ELSEVIER
Journal of Public Economics 89 (2005) 465–485
JOURNAL OF PUBLIC ECONOMICS
www.elsevier.com/locate/econbase

# Life-cycle earnings, cohort size effects and social security: a quantitative exploration

Juan A. Rojas*

*Department of Economics, Universidad Carlos III de Madrid, Calle Madrid 126, 28903, Getafe, Spain*

Received 6 December 2000; received in revised form 8 November 2003; accepted 17 April 2004
Available online 18 August 2004

**Abstract**

One of the features of the large overlapping generations model pioneered by Auerbach and Kotlikoff (1987) Dynamic Fiscal Policy is that individuals with different experience levels are perfect substitutes in production. This paper replaces this assumption with a labor market characterized by imperfect substitutability between less and more experienced workers. By comparing the quantitative properties of both cases in a calibrated model for Spain, it is found that in the model economy with imperfect substitution, the effects of aging on the financial viability of the pension system are less severe than in the standard model economy with perfect substitution.


*JEL classification:* E62; H55; J11
*Keywords:* Cohort size effects; Demographic change; Social security

## 1. Introduction

The purpose of this paper is to examine the relationship between the aging of the population and the future prospects of the social security system in Spain. Although this topic has received much attention over the last 10 years mainly due to the sharp expected increase in the share of retired individuals over the working population (see Fig. 1), the existing studies have abstracted from the interaction between the age composition of the population and the life-cycle profile of earnings. Since the pattern of pension benefits and contributions are strongly age dependent, this assumption can have important effects

* Fax: +34-91-6249875.
*E-mail address:* rojas@eco.uc3m.es (J.A. Rojas).

0047-2727/$ - see front matter 
doi:10.1016/j.jpubeco.2004.04.003

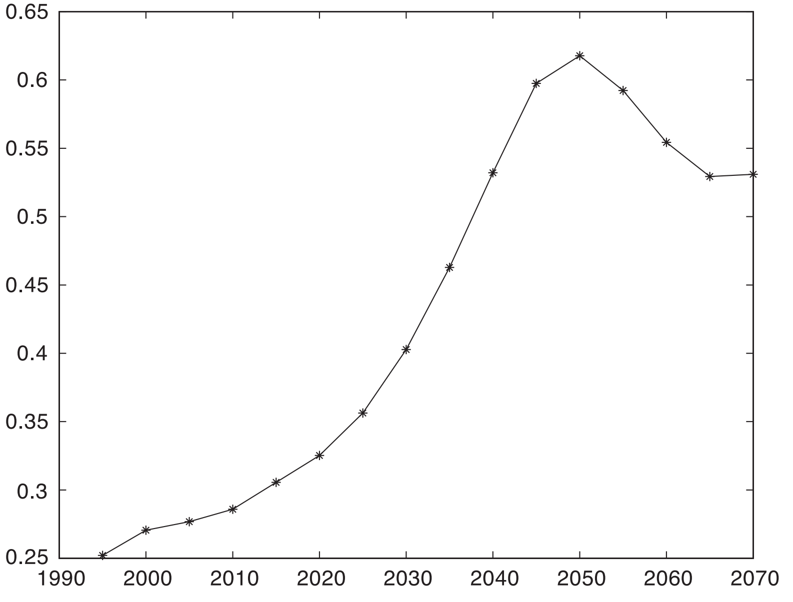


Fig. 1. Expected old dependency ratio in Spain.

on the evolution of the percentage of GDP spent on pensions. For this reason in this paper I study the following question: How do the properties of the standard large overlapping generations model compare to the properties of a model that accounts for the existence of cohort size effects? I study this issue by calibrating a computable overlapping generations model to match some key features of the Spanish economy in 1995. Then I use the demographic projections for this economy that match those from *Eurostat Demographic Statistics 1996* and analyze what should be the adjustment in the tax rate needed to keep balanced the pension system in several model economies that have different assumptions about the degree of substitution of workers with different levels of work experience.

This paper is not new in addressing the effects of the aging process upon the social security systems. The potential economic effects caused by the individuals that belong to the baby-boom generation as they enter retirement has motivated an increasing concern about the sustainability of the pension systems in developed countries. In this sense, the recent research effort on social security has mainly concentrated on the efficiency of the current pay-as-you-go pension system (e.g. Imrohoroglu et al. (1995) and Boldrin et al. (1999)), the design of a feasible reform to a funded system (e.g. Huang et al. (1997)) and the fiscal adjustments that prevents from privatization (De Nardi et al. (1999) and Montero (2000) for the Spanish economy). These studies are characterized by the perfect substitutability of workers with different levels of work experience, namely they abstract from the possible effect that an increase in the number of older workers relative to the number of young less experienced workers could have on the relative labor earnings of these workers.

In sharp contrast with this assumption, there are many empirical studies for the US economy (e.g. Berger (1985), Freeman (1979), Katz and Murphy (1992), Murphy and Welch (1992) and Welch (1979)) that have found that the age-earnings profile of workers appears to be significantly affected by the age composition of the workforce. In words of Freeman (1979), "*apparently because younger and older male workers are imperfect substitutes in production, changes in the number of young male workers relative to older male workers substantially influence the ratio of the earnings of younger men to the earnings of older men*". For the specific case of the Spanish economy the lack of data on wages by age has not allowed researchers to test these effects although there is some empirical evidence (Eguia and Echevarria (2004)) concerning the relative higher unemployment rate experienced by those individuals that belong to the baby-boom generation. In this sense, it is worth noting that in highly regulated labor markets one should expect the adjustment of the labor market to be done through a change in quantities (employment) instead of prices (wages) following a change in the relative supply of workers with different levels of work experience. Since both cases imply that the individuals of the baby-boom generation have on average lower labor earnings than individuals belonging to more scarce cohorts, we think that the assumption of perfect competitive labor markets with imperfect substitutability between young and old workers is a good starting point to tackle the question at hand.

Despite the potential implications of these cohort size effects for a variety of macroeconomic issues, there are not many studies that have attempted to introduce these effects in macroeconomic models. Some exemptions are the seminal work of Lam (1989) that studied the effects of changes in age structure on life-cycle wage profiles in stable populations. And more recently, Kremer and Thomson (1998) have studied the implications of the imperfect substitution between young and old workers for the speed of convergence of per capita output between countries and find that the existence of imperfect substitutability creates a kind of adjustment cost in human capital because total output depends positively on each generation generation's human capital but negatively on the change in human capital between generations.

Although we think that the assumption of perfect substitution between young and old workers may be useful depending on the question at hand, in the context of an aging population to legitimately abstract from the interaction between the age structure of the population and the life-cycle profile of labor earnings, the consequences of the aging of the baby-boom generation should be quantitatively similar in a model economy that abstracts from cohort size effects an one that it does not. In this paper I address this question by studying the effects of demographic projections from 1995 to 2050 on the finances of the social security system. I compare the equilibrium allocations in two model economies that only differ from each other in the degree of substitution of workers with different experience levels. Results indicate that there are quantitatively relevant differences between both economies. For instance it is found that if the rule used to compute pension benefits is left untouched, in the standard model the percentage of GDP spent on pensions will increase from 7.1% in 1995 to 17.1% in 2040. In contrast, in the model economy with cohort size effects this percentage will increase from 7.1% in 1995 to 12.8% in 2040. Similarly, under the perfect substitution case the social security tax rate has to be increased from 11.8% in 1995 to 30.5% in 2045 while with imperfect substitutability across age groups the tax rate should be increased from 11.8% in 1995 to 23.7% in 2045. The mechanism that accounts for such difference is the lower pension benefits of individuals belonging to the baby-boom generation in the model economy with cohort size effects. This is so due to the fall in the experience premium and the reduction of labor effort before retirement displayed by the members of the baby-boom generation as part of the intertemporal reallocation of hours worked in response to the fall in relative wages. The rest of the paper is organized as follows. Section 2 describes the main features of the model economies I investigate and its calibration to be a realistic description of the Spanish economy in 1995. Section 3 presents the main results of the paper. Section 4 studies the sensitivity of the results to different modelling strategies and finally Section 5 concludes.

