Documentation to accompany program *Public Sector 1*

This program calculates the taxes and expenditures of the public programs so that budget is balanced between revenues (taxes) and expenses (financing public programs, paying interest on public debt, and financing the public asset increase).

Programs:

1) Main program is PublicSector1.m and it is run by typing PublicSector1 in the Matlab Command Window.

2) All the parameters are specified in the file PublicSector_INPUT.m. Please edit this file from the Matlab Editor window and set the values of the parameters and the name of the data files before you run the main program. This file will be called from the main program.

3) time_index.m This function simply enables calculations for data that is available in steps bigger than single year.

4) plotPublicSector.m This is a combo file that contains sections to print different sets of variables. Copy desired section and paste it into Command window after running the main program to obtain sample graphs.

Parameters:
In this program we set several parameters:

1) λ - rate of technological growth
2) g_y - growth rate of labor income index $\overline{y}_l$; equal to λ
3) δ - depreciation rate
4) r_{debt} - debt interest rate
5) τ_{y_l} - labor income tax-rate in base year
6) τ_{y_A} - asset income tax-rate in base year
7) τ_C - private-consumption tax-rate in base year
8) $capital\_to\_GDP$ - ratio of public capital and GDP
9) $debt\_to\_GDP$ - ratio of public debt and GDP

Variables:

Labor Income	Y_l
Asset Income	Y_A
Private Consumption	C
GDP	GDP
Public Debt	DtG
Public Capital	KG
Public Assets	AG
Public Expenditures	$Expenditures$

Data files needed:

1) Per capita program expenditures for each of the programs (for instance, health, pension, education, long term care, other).
2) Population per age group for the simulation period
3) Private consumption profile in base year
4) Labor income profile in base year
5) Asset income profile in base year

Brief background

Value of GDP in the base year is calculated based on the total labor income in that year as

$$GDP(t_0) = \frac{3}{2} Y_l(t_0) \qquad (1.1)$$

Value of the public debt $DtG(t)$ in base year is set to -1.76 of the value of GDP in that year.

$$debt\_to\_GDP = -1.76$$
$$DtG(t_0) = debt\_to\_GDP \cdot GDP(t_0)$$

This ratio is kept constant throughout the simulation

$$DtG(t) = debt\_to\_GDP \cdot GDP(t) \qquad (1.2)$$

Similarly, the value of public capital is kept proportional to GDP.

$$capital\_to\_GDP = 0.5$$
$$KG(t) = capital\_to\_GDP \cdot GDP(t) \qquad (1.3)$$

Public assets are calculated from the equation (1.4) as

$$AG(t) = KG(t) + DtG(t) \quad (1.4)$$

Thus, public capital, public assets, and public debt are all constant relative to GDP.

We also impose budget constraint that equalizes budget revenues and expenses. Budget revenue comes from the taxes on labor income, asset income, and private consumption. Expenses are accrued from funding public programs, paying interest on public debt, and investing in public assets. The budget equation is

$$AG(t+1) = AG(t) + r_{debt} DtG(t) + Taxes(t) - Expenditures(t) \quad (1.5)$$

Total current program expenditures are equal to the sum of expenditures for all public programs.

$$Expenditures(t) = \sum_{program} Expenditures(t, program)$$

In this version, the following programs are distinguished: education, health, pension, long-term care, and other (as a joint category covering all other current program expenses).

Per capita expenditures in the base year for each program are available from the NTA project. Total spending for each program is obtained as a product of per capita expenditures and the population of the age group

$$Expenditures(t, program) = \gamma(t) \sum_{age=0}^{100} Expenditures(t, program, age) \cdot population(t, age)$$

where $\gamma(t)$ is endogenously determined to satisfy the budget constraint for the public sector.

Taxes are calculated based on the tax rates of each component (labor income, asset income, and private consumption).

$$TAX_{Y_l}(t) = \tau_{Y_l}(t) \cdot Y_l(t)$$
$$TAX_{Y_A}(t) = \tau_{Y_A}(t) \cdot Y_A(t)$$
$$TAX_C(t) = \tau_C(t) \cdot C(t)$$

$$Taxes(t) = \gamma(t)\left(TAX_C(t) + TAX_{Y_l}(t) + TAX_{Y_A}(t)\right)$$

Satisfying budget constraint in base year

Tax rates in the base year are inputs to the program. The amount of the tax collected should be balanced by the required program expenditures, asset growth, and payment of interest on public debt in the base year according to equation (1.5). If the planned taxes do not cover all the expenses, each tax-rate is increased proportionally to satisfy (1.5).

If the planned taxes exceed the planned spending, the tax-rates are decreased proportionally to satisfy (1.5).

Note that in the base year only taxes are corrected to satisfy balanced budget constraint. In the projections, half of the budget difference is financed by correcting (increasing or reducing) tax-rates, and half by correcting (reducing or increasing) planned expenditures.

Iterative projection

Labor income index y_l is increasing with the rate of technological growth, $g_y = \lambda$

$$y_l(t) = y_l(t-1)*(1+g_y)$$

Asset income is computed so that the ratio of labor income and asset income is constant and equal to the ratio in the base year.

$$\frac{Y_a(t)}{Y_l(t)} = \frac{Y_a(t_0)}{Y_l(t_0)}$$

The ratio of consumption to labor income is also assumed to be constant and equal to the ratio in the base year.

$$\frac{C(t)}{Y_l(t)} = \frac{C(t_0)}{Y_l(t_0)}$$

GDP is increasing with the GDP growth rate, which is equal to the product of technological growth and the labor index growth.

$$GDP(t+1) = (1+g_{GDP}(t))\cdot GDP(t)$$

$$1+g_{GDP}(t) = (1+g_n(t))*(1+g_y(t))$$

Satisfying budget constraint in future projections

Planned tax-rates in year t are equal to the tax-rates in the previous year. Planned per-capita program expenditures increase by the rate of technological growth compared to the previous year.

$$Expenditures(t+1, program) = Expenditures(t, program)\cdot(1+\lambda)$$

If the total taxes based on planned tax-rates do not cover all the expenses, the constraint of (1.5) is satisfied by covering half of the deficit through increasing tax-rates and the other half of the deficit by reducing per-capita expenditures for each program.

If the planned taxes exceed the planned spending, half of the difference is used to reduce the tax-rates are the other half to increase per-capita expenditures of the programs.

Note that in the base year only taxes are corrected to satisfy balanced budget constraint. In the projections, half of the budget difference is financed by correcting (increasing or reducing) tax-rates, and half by correcting (increasing or reducing) planned expenditures.

Public capital also satisfies the equation (1.6)

$$PublicCapital(t) = (1-\delta)*PublicCapital(t-1) + GDP(t-1)*i(t-1) \qquad (1.6)$$

where $i(t)$ is capital investment rate. This rate is calculated in each year from equation (1.6).