h^t\right) \geq u_i + \delta V_{i+1}^P\left(h^t, u\right)\right.\right\}\right| &\geq 2 \end{aligned}$$

Thus, $P$ is a DCW. Moreover, by construction, $V^P(h^1) = x(h^1) = w$.

■

**Proof of lemma 8.** The proof has two steps.

**Step 1** $\Psi_\delta^1(V_\delta^0) \subseteq \{w \in V_\delta^0 \,| w_i + w_j \geq 1 \text{ for all } i \neq j\}$.

Consider any $w \notin \{w \in V_\delta^0 \,| w_i + w_j \geq 1 \text{ for all } i \neq j\}$. This implies that $w_i + w_j < 1$ for some $i \neq j$. Consider a policy $u$ such that $u_i = w_i + \varepsilon$ and $u_j = w_j + \varepsilon$. Such a policy is feasible for sufficiently small positive $\varepsilon$ since $w_i + w_j < 1$. Now observe that $w_i < u_i + \delta w'_{i+1}$ and $w_j < u_j + \delta w'_{j+1}$ regardless of the values of $w'_{i+1}$ and $w'_{j+1}$. Thus $w \notin \Psi_\delta^1(V_\delta^0)$.

**Step 2** $\{w \in V_\delta^0 \,| w_i + w_j \geq 1 \text{ for all } i \neq j\} \subseteq \Psi_\delta^1(V_\delta^0)$.

Consider any $w \in \{w \in V_\delta^0 \,| w_i + w_j \geq 1 \text{ for all } i \neq j\}$. First define

$$\begin{aligned} u'' &= \left[1 - \min\left(w_2, 1 - w_3\right) - w_3, \min\left(w_2, 1 - w_3\right), w_3\right] \\ w'' &= \left[1 + \delta + \delta^2 - \frac{w_2 - u''_2}{\delta} - \frac{w_1 - u''_1}{\delta}, \frac{w_1 - u''_1}{\delta}, \frac{w_2 - u''_2}{\delta}\right] \end{aligned}$$

and note that $w_i = u''_i + \delta w''_{i+1}$. The proof of lemma 1 demonstrates that $u'' \in U$. It remains to show that $w'' \in V_\delta^0$. First note that

$$w''_3 = \max\left(0, \frac{w_2 + w_3 - 1}{\delta}\right) \leq 1$$

since $w_2 + w_3 \leq 1 + \delta$. Now note that

$$w_2'' + w_3'' = \frac{w_1 + w_2 - (1 - w_3)}{\delta} \leq 1 + \delta$$

since $w_1 + w_2 + w_3 = 1 + \delta + \delta^2$. By construction, $1 \leq w_1'' + w_2'' + w_3'' = 1 + \delta + \delta^2$. Finally, the proof of lemma 1 shows that $u_i'' \leq w_i$ for $i = 1, 2$. This implies that $w'' \geq 0$. Thus, $w'' \in V_\delta^0$.

Now consider any $u \in U$. For all $i$, define

$$z_i = \frac{w_i - u_i}{\delta}.$$

Note first that $w_i \geq u_i + \delta w_{i+1}'$ if and only if $w_{i+1}' \leq z_i$. Second, note that $z_i \geq 0$ for at least two individuals; if not, then for two individuals, $i$ and $j$, it must be the case that $w_i + w_j < u_i + u_j \leq 1$, which contradicts $w \in \{w \in V_\delta^0 \,|w_i + w_j \geq 1 \text{ for all } i \neq j\}$. There are two cases:

**Case 1** $z_3 < 0$. In this case, we must have $z_1, z_2 \geq 0$. Define $w'$ as follows

$$\begin{aligned} w_3' &= \min(z_2, 1) \\ w_2' &= \min(z_1, 1 + \delta - w_3') \\ w_1' &= 1 + \delta + \delta^2 - w_2' - w_3' \end{aligned}$$

It is straightforward to see that $w' \in V_\delta^0$. First note that $w_3' \leq 1$, $w_2' + w_2' \leq 1 + \delta$, and $1 \leq w_1 + w_2 + w_3 \leq 1 + \delta + \delta^2$. Second, note that $z_1, z_2 \geq 0$ guarantees that $w' \geq 0$. It is also straightforward to see that $w_2 \geq u_2 + \delta w_3'$ and $w_1 \geq u_1 + \delta w_2'$ since $w_3' \leq z_2$ and $w_2' \leq z_1$.