## 2. The model

### 2.1. Demographics

The economy is populated by agents that live a maximum of $I$ periods. Each type of agent is indexed by age $i$ and time $t$. Upon arrival at the age of $I_A$ an agent starts taking decisions. Each individual is endowed with 1 unit of time that can be allocated to work or leisure up to age $I_{R-1}$. After this age agents retire. Each agent faces an age dependent probability of surviving between age $i$ and age $i+1$ at $t$ denoted by $S_{i,t}$. Then the unconditional probability of reaching age $i$ for an individual that has age $v$ at $t$ is $\pi^i_{v,t} = \prod_{k=v+1}^{i} S_{k-1,t+k-v-1}$ with $\pi^v_{v,t} = 1$. Let $\mu_{i,t}$ be the share of age-i individuals over the total population at time $t$. Agents reach adulthood at 20 and live up to age 95, after which death is certain. Each model period corresponds to 5 years. We take the age structure of the population of 1995 as the initial condition. The procedure to propagate the economy after this year uses the law of motion of the population characterized by equations (A.4) and (A.5) (see Appendix A). We use the database Eurostat Demographic Statistics 1996 to obtain the expected evolution of the population growth rate, the mortality rates and the migration rates from 1995 to 2050. These estimates assume that in Spain, the average life expectancy at birth will increase from 77.2 years in 1995 to 82.7 years in 2050. Finally in order to compute the final steady state, after 2050 I fix the net migration rates and the survival probabilities of that year and let the age structure run until it reaches a stationary structure characterized by a population growth rate equal to zero. Figs. 1 and 2 shows the dynamics of the dependency ratio (the share of those aged +65 over those with age 20–64).

### 2.2. Preferences

At each point in time agents are assumed to maximize lifetime utility. Hence the problem of the typical agent that at $t$ has age $I=v$ ($v \geq I_A$) is to choose consumption and leisure $l_{i,t}=1-h_{i,t}$ to solve the problem

$$Max \sum_{i=v}^{I} \beta^{i-v} \pi^i_{v,t} U(c_{i,t+i-v}, h_{i,t+i-v})$$

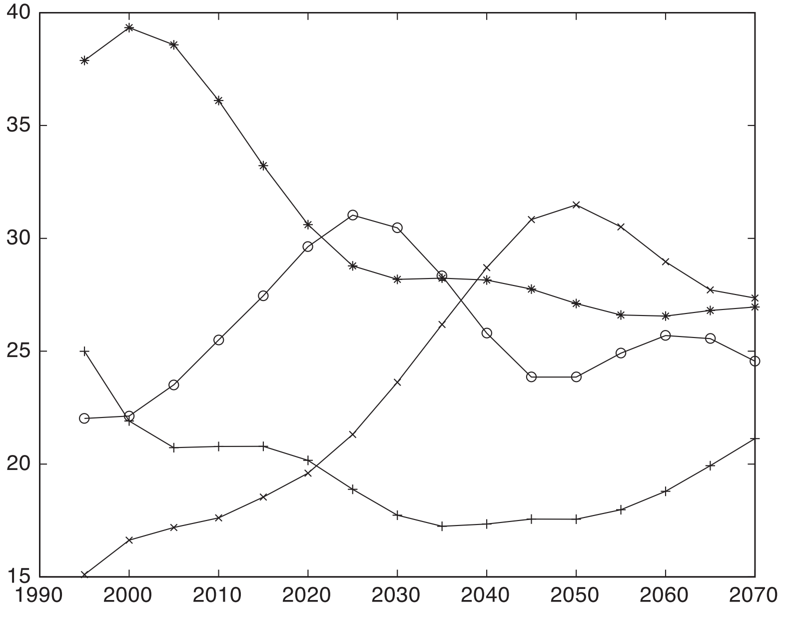


Fig. 2. Dynamics of age groups as % of total population. '+':(0–19), '*':(20–44), 'o':(4–64), 'x':(65+).

subject to the following period-by-period constraint

$$a_{i+1,t+1} = (1 + r_t(1 - \tau_k))a_{i,t} + y_{i,t} - c_{i,t}$$

with $a_{i+1,t+1} \geq 0$, $a_{1,t} = 0$, $a_{I+1,t} = 0$. The discount parameter is $\beta$, and is assumed to be the same for all agents. Borrowing is not possible and agents accumulate asset holdings to smooth consumption over time $r_t$ is the interest rate net of depreciation, $a_{i+1,\ t+1}$ denotes next period asset holdings, $y_{i,\ t}$ is labor income net of taxes plus transfers and $\tau_k$ is a proportional capital income tax. Let $e_i$ be the efficiency index, $\tau_{ss,t}$ the social security proportional tax, $\tau_l$ a proportional labor income tax and $d_{i,t}$ the social security benefits. Finally $w_{i,t}$ denotes real wages, that are indexed by age in order to allow for the possibility of wage heterogeneity in terms of experience, and $B_t$ is the accidental bequest received at t. These considerations allow us to define the labor income net of taxes plus transfers as $y_{i,t} = w_{i,t}e_ih_{i,t}(1 - \tau_l - \tau_{ss,t}) + d_{i,t} + B_t$. In the initial experiment I use a utility function of the constant relative risk-aversion class (to be generalized afterwards)

$$u(c,\ l) = \frac{(c^{\theta}l^{1-\theta})^{1-\sigma}}{1-\sigma} \tag{1}$$

where the inverse of the elasticity of substitution $\sigma$ and the share of consumption  has been set such that the average time spent working is around 1/3 and the intertemporal elasticity of substitution is consistent with the empirical estimates reviewed in Auerbach and Kotlikoff (1987). Hence we use $\sigma = 2$ and $\theta = 0.33$. The discount rate parameter is set equal to $\beta = 0.987$ so as to reproduce a private capital-output ratio of 2.5 in the Spanish economy as reported by Puch and Licandro (1997). Notice that although the discount parameter is the same for all agents, the effective discounting is heterogeneous and age dependent due to the existence of mortality risk.

### *2.3. Efficiency units profile and production technology*

As is standard in large overlapping generations models, in order to allow for the fact that earnings grow with experience agents are endowed with an exogenous profile of age specific efficiency units $e_i$. This has been done using the cross-sectional distribution of gross hourly wages in 1993 available in the European Household Panel (1994). The endowment of efficiency units is determined by dividing each cohort's average wage by the average of the sample and then by smoothing the wage profile with a polynomial of degree two.

Production in period t is given by a standard constant returns to scale production function that converts capital $K_t$ and labor $N_t$ into output. The technology $A_t$ improves over time at a constant rate because of labor augmenting technological change, $A_{t+1} = (1+\lambda)A_t$. The capital share parameter is $\alpha = 0.375$ following the estimates of Domenech and Taguas (1995) for the Spanish economy. The productivity growth has been set to $\lambda = 1.5\%$ in annual terms which is the average growth of per-capita consumption over the period 1960–1995. Hence,

$$Y_t = F(K_t,\ A_tN_t) = K_t^{\alpha}(A_tN_t)^{1-\alpha} \tag{2}$$

with

$$N_t = B(\gamma L_t^{1-\rho} + (1-\gamma)H_t^{1-\rho})^{\frac{1}{1-\rho}} \tag{3}$$

where $L_t$ and $H_t$ denotes less and more experienced workers, meaning workers with less and more than 25 years of working experience. The procedure to set the values of the inverse of the elasticity of substitution $\rho$, the parameter $B$ and the share parameter $\gamma$ is as follows. In the model economy with perfect substitution, a change in the relative supply of experienced workers does not translate into changes in the relative wages of individuals by age. Consequently, this is the case where $\rho = 0$. In addition, the value that governs the overall efficiency of labor input is set to a normalized value of $B = 1$. Finally, the value of the share parameter $\gamma$ is set such that $\frac{w_h}{w_l} = 1$. Notice that this means that the age-profile of wages in the initial steady state of the model economy which consists of a product of the market wage $w_i$ and the efficiency index $e_i$ resembles the age-specific profile of hourly wages computed from the data $e_i$. Since the relative wage is given by

$$\frac{w_h}{w_l} = \frac{\gamma}{1-\gamma}\left(\frac{H}{L}\right)^{-\rho} \tag{4}$$

then, when $\rho = 0$, $\frac{w_h}{w_l} = 1$ if $\gamma = 0.5$. On the other hand, Murphy and Welch (1992), among others, have studied the existence of imperfect substitutability among workers with different levels of working experience. Their estimates of the elasticities of complementarity imply values of the $\rho$ parameter between 0.8 and 2. In this paper we use $\rho = 1.2$ as our benchmark case for the case of imperfect substitution. Then, the share parameter $\gamma$ is set such that $\frac{w_h}{w_l} = 1$, yielding $\gamma = 0.717$, and the parameter that governs the overall efficiency of the labor input $B$ is set so that the *level* of wages equals the level of spot wages in the benchmark model economy with perfect substitutability between less and more experienced workers, hence both model economies share the same features in the initial steady state. This yields $B = 0.9120$. Finally, firms rent labor and capital at given wages and net interest rate to maximize

$$F(K_t,\ A_t N_t) - (r_t + \delta)K_t - w_{l,t}L_t - w_{h,t}H_t \tag{5}$$

where $\delta$ is the depreciation rate for capital and is set to match the average ratio of gross investment over output I/Y = 24%. This yields a value of $\delta = 9\%$ in annual terms. These values are also used by Conesa and Garriga (2003).