**Case 2** $z_3 \geq 0$. In this case, either $z_1 \geq 0$ or $z_2 \geq 0$. If $z_1 \geq 0$, define $w' = \left(\delta^2 + \delta^3, 0, 1\right)$. It is easy to see that $w' \in V_\delta^0$, and that $w_2' \leq z_1$. Moreover, $z_3 \geq 0$ implies that $w_3 \geq u_3$. Thus, $w_i \geq u_i + \delta w_{i+1}'$ for $i = 1, 3$. If $z_1 < 0$ and $z_2 \geq 0$, then let $w' = \left(1, \delta^2 + \delta, 0\right)$. Clearly $w' \in V_\delta^0$. Moreover, $w_i \geq u_i + \delta w_{i+1}'$ for $i = 2, 3$.

Thus, $w \in \Psi_\delta^1 (V_\delta^0)$, as required. ■

**Proof of lemma 9.** The proof has two steps.

**Step 1** $\Psi_\delta^2(V_\delta^0) \subseteq \left\{ w \in V_\delta^0 \;\middle|\; \begin{array}{c} w_i + w_j \geq 1 \text{ for all } i \neq j \\ w_1 + w_2 + w_3 \geq 1 + \delta \\ w_1 + w_3 \leq 1 + \delta^2 + \delta^3 \end{array} \right\}$.

Consider any $w \in \Psi_\delta^2 (V_\delta^0)$. Since $\Psi_\delta^2 (V_\delta^0) \subseteq \Psi_\delta^1 (V_\delta^0)$, we know that the first condition defining the right-hand-side set ($w_i + w_j \geq 1$ for all $i \neq j$) holds.

We also know that there exist $u'' \in U$ and $w'' \in \Psi_\delta^1(V_\delta^0)$ such that $w_i = u_i'' + \delta w_{i+1}''$ for all $i$. Thus,

$$w_1 + w_2 + w_3 = u_1'' + u_2'' + u_3'' + \delta(w_2'' + w_3'') = 1 + \delta(w_2'' + w_3'') \geq 1 + \delta$$

where the last inequality follows from $w'' \in \Psi_\delta^1(V_\delta^0)$. We also know that

$$1 \leq w_1'' + w_3'' \leq 1 + \delta + \delta^2 - w_2''$$

where the first inequality follows from $w'' \in \Psi_\delta^1(V_\delta^0)$ and the second from $w_1'' + w_2'' + w_3'' \leq 1 + \delta + \delta^2$. Hence $w_2'' \leq \delta + \delta^2$, which implies that $w_1 - (1 - w_3) \leq w_1 - u_1'' \leq \delta^2 + \delta^3$ (since $w_2'' = (w_1 - u_1'')/\delta$ and $u_3'' = w_3$). Rearranging this inequality results in the requirement that $w_1 + w_3 \leq 1 + \delta^2 + \delta^3$. Thus, $w$ satisfies all three conditions defining the right-hand-side set.

**Step 2** $\left\{ w \in V_\delta^0 \,\middle|\, \begin{array}{c} w_i + w_j \geq 1 \text{ for all } i \neq j \\ w_1 + w_2 + w_3 \geq 1 + \delta \\ w_1 + w_3 \leq 1 + \delta^2 + \delta^3 \end{array} \right\} \subseteq \Psi_\delta^2(V_\delta^0)$.

Consider any $w$ that is an element of the set on the left hand side. I will show that $w \in \Psi_\delta^2(V_\delta^0)$. First define

$$\begin{aligned} u'' &= [1 - w_3 - \max(w_2 - \delta, 0), \max(w_2 - \delta, 0), w_3] \\ w'' &= \begin{bmatrix} \min\left(1 + \delta + \delta^2 - \frac{w_1 - u_1''}{\delta} - \frac{w_2 - u_2''}{\delta}, 1 + \delta^2 + \delta^3 - \frac{w_2 - u_2''}{\delta}\right) \\ \frac{w_1 - u_1''}{\delta} \\ \frac{w_2 - u_2''}{\delta} \end{bmatrix}' \end{aligned}$$