## 2.4. Government

The government levies a proportional social security tax on labor income $\tau_{ss,t}$ to finance a benefit $d_{i,t}$ per retiree. This system is assumed to be self-financed, i.e.

$$\sum_{i=I_A}^{I_R-1} \mu_{i,t} w_{i,t} h_{i,t} e_i \tau_{ss,t} = \sum_{i=I_R}^{I} \mu_{i,t} d_{i,t}$$

where benefits are computed as follows. Upon retirement an individual's pension is computed applying a replacement rate over the average of earnings of the last 8 years before retirement. This replacement rate is 100% if the individual has contributed for at least 35 years. The pension system in Spain also includes a maximum and a minimum pension level but given that individuals of the same generation are assumed to be homogeneous we abstract from this feature for the moment and leave it for the sensitivity analysis. Hence in age $I_R$ benefits are given by,

$$d_{I_R,t} = \frac{rep}{1+\lambda} w_{av,t}$$

where $\lambda$, *rep* and $w_{av}$ are the productivity growth, the legal replacement rate and some average of past earnings respectively. From age $I_R + 1$ to $I$, the pension benefit is normalized by productivity growth $(1 + \lambda)$, since new pensions are greater than old ones, i.e. $d_{i,t,j,n} = \frac{d_{i-1,t,j,n}}{1+\lambda}$. The government also levies a proportional tax on capital $\tau_k$ and labor $\tau_l$ income to finance per capita government consumption $G_t$ such that

$$\sum_{i=I_A}^{I} \mu_{i,t}(r_t a_{i,t} \tau_k + w_{i,t} h_{i,t} e_i \tau_l) = G_t.$$

In particular, we use a value of $\tau_k = 0.186$ and $\tau_l = 0.17$ as reported by Bosca et al. (1999). These values generate a government to output ratio of $G/Y = 0.13$ which is consistent with the average of this number from 1970 to 1994 in Spain.

## 3. Findings

Before describing the findings of the paper, I first explain what do I mean by the baby-boom generation. The term baby-boom generation in Spain refers to those individuals that were born between 1960 and 1978. In that period, the average number of children per women was around 2.8. In contrast, the average number of children averaged 2.5 from 1950 to 1959 and 1.5 from 1980 to 1995. This recent fertility pattern has motivated an increasing concern about the sustainability of the pension system as the dependency ratio is expected to increase dramatically with the aging of the baby-boom generation. In particular, the number of retirees over the working population (old dependency ratio) is expected to increase from 0.252 in 1995 to 0.462 and 0.597 in 2035 and 2045 respectively. The effects of this ongoing process are the following. In both model economies, the aging of the baby boom generation increases the number of the retirees over the total population and given the legal rule used to compute pension benefits, this process induces an increase in the percentage of GDP spent on pensions and the social security tax rate needed to balance the government budget. The rising trend in social security taxes reduces aggregate labor supply give individuals incentives to reallocate work effort from later years of the lifecycle to early ones in an attempt to avoid the distortions associated to higher taxes (see Fig. 3 and Table 1). In addition, with the aging of the population the share of the population with high wealth holdings but lower saving rates (the old) increases, and both factors imply a rising trend in the capital labor ratio and a fall in the aggregate saving rate in the economy. This induces a higher wage rate and a lower market return on capital. Notice that the increase in the social security tax rate due to the demographic change is partially compensated by two facts. First, as the capital-labor ratio grows, new wages are greater than old ones and consequently the pension burden is more easily sustainable. An second, the reduction in hours worked when old translates into lower pension benefits, since the years before retirement are crucial for the determination of these benefits.

The overall effect of aging on the pension budget in the economy with perfect substitution between less and more experienced workers is the following. There is an increase in the share of GDP spent on pensions from 7.12% in 1995 to 12.5% and 19.35% in 2030 and 2050 respectively, and an increase in the social security tax rate from 11.76% in 1995 to 30.96% by 2040.

### *3.1. Imperfect substitutability model*

The main feature of this model economy is that, in contrast to the previous case, the changes in the relative supply of workers with different levels of experience induce changes in the relative wage by age. Before analyzing the effects of this mechanism on the social security tax rate, it is useful to describe the dynamics of the relative supply of less and more experienced workers. Notice firstly that in 1995 the baby-boom generation are between 15 and 34 years old and consequently they are mostly working as less experienced workers. By 2010 some baby-boomers (those born by 1960) start working as experienced workers and some members of the baby-bust generation enter the labor force pushing down the relative wage of old workers. This is so because the members of the baby-bust generation that enter the labor force have a smaller size than the members of

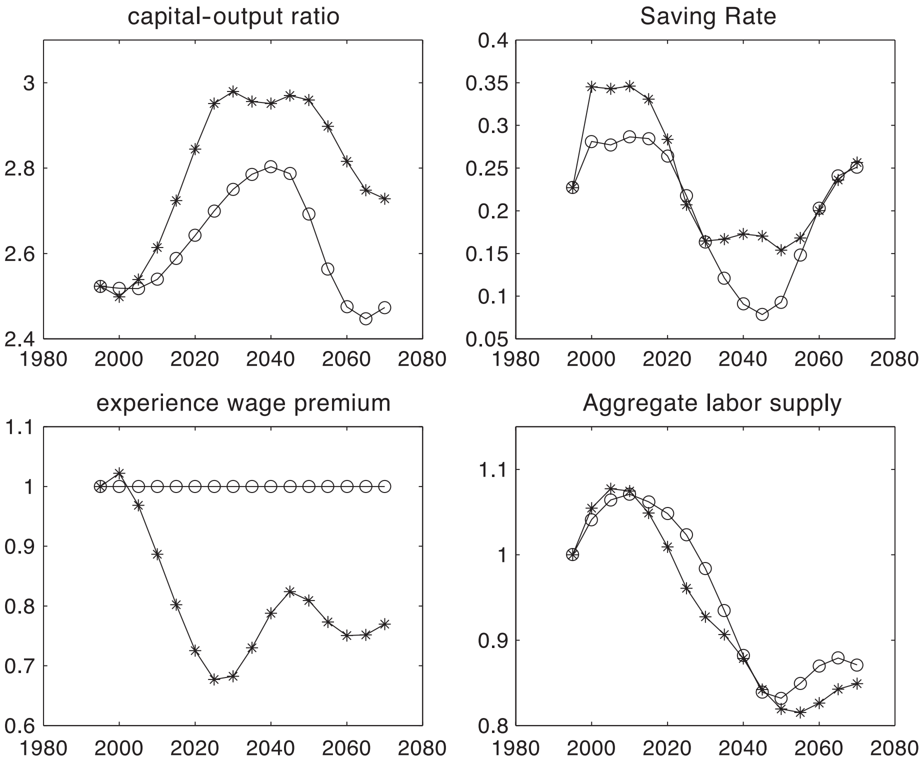


Fig. 3. Aggregate effects of demographic change. Experiment 1. 'o': Perfect subs., '*': Imperfect subs.

the baby-boom cohort that start working in the group of more experienced workers. This process lasts until 2025 when all the baby-boomers are working as more skilled workers. Finally, by 2030 the baby-boom generation enter retirement and the relative wage of older workers slightly increases. By 2045 the entire baby-boom generation is retired.