Note that $u'' \in U$ because $u_1'' + u_2'' + u_3'' = 1$; and

$$u_2'' + u_3'' = \max(w_3 + w_2 - \delta, w_3) \leq 1,$$

implying $u'' \geq 0$. It is straightforward to see that $w'' \in V_\delta^0$. First, note that $w_3'' \leq 1$ since $u_2'' \geq w_2 - \delta$. Second, note that

$$w_2'' + w_3'' = \frac{w_1 + w_2 - (1 - w_3)}{\delta} \leq 1 + \delta,$$

and that $1 \leq w_1'' + w_2'' + w_3'' \leq 1 + \delta + \delta^2$. Finally, observe that $w'' \geq 0$ because $u_2'' \leq w_2$, $u_1'' \leq w_1$ (which follows from the fact that $w_1 + w_3 \geq 1 - \max(w_2 - \delta, 0)$), and

$$\begin{aligned} w_1'' &= \min\left(1 + \delta + \delta^2 - w_2'' - w_3'', 1 + \delta^2 + \delta^3 - w_3''\right) \\ &\geq \min\left(\delta^2, \delta^2 + \delta^3\right) \\ &\geq 0. \end{aligned}$$

where the first inequality follows from $w_3'' \leq 1$ and $w_2'' + w_3'' \leq 1 + \delta$. It remains to be seen that $w'' \in \Psi_\delta^1 (V_\delta^0)$. To see this, note that

$$w_2'' + w_3'' = \frac{w_1 + w_2 - (1 - w_3)}{\delta} \geq 1$$

since $w_1 + w_2 + w_3 \geq 1 + \delta$. Now observe that

$$w_1 + w_3 = \min\left(1 + \delta + \delta^2 - w_2'', 1 + \delta^2 + \delta^3\right)$$

Thus, to establish that $w_1 + w_3 \geq 1$, it suffices to show that $w_2'' \leq \delta + \delta^2$. Consider two possibilities. First, we could have $w_2 > \delta$. In this case, $u_2'' = w_2 - \delta$ and $u_1'' = 1 - w_3 - w_2 + \delta$. This implies that

$$w_2'' = \frac{w_1 + w_2 + w_3 - (1 + \delta)}{\delta} \leq \delta$$

where the inequality follows from $w_1 + w_2 + w_3 \leq 1 + \delta + \delta^2$. Second, we could have $w_2 \leq \delta$. In this case, $u_2'' = 0$ and $u_1'' = 1 - w_3$. This implies that $w_2'' = \frac{w_1 + w_3 - 1}{\delta} \leq \delta + \delta^2$ where the inequality follows from $w_1 + w_3 \leq 1 + \delta^2 + \delta^3$. Thus, $w_1'' + w_3'' \geq 1$. Finally, note that

$$\begin{aligned} w_1'' + w_2'' &= \min\left(1 + \delta + \delta^2 - w_3'', 1 + \delta^2 + \delta^3 - w_3'' + w_2''\right) \\ &\geq \min\left(\delta + \delta^2, \delta^2 + \delta^3 + w_2''\right) \\ &\geq 1 \end{aligned}$$

where the first inequality follows from $w_3'' \leq 1$ and the second from $\delta^2 + \delta^3 \geq 1$.

Now consider any $u \in U$. For all $i$, define

$$z_i = \frac{w_i - u_i}{\delta}$$

and note that $w_i \geq u_i + \delta w_{i+1}'$ if and only if $w_{i+1}' \leq z_i$. As demonstrated in the proof of lemma 8, $z_i \geq 0$ for at least two individuals. Moreover, in this case,

$$z_1 + z_2 + z_3 = \frac{w_1 + w_2 + w_3 - 1}{\delta} \geq 1$$

since $w_1 + w_2 + w_3 \geq 1 + \delta$. There are two cases.

**Case 1** $z_3 < 0$. In this case, $z_1, z_2 \geq 0$ and $z_1 + z_2 \geq 1$. Let

$$w' = \left[\delta^2 + \delta^3, \frac{z_1}{z_1 + z_2}, \frac{z_2}{z_1 + z_2}\right].$$

Note that $w' \in V_\delta^0$ because $w' \geq 0$, $w_3' \leq 1$, $w_2' + w_3' \leq 1 + \delta$, and $1 \leq w_1' + w_2' + w_3' \leq 1 + \delta + \delta^2$. Moreover, it is straightforward to verify that $w' \in \Psi_\delta^1 (V_\delta^0)$ provided $\delta^2 + \delta^3 \geq 1$. Finally, notice that $w_{i+1}' \leq z_i$ for $i = 1, 2$ since $z_1 + z_2 \geq 1$.

**Case 2** $z_3 \geq 0$. In this case, define $w'$ exactly as in case 2 of the proof of lemma 8. It has already been demonstrated that $w' \in V_\delta^0$, and that $w'_{i+1} \geq z_i$ for at least two individuals. Additionally, it is straightforward to verify that $w' \in \Psi_\delta^1 \left(V_\delta^0\right)$ provided $\delta^3 + \delta^2 \geq 1$.

Thus, $w \in \Psi_\delta^2 \left(V_\delta^0\right)$, as required. ■

**Proof of proposition 1.** We already know that $\Psi_\delta^\infty \left(V_\delta^0\right) \subseteq \Psi_\delta^2 \left(V_\delta^0\right)$. To establish that $\Psi_\delta^2 \left(V_\delta^0\right) \subseteq \Psi_\delta^\infty \left(V_\delta^0\right)$, it suffices to show that $\Psi_\delta^2 \left(V_\delta^0\right) \subseteq \Psi_\delta \left(\Psi_\delta^2 \left(V_\delta^0\right)\right)$. This will then imply that $\Psi_\delta^2 \left(V_\delta^0\right) \subseteq \Psi_\delta^\infty \left(V_\delta^0\right)$ and therefore $\Psi_\delta^2 \left(V_\delta^0\right) = \Psi_\delta^\infty \left(V_\delta^0\right)$. Consider any $w \in \Psi_\delta^2 \left(V_\delta^0\right)$. Define $w''$ and $u''$ exactly as in the proof of lemma 9. It has already been shown that $u'' \in U$ and $w'' \in \Psi_\delta^1 \left(V_\delta^0\right)$. It remains to be shown, therefore, that $w'' \in \Psi_\delta^2 \left(V_\delta^0\right)$. This is easy to see because $w''_1 + w''_2 + w''_3 = \min\left(1 + \delta + \delta^2, 1 + \delta^2 + \delta^3 + w''_2\right) \geq 1 + \delta$ since $\delta + \delta^2 \geq 1$. Moreover, by construction, $w''_1 + w''_3 \leq 1 + \delta^2 + \delta^3$. Thus $w'' \in \Psi_\delta^2 \left(V_\delta^0\right)$.

Now consider any $u \in U$ and define $w'$ exactly as in the proof of lemma 9. I have already shown that $w' \in \Psi_\delta^1 \left(V_\delta^0\right)$ and that $w_i \geq u_i + \delta w''_{i+1}$ for at least two individuals. To see that $w' \in \Psi_\delta^2 \left(V_\delta^0\right)$, it is easy to check that in all cases, $w''_1 + w''_2 + w''_3 \geq \delta^2 + \delta^3 + 1 \geq 1 + \delta$ for $\delta + \delta^2 \geq 1$. Moreover, $w'_1 + w'_3 \leq 1 + \delta^2 + \delta^3$ as well. Thus $w' \in \Psi_\delta^2 \left(V_\delta^0\right)$.

This argument shows that $w \in \Psi_\delta \left(\Psi_\delta^2 \left(V_\delta^0\right)\right)$, as required. ■

**Proof of proposition 2.** I prove each part separately.

**Part (i)** For any $w \in V_\delta^*$, $w_1 + w_2 + w_3 \leq 1 + \delta + \delta^2$ and $w_2 + w_3 \geq 1$. Thus, $w_1 \leq \delta + \delta^2$, which in turn implies that $\pi_1^{\max}(\delta) \leq \left(\delta + \delta^2\right) / \left(1 + \delta + \delta^2\right)$. To see that, in fact, $\pi_1^{\max}(\delta) = \left(\delta + \delta^2\right) / \left(1 + \delta + \delta^2\right)$, consider the payoffs $w = \left(\delta + \delta^2, 1, 0\right)$. Clearly $w \in V_\delta^0$. It is also easy to check that $w_i + w_j \geq 1$ for all $i \neq j$, $w_1 + w_2 + w_3 \geq 1 + \delta$, and $w_1 + w_3 \leq \delta^3 + \delta^2 + 1$. Thus $w \in V_\delta^*$ and generates a normalized payoff of $\left(\delta + \delta^2\right) / \left(1 + \delta + \delta^2\right)$ for the young.