The consideration of the dynamics of relative wages changes dramatically the pattern of pension benefits as compared to the standard model economy. The pension benefit, as it stands in the Spanish economy, is computed applying a 100% to the average earnings of the last 8 years before retirement. Consequently, the dynamics of earnings before retirement are crucial for the differential behavior between the model economies. We have already seen that as the baby-boom generation starts working as experienced workers, the wage of those workers declines in response to the increase in the relative supply of more skilled people. As this process happens from 2000, it affects to all workers that start retiring from this date even if they do not belong to the baby-boom cohort. This explains why the less pronounced increase of the share of GDP spent on pensions in the initial periods of the transition. As more members of the baby-boom generation belong to the more experienced category the relative wage of this category declines until it reaches its lowest value in 2025. From this date onwards the baby-boomers start retiring but in contrast to the benchmark case without cohort size effects, the pension level to which

Table 1
Aggregate effects of demographic change

| *Model economy without cohort effects* $\rho=0$ $B=1$ $\gamma=0.5$ $\sigma=2$ $\mu=1$ $\theta=0.33$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Lifetime work effort by Age | | $\tau_{ss}$ | Pension/Y | Saving rate | K/Y | Wage premium |
| | 20–44 | 45–64 | | | | | |
| 1995 | 0.377 | 0.250 | 0.118 | 0.071 | 0.227 | 2.523 | 1.000 |
| 2005 | 0.381 | 0.249 | 0.127 | 0.079 | 0.277 | 2.518 | 1.000 |
| 2010 | 0.385 | 0.243 | 0.131 | 0.082 | 0.287 | 2.539 | 1.000 |
| 2020 | 0.385 | 0.242 | 0.151 | 0.094 | 0.264 | 2.642 | 1.000 |
| 2025 | 0.381 | 0.248 | 0.171 | 0.107 | 0.217 | 2.700 | 1.000 |
| 2030 | 0.376 | 0.255 | 0.200 | 0.125 | 0.164 | 2.750 | 1.000 |
| 2035 | 0.374 | 0.258 | 0.236 | 0.147 | 0.121 | 2.786 | 1.000 |
| 2040 | 0.375 | 0.258 | 0.274 | 0.171 | 0.091 | 2.803 | 1.000 |
| 2045 | 0.377 | 0.255 | 0.305 | 0.191 | 0.078 | 2.787 | 1.000 |
| 2050 | 0.379 | 0.252 | 0.310 | 0.194 | 0.093 | 2.692 | 1.000 |
| 2055 | 0.380 | 0.251 | 0.290 | 0.181 | 0.148 | 2.564 | 1.000 |
| *Model economy with cohort effects* $\rho=1.2$ $B=0.912$ $\gamma=0.717$ $\sigma=2$ $\mu=1$ $\theta=0.33$ | | | | | | | |
| 1995 | 0.377 | 0.250 | 0.118 | 0.071 | 0.227 | 2.523 | 1.000 |
| 2005 | 0.390 | 0.219 | 0.124 | 0.078 | 0.343 | 2.539 | 0.969 |
| 2010 | 0.397 | 0.215 | 0.126 | 0.079 | 0.346 | 2.614 | 0.887 |
| 2020 | 0.405 | 0.214 | 0.135 | 0.084 | 0.284 | 2.844 | 0.725 |
| 2025 | 0.402 | 0.218 | 0.145 | 0.090 | 0.207 | 2.951 | 0.677 |
| 2030 | 0.396 | 0.225 | 0.159 | 0.099 | 0.165 | 2.979 | 0.682 |
| 2035 | 0.391 | 0.231 | 0.178 | 0.111 | 0.167 | 2.956 | 0.730 |
| 2040 | 0.389 | 0.233 | 0.206 | 0.128 | 0.173 | 2.951 | 0.788 |
| 2045 | 0.391 | 0.232 | 0.237 | 0.148 | 0.170 | 2.970 | 0.824 |
| 2050 | 0.394 | 0.229 | 0.254 | 0.159 | 0.154 | 2.959 | 0.809 |
| 2055 | 0.394 | 0.228 | 0.248 | 0.155 | 0.168 | 2.898 | 0.773 |

Experiment 1: Baseline.

baby-boomers qualify for is lower since they have lower labor earnings before retirement. The reason is two fold. The first one is the reduction of wages before retirement associated to the decrease in the experience premium and the second is based on the behavioral response to this change in the experience premium. Notice that in the benchmark case, individuals tend to reallocate work effort towards the early ages of the lifecycle in order to avoid the distortions associated with higher social security taxes. In the model economy with cohort size effects, individuals foresee the fall in wages when old and react by working harder when young and less before retirement (see Table 1 where it is reported the lifetime work effort by generations born in a particular year). Hence, the arbitrage between consumption and leisure plays an important role in generating the labor supply response to anticipated wage changes due to cohort size effects. In particular in our simulations the intratemporal elasticity of substitution is one due to the assumption of a Cobb-Douglas form between consumption and leisure. With a lower elasticity of substitution between consumption and leisure one should expect both model economies to be more similar due to the fact that although the reduction in the experience premium would still be present (it is caused by the pure demographic process), individuals would not substitute so easily consumption for leisure before retirement when facing lower wages. This possibility is explored later on in this section using a more general CES utility function.

In the model economy with cohort effects the anticipated fall of wages at older ages allows households to make prior saving adjustments and to work harder when is more profitable to do so (in the initial ages of the lifecycle). In fact, we can observe that in the model economy with cohort size effects the saving rate is higher (see Table 1) and households that belong to the baby-boom cohort end up working less before retirement as compared to the benchmark case. In summary, findings indicate that in contrast to the standard model economy where the social security tax rate increases from 11.8% in 1995 to 30.5% in 2045, in the model economy with cohort size effects the increase in labor taxation is less severe. In particular it should be increased from 11.8% in 1995 to 23.7% in 2045. In addition, in the model economy with cohort size effects the Pensions/GDP ratio would grow from 7.1% in 1995 to 12.8% in 2040, implying a substantial difference between this model economy and the standard one without cohort size effects.

### *3.1.1. A decomposition analysis*

The purpose of this section is to perform a decomposition analysis in order to have a clear understanding of what is driving the different behavior between both model economies. Hence, we first compute de difference between the social security tax rate in the benchmark model economy and the model economy with cohort size effect. Secondly, we take the social security budget and ask what is the tax rate that balances this budget assuming that the only departure from the standard economy is respectively the pension benefits $d_{i,\,t}$ and taxable labor income $w_{i,t}\,e_{i,t}\,h_{i,t}$ of the model with cohort size effects. Finally, we compute the percentage that represents this last measure over the total difference between both model economies, ending up with an idea of how much of the difference between both models can be accounted for by the evolution of the pensions benefits and taxable labor income in the cohort size economy. The results of this exercise for selected years is shown in Table 2 and indicate that most of the different behavior between the model economies is driven by the lower labor earnings that the members of the baby-boom generation experience before retirement, since these years are critical for the determination of the pension level.

Table 2
Decomposition analysis

| Year | Pension benefits | Taxable labor income |
|---|---|---|
| 2010 | 45.5% | 54.5% |
| 2015 | 75.0% | 25.0% |
| 2020 | 94.8% | 5.20% |
| 2035 | 98.3% | 1.70% |
| 2045 | 85.0 | 15.0% |
| 2050 | 80.7 | 19.3% |

Baseline experiment (selected years).

### 3.2. Experiment 2: low substitution consumption-leisure

In this experiment a more general utility function of the CES type is used. In particular the functional form is

$$U(c,\ l) = \frac{1}{1-\sigma}(\theta c^{1-\mu} + (1-\theta)l^{1-\mu})^{\frac{1-\sigma}{1-\mu}} \tag{6}$$

where $\mu$ is the inverse of the intratemporal elasticity of substitution. Notice, that in the previous experiment $\mu = 1$ i.e the Cobb Douglas case or unitary elasticity of intratemporal substitution. In contrast, now we used a lower value of this elasticity or equivalently $\mu = 2.5$. With this lower value of elasticity, the share parameter has to be recalibrated in order to match an average working time of 1/3. The value that satisfies this condition is now $\theta = 0.0065$. The rest of the parameters are the same as in the previous experiment. The results are shown in Table 3. In this case, individuals are not so much willing to substitute consumption for leisure, and consequently in the model economy with cohort size effects,

Table 3
Aggregate effects of demographic change

| *Model economy without cohort effects* $\rho = 0$ $B = 1$ $\gamma = 0.5$ $\sigma = 2$ $\mu = 2.5$ $\theta = 0.007$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Lifetime work effort by Age | | $\tau_{ss}$ | Pension/Y | Saving rate | K/Y | Wage premium |
| | 20–44 | 45–64 | | | | | |
| 1995 | 0.374 | 0.243 | 0.120 | 0.075 | 0.241 | 2.673 | 1.000 |
| 2005 | 0.381 | 0.249 | 0.126 | 0.079 | 0.253 | 2.564 | 1.000 |
| 2010 | 0.385 | 0.243 | 0.131 | 0.082 | 0.270 | 2.572 | 1.000 |
| 2020 | 0.385 | 0.242 | 0.153 | 0.096 | 0.255 | 2.661 | 1.000 |
| 2025 | 0.381 | 0.248 | 0.173 | 0.108 | 0.210 | 2.713 | 1.000 |
| 2030 | 0.377 | 0.255 | 0.202 | 0.126 | 0.159 | 2.759 | 1.000 |
| 2035 | 0.375 | 0.258 | 0.237 | 0.148 | 0.119 | 2.790 | 1.000 |
| 2040 | 0.375 | 0.258 | 0.274 | 0.171 | 0.091 | 2.804 | 1.000 |
| 2045 | 0.377 | 0.255 | 0.304 | 0.190 | 0.081 | 2.787 | 1.000 |
| 2050 | 0.380 | 0.252 | 0.308 | 0.193 | 0.097 | 2.694 | 1.000 |
| 2055 | 0.381 | 0.251 | 0.289 | 0.181 | 0.151 | 2.568 | 1.000 |
| *Model economy with cohort effects* $\rho = 1.2$ $B = 0.947$ $\gamma = 0.717$ $\sigma = 2$ $\mu = 2.5$ $\theta = 0.007$ | | | | | | | |
| 1995 | 0.374 | 0.243 | 0.120 | 0.075 | 0.241 | 2.673 | 1.000 |
| 2005 | 0.373 | 0.222 | 0.125 | 0.078 | 0.389 | 2.638 | 0.964 |
| 2010 | 0.375 | 0.220 | 0.129 | 0.080 | 0.405 | 2.742 | 0.850 |
| 2020 | 0.385 | 0.227 | 0.143 | 0.089 | 0.332 | 3.042 | 0.649 |
| 2025 | 0.386 | 0.234 | 0.156 | 0.097 | 0.237 | 3.167 | 0.597 |
| 2030 | 0.385 | 0.241 | 0.174 | 0.109 | 0.183 | 3.193 | 0.606 |
| 2035 | 0.385 | 0.244 | 0.197 | 0.123 | 0.184 | 3.155 | 0.662 |
| 2040 | 0.387 | 0.244 | 0.227 | 0.142 | 0.188 | 3.135 | 0.729 |
| 2045 | 0.390 | 0.240 | 0.260 | 0.163 | 0.177 | 3.134 | 0.771 |
| 2050 | 0.392 | 0.236 | 0.276 | 0.173 | 0.153 | 3.100 | 0.752 |
| 2055 | 0.391 | 0.234 | 0.271 | 0.169 | 0.162 | 3.022 | 0.707 |

Experiment 2: low substitution consumption-leisure.

individuals do not reallocate so heavily work effort from later ages to younger ones over the lifecycle with the expected fall in the experience premium. Instead, they save relatively less early in life. Hence in this economy individuals qualify for higher social security benefits, and then the differences between the model economy with and without cohort effects are less pronounced. In particular, in terms of social security tax rate, the maximum difference is now 5 percentage points as compare to previous case where this difference was almost 8 percentage points.

### 3.3. Experiment 3: Low substitution consumption-leisure and high substitution across experience groups

Now, in addition to considering a low substitution between consumption and leisure, we add the effect of a higher elasticity of substitution across experience groups. Then, in this case the experience premium does not fall so much as the relative share of old workers increases. In the model economy with cohort size effects of the previous

Table 4
Aggregate effects of demographic change

| *Model economy without cohort effects* $\rho=0$ $B=1$ $\gamma=0.5$ $\sigma=2$ $\mu=2.5$ $\theta=0.007$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Lifetime work effort by Age | | $\tau_{ss}$ | Pension/Y | Saving rate | K/Y | Wage premium |
| | 20–44 | 45–64 | | | | | |
| 1995 | 0.374 | 0.243 | 0.120 | 0.075 | 0.241 | 2.673 | 1.000 |
| 2005 | 0.381 | 0.249 | 0.126 | 0.079 | 0.253 | 2.564 | 1.000 |
| 2010 | 0.385 | 0.243 | 0.131 | 0.082 | 0.270 | 2.572 | 1.000 |
| 2020 | 0.385 | 0.242 | 0.153 | 0.096 | 0.255 | 2.661 | 1.000 |
| 2025 | 0.381 | 0.248 | 0.173 | 0.108 | 0.210 | 2.713 | 1.000 |
| 2030 | 0.377 | 0.255 | 0.202 | 0.126 | 0.159 | 2.759 | 1.000 |
| 2035 | 0.375 | 0.258 | 0.237 | 0.148 | 0.119 | 2.790 | 1.000 |
| 2040 | 0.375 | 0.258 | 0.274 | 0.171 | 0.091 | 2.804 | 1.000 |
| 2045 | 0.377 | 0.255 | 0.304 | 0.190 | 0.081 | 2.787 | 1.000 |
| 2050 | 0.380 | 0.252 | 0.308 | 0.193 | 0.097 | 2.694 | 1.000 |
| 2055 | 0.381 | 0.251 | 0.289 | 0.181 | 0.151 | 2.568 | 1.000 |
| *Model economy with cohort effects* $\rho=0.6$ $B=0.9681$ $\gamma=0.6157$ $\sigma=2$ $\mu=2.5$ $\theta=0.007$ | | | | | | | |
| 1995 | 0.374 | 0.243 | 0.120 | 0.075 | 0.241 | 2.673 | 1.000 |
| 2005 | 0.374 | 0.238 | 0.133 | 0.083 | 0.299 | 2.705 | 0.981 |
| 2010 | 0.378 | 0.235 | 0.136 | 0.085 | 0.334 | 2.739 | 0.915 |
| 2020 | 0.388 | 0.238 | 0.152 | 0.095 | 0.309 | 2.918 | 0.781 |
| 2025 | 0.389 | 0.245 | 0.167 | 0.104 | 0.240 | 3.005 | 0.739 |
| 2030 | 0.390 | 0.252 | 0.189 | 0.118 | 0.189 | 3.033 | 0.744 |
| 2035 | 0.391 | 0.256 | 0.217 | 0.135 | 0.172 | 3.020 | 0.785 |
| 2040 | 0.394 | 0.256 | 0.250 | 0.156 | 0.160 | 3.009 | 0.834 |
| 2045 | 0.397 | 0.252 | 0.283 | 0.177 | 0.142 | 2.999 | 0.862 |
| 2050 | 0.400 | 0.249 | 0.298 | 0.186 | 0.127 | 2.941 | 0.846 |
| 2055 | 0.399 | 0.246 | 0.288 | 0.180 | 0.152 | 2.843 | 0.813 |

Experiment 3: low substitution consumption-leisure and high substitution across experience groups.

experiments, the inverse of the elasticity of substitution across experience groups was $\rho = 1.2$. Now, consider a lower value of $\rho = 0.6$. Following the same procedure explained in the calibration, the parameters $\gamma$ and $B$ have to be recalibrated so that the model economy with and without cohort size effects display the same features in the initial steady state. The new values are $\gamma = 0.6157$ and $B = 0.9681$. The results reported in Table 4 show that when the elasticity of intratemporal substitution is low and the elasticity of substitution across experience groups is high, then in both model economies the change in the lifecycle profile of work effort and the social security tax rate is very similar. In particular, the maximum difference between both model economies in terms of the social security tax rate and the evolution of the percentage of GDP spent on pensions is just 2 percentage points. The reason is that the driving force operating in the model economy with cohort effects is less important (since the change in the experience premium is less pronounced) and the individuals' reaction to this change (through the reallocation of work effort) is also downsized.

## 4. Sensitivity analysis

I now examine whether the findings on the effects of introducing imperfect substitutability in an overlapping generations model are robust to alternative assumptions about the degree of substitution of workers with different levels of experience, the number of experience groups used to sort the working population, the way in which pensions are updated to productivity growth and finally the degree of intragenerational heterogeneity of the model.

### *4.1. Experiment 4: Three experience groups and $\rho = 0.6$*

In this case the working population is sorted into three groups. The first group $L_t$ contains those individuals with less than 14 years of experience, i.e. those aged between 20 and 34. The second group $M_t$ includes those individuals with more than 15 years of experience but less than 30. And the third one $H_t$ refers to those workers with more than 30 years of work experience. Given this way of partitioning the working population the elasticity of substitution across groups is taken to be higher than the value used when using two groups. The aggregate labor input is

$$N_t = B(\gamma_L L_t^{1-\rho} + \gamma_M M_t^{1-\rho} + (1 - \gamma_L - \gamma_M) H_t^{1-\rho})^{\frac{1}{1-\rho}}. \tag{7}$$

The procedure to calibrate this model economy follow the lines explained in Section 2. In particular, the perfect substitution model economy is characterized by $\rho = 0$, $\gamma_L = 0.3333$, $\gamma_M = 0.3333$ and $B = 1$. And in the model economy which accounts for the existence of cohort size effects the corresponding parameters are $\rho = 0.6$, $\gamma_L = 0.359$, $\gamma_M = 0.397$ and $B = 1$. The results reported in Table 5 show that both model economies perform differently in terms of tax rate needed to keep balanced the social security system.