**Part (ii)** We know that for any $w \in V_\delta^*$, $w_1 + w_2 + w_3 \leq 1 + \delta + \delta^2$ and $w_1 + w_3 \geq 1$. Together, these imply that $w_2 \leq \delta + \delta^2$ and therefore $\pi_2^{\max}(\delta) \leq \left(\delta + \delta^2\right) / (1 + \delta) = \delta$. To see that $\pi_2^{\max}(\delta) = \delta$, consider the continuation payoffs $w = \left(\delta^2, \delta^2 + \delta, 1 - \delta^2\right)$. It is easy to verify that $w \in V_\delta^*$ provided $\delta^3 + \delta^2 \geq 1$, and that $\pi_2(w; \delta) = \delta$.

**Part (iii)** Mechanical feasibility requires that $w_3 = \pi_3(w; \delta) \leq 1$. Moreover, it is easy to check that the payoffs $w = \left(\delta^2, \delta, 1\right)$ lie in $V_\delta^*$ provided $\delta^3 + \delta^2 \geq 1$. Thus, $\pi_3^{\max}(\delta) = 1$.

■

**Proof of proposition 3.** Consider any two policy sequences $(u^1, u^2, \ldots)$ and $(\widehat{u}^1, \widehat{u}^2, \ldots)$. Suppose that

$$\left|\left\{i \in I \,\middle|\, V_i(u^t, u^{t+1}, \ldots; \delta, g) \geq V_i(\widehat{u}^t, \widehat{u}^{t+1}, \ldots; \delta, g)\right\}\right| \geq 2.$$

This implies that for some $S \subseteq I$ with $|S| \geq 2$,

$$\sum_{j=0}^{3-i} \delta^j u_{i+j}^{t+j}(1+g)^{t+j-1} \geq \sum_{j=0}^{3-i} \delta^j \widehat{u}_{i+j}^{t+j}(1+g)^{t+j-1} \tag{14}$$

for all $i \in S$. Consider any $\hat{g}$ and $\hat{\delta}$ such that $(1+\hat{g})\hat{\delta} = (1+g)\delta$. Equation (14) implies that

$$\begin{aligned} (1+g)^{t-1} \sum_{j=0}^{3-i} \left((1+g)\delta\right)^j u_{i+j}^{t+j} &\geq (1+g)^{t-1} \sum_{j=0}^{3-i} \left((1+g)\delta\right)^j \widehat{u}_{i+j}^{t+j} \\ (1+\hat{g})^{t-1} \sum_{j=0}^{3-i} \left((1+\hat{g})\hat{\delta}\right)^j u_{i+j}^{j} &\geq (1+\hat{g})^{t-1} \sum_{j=0}^{3-i} \left((1+\hat{g})\hat{\delta}\right)^j \widehat{u}_{i+j}^{j+1} \end{aligned}$$

for all $i \in S$. Thus, $\left|\left\{i \in I \,\middle|\, V_i(u^t, u^{t+1}, \ldots; \hat{\delta}, \hat{g}) \geq V_i(\widehat{u}^t, \widehat{u}^{t+1}, \ldots; \hat{\delta}, \hat{g})\right\}\right| \geq 2$.

Now let us look at policy programs, rather than policy sequences. Suppose $P \in \mathcal{P}^*(\delta, g)$. This implies that for any $h^t$ and any $u \in U$,

$$\left|\left\{i \in I \,\middle|\, V_i^P(h^t; \delta, g) \geq u_i + \delta V_{i+1}^P(h^t, u; \delta, g))\right\}\right| \geq 2,$$

or, alternatively,

$$\left|\left\{i \in I \,\middle|\, V_i^P(h^t; \delta, g) \geq V_i(u, C^P\left(h^t, u\right); \delta, g))\right\}\right| \geq 2.$$

As seen above,

$$\left|\left\{i \in I \,\middle|\, V_i^P(h^t; \hat{\delta}, \hat{g}) \geq V_i(u, C^P\left(h^t, u\right); \hat{\delta}, \hat{g}))\right\}\right| \geq 2$$

as well. Thus, $P \in \mathcal{P}^*(\hat{\delta}, \hat{g})$, as required. Since this result holds for any $g$ and $\delta$, it is also true that $\mathcal{P}^*(\hat{\delta}, \hat{g}) \subseteq \mathcal{P}^*(\delta, g)$. ∎

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