Table 5
Aggregate effects of demographic change

| | Lifetime work effort by Age | | $\tau_{ss}$ | Pension/Y | Saving rate | K/Y | Wage premium |
|---|---|---|---|---|---|---|---|
| *Model economy without cohort effects* $\rho=0$ $B=1$ $\gamma_L=0.333$ $\gamma_M=0.333$ $\sigma=2$ $\mu=1$ $\theta=0.33$ | | | | | | | |
| | 20–44 | 45–64 | | | | | |
| 1995 | 0.377 | 0.256 | 0.117 | 0.073 | 0.227 | 2.522 | 1.000 |
| 2005 | 0.381 | 0.249 | 0.129 | 0.081 | 0.246 | 2.567 | 1.000 |
| 2010 | 0.385 | 0.243 | 0.134 | 0.083 | 0.266 | 2.571 | 1.000 |
| 2020 | 0.385 | 0.242 | 0.155 | 0.097 | 0.253 | 2.656 | 1.000 |
| 2025 | 0.381 | 0.248 | 0.174 | 0.109 | 0.209 | 2.707 | 1.000 |
| 2030 | 0.377 | 0.254 | 0.203 | 0.127 | 0.159 | 2.753 | 1.000 |
| 2035 | 0.374 | 0.258 | 0.238 | 0.148 | 0.120 | 2.784 | 1.000 |
| 2040 | 0.375 | 0.258 | 0.274 | 0.171 | 0.092 | 2.798 | 1.000 |
| 2045 | 0.377 | 0.255 | 0.305 | 0.191 | 0.080 | 2.782 | 1.000 |
| 2050 | 0.380 | 0.252 | 0.309 | 0.193 | 0.094 | 2.689 | 1.000 |
| 2055 | 0.381 | 0.250 | 0.290 | 0.181 | 0.149 | 2.562 | 1.000 |
| *Model economy with cohort effects* $\rho=0.6$ $B=0.9681$ $\gamma_L=0.3592$ $\gamma_M=0.3976$ $\sigma=2$ $\mu=1$ $\theta=0.33$ | | | | | | | |
| 1995 | 0.377 | 0.256 | 0.117 | 0.073 | 0.227 | 2.522 | 1.000 |
| 2005 | 0.380 | 0.243 | 0.126 | 0.079 | 0.311 | 2.570 | 0.917 |
| 2010 | 0.388 | 0.235 | 0.130 | 0.081 | 0.318 | 2.628 | 0.845 |
| 2020 | 0.396 | 0.231 | 0.147 | 0.092 | 0.257 | 2.819 | 0.762 |
| 2025 | 0.391 | 0.238 | 0.159 | 0.099 | 0.213 | 2.878 | 0.813 |
| 2030 | 0.384 | 0.246 | 0.176 | 0.110 | 0.173 | 2.918 | 0.891 |
| 2035 | 0.380 | 0.250 | 0.196 | 0.122 | 0.160 | 2.928 | 0.927 |
| 2040 | 0.382 | 0.250 | 0.220 | 0.138 | 0.159 | 2.929 | 0.893 |
| 2045 | 0.385 | 0.247 | 0.248 | 0.155 | 0.153 | 2.940 | 0.843 |
| 2050 | 0.387 | 0.245 | 0.260 | 0.163 | 0.146 | 2.905 | 0.819 |
| 2055 | 0.386 | 0.244 | 0.252 | 0.157 | 0.176 | 2.821 | 0.835 |

Experiment 4: three experience groups.

For instance in the model economy with cohort size effects the tax rate has to increase by 3.6 and around 6 percentage points less than the standard model in 2030 and 2045 respectively. Consequently, the results are robust to alternative ways of sorting the working population into experience groups.

### *4.2. Experiment 5: No substitution across experience groups*

For comparative purposes I have also considered the case of no substitutability across experience groups. This case has been approximated using an very high value of the inverse of the elasticity of substitution across the two experience groups. In particular, the value used is $\rho=15$ which corresponds to an elasticity of substitution of 0.066. The rest of parameters needed to make both model economies (perfect vs. imperfect substitution) comparable are $B=0.7$ and $\gamma=0.999997$. The results of this experiment are shown in Table 6. It is worth noting that in this case, in the model economy with (virtually) no substitution across experience groups, the social security tax rate would stay roughly constant over the next 15 years, and the aging of the population would

Table 6
Aggregate effects of demographic change

| | Lifetime work effort by Age | | $\tau_{ss}$ | Pension/Y | Saving rate | K/Y | Wage premium |
|---|---|---|---|---|---|---|---|
| | 20–44 | 45–64 | | | | | |
| *Model economy without cohort effects* $\rho=0$ $B=1$ $\gamma=0.5$ $\sigma=2$ $\mu=1$ $\theta=0.33$ | | | | | | | |
| 1995 | 0.377 | 0.250 | 0.118 | 0.071 | 0.227 | 2.523 | 1.000 |
| 2005 | 0.381 | 0.249 | 0.127 | 0.079 | 0.277 | 2.518 | 1.000 |
| 2010 | 0.385 | 0.243 | 0.131 | 0.082 | 0.287 | 2.539 | 1.000 |
| 2020 | 0.385 | 0.242 | 0.151 | 0.094 | 0.264 | 2.642 | 1.000 |
| 2025 | 0.381 | 0.248 | 0.171 | 0.107 | 0.217 | 2.700 | 1.000 |
| 2030 | 0.376 | 0.255 | 0.200 | 0.125 | 0.164 | 2.750 | 1.000 |
| 2035 | 0.374 | 0.258 | 0.236 | 0.147 | 0.121 | 2.786 | 1.000 |
| 2040 | 0.375 | 0.258 | 0.274 | 0.171 | 0.091 | 2.803 | 1.000 |
| 2045 | 0.377 | 0.255 | 0.305 | 0.191 | 0.078 | 2.787 | 1.000 |
| 2050 | 0.379 | 0.252 | 0.310 | 0.194 | 0.093 | 2.692 | 1.000 |
| 2055 | 0.380 | 0.251 | 0.290 | 0.181 | 0.148 | 2.564 | 1.000 |
| *Model economy with cohort effects* $\rho=15$ $B=0.7$ $\gamma=0.999997$ $\sigma=2$ $\mu=1$ $\theta=0.33$ | | | | | | | |
| 1995 | 0.377 | 0.250 | 0.118 | 0.071 | 0.227 | 2.523 | 1.000 |
| 2005 | 0.414 | 0.176 | 0.119 | 0.075 | 0.743 | 2.510 | 0.901 |
| 2010 | 0.419 | 0.177 | 0.118 | 0.074 | 0.539 | 2.921 | 0.755 |
| 2020 | 0.432 | 0.186 | 0.126 | 0.078 | 0.322 | 3.480 | 0.536 |
| 2025 | 0.432 | 0.188 | 0.131 | 0.082 | 0.185 | 3.662 | 0.486 |
| 2030 | 0.427 | 0.190 | 0.137 | 0.086 | 0.154 | 3.662 | 0.484 |
| 2035 | 0.421 | 0.192 | 0.141 | 0.088 | 0.228 | 3.547 | 0.520 |
| 2040 | 0.417 | 0.194 | 0.150 | 0.094 | 0.285 | 3.490 | 0.571 |
| 2045 | 0.417 | 0.197 | 0.164 | 0.103 | 0.300 | 3.511 | 0.612 |
| 2050 | 0.418 | 0.199 | 0.176 | 0.110 | 0.246 | 3.569 | 0.610 |
| 2055 | 0.417 | 0.202 | 0.179 | 0.112 | 0.209 | 3.579 | 0.580 |

Experiment 5: no substitution across experience groups.

require adjustments of just 6 percentage points by 2050 as compared to the 20 percentage points of adjustment needed to cope with the aging of the baby-boom generation in the standard model economy with perfect substitution.

### *4.3. Experiment 6: Fully indexed pensions to productivity growth*

Following the existing pension's rules of the Spanish social security system, the level of pensions were not indexed to the growth of productivity in the previous experiments. In order to check the sensitivity of the main results to this assumption, in the present experiment we update the level of pensions to productivity growth. The results are reported in Table 7. It is found that, if pensions are indexed to productivity, then the adjustment required in the social security budget is more pronounced both in the model economy with and without cohort size effects. This is so, because the pension expenditure is now higher than in the case of non-indexed pensions. However, the differences between the model economy with perfect and imperfect substitution across experience groups are similar (see Table 7) to the ones obtained in Experiment 1, since the mechanism that is at

Table 7
Aggregate effects of demographic change

| | Lifetime work effort by Age | | $\tau_{ss}$ | Pension/Y | Saving rate | K/Y | Wage premium |
|---|---|---|---|---|---|---|---|
| | 20–44 | 45–64 | | | | | |
| *Model economy without cohort effects* $\rho=0$ $B=1$ $\gamma=0.5$ $\sigma=2$ $\mu=1$ $\theta=0.33$ | | | | | | | |
| 1995 | 0.377 | 0.250 | 0.118 | 0.071 | 0.227 | 2.523 | 1.000 |
| 2005 | 0.385 | 0.239 | 0.135 | 0.085 | 0.245 | 2.535 | 1.000 |
| 2010 | 0.390 | 0.232 | 0.142 | 0.089 | 0.262 | 2.543 | 1.000 |
| 2020 | 0.390 | 0.230 | 0.164 | 0.102 | 0.250 | 2.626 | 1.000 |
| 2025 | 0.385 | 0.237 | 0.183 | 0.115 | 0.207 | 2.676 | 1.000 |
| 2030 | 0.380 | 0.245 | 0.213 | 0.133 | 0.158 | 2.721 | 1.000 |
| 2035 | 0.377 | 0.250 | 0.249 | 0.156 | 0.117 | 2.755 | 1.000 |
| 2040 | 0.378 | 0.250 | 0.289 | 0.181 | 0.088 | 2.773 | 1.000 |
| 2045 | 0.381 | 0.247 | 0.323 | 0.202 | 0.072 | 2.761 | 1.000 |
| 2050 | 0.384 | 0.242 | 0.331 | 0.207 | 0.081 | 2.670 | 1.000 |
| 2055 | 0.385 | 0.240 | 0.312 | 0.195 | 0.136 | 2.535 | 1.000 |
| *Model economy with cohort effects* $\rho=1.2$ $B=0.912$ $\gamma=0.717$ $\sigma=2$ $\mu=1$ $\theta=0.33$ | | | | | | | |
| 1995 | 0.377 | 0.250 | 0.118 | 0.071 | 0.227 | 2.523 | 1.000 |
| 2005 | 0.391 | 0.213 | 0.134 | 0.084 | 0.302 | 2.555 | 0.987 |
| 2010 | 0.398 | 0.209 | 0.139 | 0.087 | 0.313 | 2.610 | 0.905 |
| 2020 | 0.407 | 0.206 | 0.150 | 0.094 | 0.262 | 2.807 | 0.745 |
| 2025 | 0.403 | 0.210 | 0.160 | 0.100 | 0.192 | 2.899 | 0.698 |
| 2030 | 0.397 | 0.219 | 0.174 | 0.109 | 0.155 | 2.918 | 0.705 |
| 2035 | 0.392 | 0.226 | 0.194 | 0.121 | 0.158 | 2.891 | 0.755 |
| 2040 | 0.390 | 0.229 | 0.223 | 0.139 | 0.165 | 2.885 | 0.815 |
| 2045 | 0.391 | 0.227 | 0.258 | 0.161 | 0.161 | 2.907 | 0.852 |
| 2050 | 0.394 | 0.224 | 0.279 | 0.174 | 0.142 | 2.900 | 0.839 |
| 2055 | 0.395 | 0.221 | 0.276 | 0.172 | 0.156 | 2.838 | 0.802 |

Experiment 6: fully indexed pensions

the root of the differential behavior (the change in relative wages) is hardly affected by the indexation characteristics of social security benefits.

### *4.4. Experiment 8: Intragenerational heterogeneity*

In the previous experiments, we abstracted from some particular features of the Spanish social security system such as the existence of a minimum and maximum pension floors. This is a legitimate abstraction since in the model economies studied individuals of the same generation were assumed to be homogeneous. In order to check whether the results of the previous exercises are sensitive to neglecting these additional features of the pension system, in this section we repeat the same exercise but with enough heterogeneity among individuals of the same generation so as to be able to model the existence of individuals being affected by the above mentioned pension floors.

In order to generate enough heterogeneity across the individuals of the same generation, the age specific labor productivities are set following the procedure used by Huggett and

Table 8
Aggregate effects of demographic change

| *Model economy without cohort effects* $\rho=0$ $B=1$ $\gamma=0.5$ $\sigma=2$ $\mu=1$ $\theta=0.33$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Lifetime work effort by Age | | $\tau_{ss}$ | Pension/Y | Saving rate | K/Y | Wage premium |
| | 20–44 | 45–64 | | | | | |
| 1995 | 0.376 | 0.250 | 0.118 | 0.074 | 0.224 | 2.493 | 1.000 |
| 2005 | 0.380 | 0.248 | 0.130 | 0.081 | 0.269 | 2.530 | 1.000 |
| 2010 | 0.384 | 0.242 | 0.134 | 0.084 | 0.281 | 2.548 | 1.000 |
| 2020 | 0.384 | 0.240 | 0.155 | 0.097 | 0.260 | 2.647 | 1.000 |
| 2025 | 0.380 | 0.246 | 0.174 | 0.109 | 0.215 | 2.703 | 1.000 |
| 2030 | 0.376 | 0.253 | 0.202 | 0.126 | 0.162 | 2.751 | 1.000 |
| 2035 | 0.373 | 0.257 | 0.237 | 0.148 | 0.122 | 2.785 | 1.000 |
| 2040 | 0.373 | 0.257 | 0.274 | 0.172 | 0.093 | 2.803 | 1.000 |
| 2045 | 0.376 | 0.254 | 0.306 | 0.191 | 0.081 | 2.789 | 1.000 |
| 2050 | 0.378 | 0.251 | 0.311 | 0.195 | 0.096 | 2.697 | 1.000 |
| 2055 | 0.379 | 0.250 | 0.293 | 0.183 | 0.150 | 2.571 | 1.000 |
| *Model economy with cohort effects* $\rho=1.2$ $B=0.912$ $\gamma=0.717$ $\sigma=2$ $\mu=1$ $\theta=0.33$ | | | | | | | |
| 1995 | 0.376 | 0.250 | 0.118 | 0.074 | 0.224 | 2.493 | 1.000 |
| 2005 | 0.390 | 0.217 | 0.127 | 0.080 | 0.329 | 2.550 | 0.971 |
| 2010 | 0.397 | 0.213 | 0.130 | 0.081 | 0.336 | 2.618 | 0.892 |
| 2020 | 0.406 | 0.212 | 0.142 | 0.089 | 0.276 | 2.840 | 0.734 |
| 2025 | 0.402 | 0.217 | 0.153 | 0.096 | 0.201 | 2.944 | 0.685 |
| 2030 | 0.396 | 0.224 | 0.169 | 0.106 | 0.160 | 2.970 | 0.691 |
| 2035 | 0.391 | 0.230 | 0.190 | 0.119 | 0.162 | 2.946 | 0.739 |
| 2040 | 0.389 | 0.232 | 0.217 | 0.136 | 0.168 | 2.939 | 0.798 |
| 2045 | 0.391 | 0.230 | 0.248 | 0.155 | 0.166 | 2.954 | 0.835 |
| 2050 | 0.393 | 0.227 | 0.263 | 0.165 | 0.153 | 2.939 | 0.819 |
| 2055 | 0.394 | 0.227 | 0.256 | 0.160 | 0.169 | 2.875 | 0.782 |

Experiment 7: intragenerational heterogeneity.

Ventura (1999) and is as follows. I first use the age specific profile of efficiency units computed from the European Community Household Panel (1994) used in the previous exercises and compute the logarithm of this profile denoted by $\hat{y}_i$ for workers aged between 20 and 64. Then, it is assumed that upon birth an agent faces a permanent individual shock $z$ to its log efficiency which determines its working productivity over its career. This shock is normally distributed as $z \sim N(0, \sigma_z^2)$ and the log efficiency parameter at age 1 is $y_1 = \hat{y}_1 + z$. Then an agent's log lifetime efficiency profile evolves according to $y_i - \hat{y}_i = y_{i-1} - \hat{y}_{i-1}$. Finally the efficiency profile is $e_{i,j} = \exp(\hat{y}_i + z)$. For computational purposes I follow Huggett and Ventura (1998) and approximate the shock process $z$ with 21 evenly-space values between $-4\sigma_z$ and $4\sigma_z$. The probabilities are calculated by integrating the area under the normal distribution and the standard deviation of the stochastic process $\sigma_z$ is set to 0.532 so that the Gini index of the distribution of gross hourly wages of the model economy in the initial steady state matches that of the ECHP data, being equal to 0.31. Then, in this model economy the way of computing pensions is the same as in the previous experiments except for the fact that we add the existence of a maximum and minimum pension limits equal to 1.85 and 0.44 times the per-capita output respectively, which corresponds to their empirical counterparts in the Spanish economy in 1995. Hence in age $I_R$ benefits are given by,

$$d_{I_R,t,j} = max(P_{min},\ min(P_{max},\ \frac{rep}{1+\lambda} w_{av,j}))$$

Given the heterogeneity introduced in the model economies, in the initial steady steady the percentage of retirees receiving the minimum and maximum pension limit is 27.43% and 1.39% respectively. Their empirical counterparts are 23% and 0.015%, consequently the model economies considered slightly overestimates the number of retirees affected by such limits. The aggregate effects of aging in this case are shown in Table 8. For illustrative purposes Table 8 reports only the average lifecycle profile of work effort for those individuals endowed with the median ability level. Recall that in this experiment there are 21 types in each generation, then the median ability corresponds to ability 11. Notice that as in the previous experiments, the fall in the experience premium brings about a reallocation of work effort towards early ages of the lifecycle, which is one of the reasons that explains the lower pension burden in the model economy with cohort size effects. The other features are pretty much similar to the ones reported in other experiments.

## 5. Concluding remarks

This paper has extended the standard large overlapping generations model to allow for the interaction between changes in the age composition of the workforce and the shape of life-cycle earnings. We have found that the effect of aging on the sustainability of the social security system depends critically on the degree of substitution of labor at different ages. In particular, for the empirically plausible parameter space used in this paper we find that the adjustment of the tax rate needed to left untouched the pension system in Spain is less severe in a model that accounts for the existence of cohort size effects than in a model that it does not. However, despite the potential implications of the present paper for the political discussion about the desirability and the magnitude of the reform of the existing pay-as-you-go systems in developed countries, one should bear in mind that in the present analysis it is crucial to have a good estimation of the degree of substitution between young and old workers. And for this to be the case, more empirical research is needed for countries other than the US.


## Acknowledgements

Financial support from a DGESIC grant PB98-1058-C03 is gratefully acknowledged. I have received very helpful comments from two anonymous referees, my supervisor at the European University Institute Ramon Marimon, M. Boldrin, L. Bovemberg, Javier Diaz, J.J. Dolado, O. Licandro, J.F. Jimeno, Carlos Urrutia and seminar participants at Universidad Carlos III de Madrid, the OCFEB (Erasmus University of Rotterdam), FEDEA and the Universidade Nova de Lisboa. All errors are my own.

## Appendix A. The Equilibrium

In this economy a *Competitive Equilibrium* is a list of sequences of quantities $c_{i,t}$, $h_{i,t}$, $a_{i,t}$, $\mu_{i,t}$, $d_{i,t}$, $L_t$, $N_t$, $K_t$, prices $w_{l,t}$, $w_{h,t}$, $r_t$, social security tax rates $\tau_{ss,\,t}$ and an income tax rates such that, at each point in time $t$:

1. firms maximize profits setting wages and the interest rate equal to marginal products,

$$w_{l,t} = F_L(K_t,\ L_t,\ H_t) \tag{8}$$

$$w_{h,t} = F_H(K_t,\ L_t,\ H_t) \tag{9}$$

$$r_t = F_K(K_t,\ L_t,\ H_t) - \delta \tag{10}$$

2. agents maximize lifetime utility subject to the period budget constraints taking wages, the interest rate, taxes, transfers, survival probabilities and the age structure of the population as given,
3. the age structure of the population $\{\mu_{i,\,t}\}$ is generated by the following aggregate law of motion given initial conditions $\mu_{i,\,0}$,

$$\mu_{i+1,t+1} = \frac{s_{i,t}\mu_{i,t}}{1+n_t} + p_{i+1,t+1} \tag{11}$$

where $n_t$ is the population growth rate and $p_{i+1,t+1}$ is the age specific immigration rate. Finally, the next period share of newly born agents $\mu_{1,t+1}$ is given by

$$\mu_{1,t+1} = 1 - \sum_{i=2}^{I} \mu_{i,t+1}. \tag{12}$$

4. Market clearing conditions for capital and each type of labor,

$$K_t = \sum_{i=I_A}^{I} \mu_{i,t} a_{i,t} \tag{13}$$

$$H_t = \sum_{i=I_E}^{I_R-1} \mu_{i,t} e_{i,t} h_{i,t} \tag{14}$$

$$L_t = \sum_{i=I_A}^{I_{E-1}} \mu_{i,t} e_{i,t} h_{i,t} \tag{15}$$

where $I_E$ denotes the age at which an individual starts being considered as an experienced worker.

5. Finally, the budget constraint of the government is satisfied period by period.

Hence with these conditions the goods market clears every period,

$$F(K_t,\ L_t,\ H_t) + (1-\delta)K_t = K_{t+1} + G_t + \sum_i \mu_{i,\ t} c_{i,\ t}. \tag{16}$$

## References


Auerbach, A., Kotlikoff, L., 1987. Dynamic Fiscal Policy. Cambridge University Press, Cambridge.

Berger, M., 1985. The effect of cohort size on earnings growth: a reexamination of the evidence. Journal of Political Economy 93, 561–573.

Boldrin, M., Dolado, J.J., Jimeno, J.F., Peracchi, F., 1999. The future of pensions in Europe: a reappraisal. Economic Policy 29, 289–322.

Bosca, J.E., Fernandez, M., Taguas, D., 1999. Estructura impositiva en los paises de la OCDE. Ministerio de Economia y Hacienda, Madrid. Unpublished Manuscript.

Conesa, J.C., Garriga, C., 2003. Status quo problem in social security reforms. Macroeconomic Dynamics 7, 691–710.

De Nardi, M., Imrohoroglu, S., Sargent, T., 1999. Projected U.S. demographics and social security. Review of Economic Dynamics 2 (3), 575–615.

Domenech, R., Taguas, D., 1995. Potential ouput estimates for the Spanish economy. Ministerio de Economia y Hacienda, Madrid. Unpublished Manuscript.

Eguia, B., Echevarria, C.A., 2004. Unemployment rates and population changes in Spain. Journal of Applied Economics XII (1), 47–76.

Freeman, R., 1979. The effect of demographic factors on age-earnings profiles. Journal of Human Resources 14, 289–318.

Huang, Imrohoroglu, and Sargent, (1997). "Two Computations to Fund Social Security", *Macroeconomic Dynamics* 1, 7–44.

Huggett, M., Ventura, G., 1999. On the distributional effects of social security reform. Review of Economic Dynamics 2 (3), 498–531.

Imrohoroglu, S., Jones, D., 1995. A life cycle analysis of social security. Economic Theory 6, 83–114.

Katz, L., Murphy, K., 1992. Changes in relative wages, 1963–1987: supply and demand factors. The Quarterly Journal of Economics CVII, 35–78.

Kremer, M., Thomson, J., 1998. Why isn't convergence instantaneous? Young workers, old workers, and gradual adjustment. Journal of Economic Growth 3 (1), 5–28.

Lam, D., 1989. Population growth, age structure and age-specific productivity. Journal of Population Economics 2, 189–210.

Montero, M., 2000. Estructura demografica y sistemas de pensiones. Un analisis de equilibrio general aplicado a la economia espanola. Investigaciones Economicas 24 (2), 297–327.

Murphy, K., Welch, F., 1992. The Structure of Wages. The Quarterly Journal of Economics CVII, 285–326.

Puch, L., Licandro, O., 1997. Are there any special features in the Spanish business cycle? Investigaciones Economicas XXI (2), 361–394.

Welch, F., 1979. Effects of cohort size on earnings: the baby boom babies financial bust. Journal of Political Economy 87, S65–S97.