Some Economic Impacts of Changing Population Age Distributions

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Abstract

Starting from a standard growth model and an arbitrary initial population age distribution, I consider the consequences of an arbitrary but small perturbation of this initial age distribution. This perturbation might be in the direction of population aging or the dividend phase, or it could reflect comparative steady state analysis.

Effects arise through the age profiles of labor income, consumption, asset holding, and saving, which can be taken from NTA. These effects can be assessed for different outcomes of interest, including per capita income growth, factor price ratios, and consumption per effective consumer (for which the outcome is very similar to the Generalized Support Ratio).

Uses of NTA

- NTA are descriptive accounts with many uses
- Reveal rich patterns of many kinds, otherwise invisible
  - Gender accounts showing conditions of work and gender equity
  - Generational accounts showing intergenerational equity
  - Age accounts showing patterns of earning, consuming and sharing
  - Family transfer accounts showing altruistic linkages
  - Public sector accounts of many kinds
- Also valuable for observing and monitoring progress and change
- Point to particular features of countries or regions

Often we want to go further and use NTA to shed light on dynamic issues

- Without additional assumptions and modeling, accounting identities cannot tell us what would happen if any ingredient were to change
- But the need for dynamic insights is great, so take various approaches
- The most basic approaches stay very close to NTA age profiles and make only minimal behavioral assumptions
  - Support ratios, demographic dividend calculations
  - Classic fiscal or other projections using NTA profiles and population projections
- Others approaches rely more heavily on assumptions about behavior and macroeconomics
  - General equilibrium models from Spanish team

- Some approaches are situated between the two, using limited behavioral assumptions and economic feedbacks, but staying close to the detailed NTA age profiles
    - Andy Mason's fiscal projections with interactions between public and private transfers
- My approach here is somewhere in this intermediate range
- My goal is a simple measure that builds on the Support Ratio and the General Support Ratio, but is more closely rooted in a standard economic model
- I apply here to US, but it should be equally applicable to developing countries in the dividend phase; an alternative measure of DD

The support ratio

Usual definition, as given in the NTA Manual, is the ratio of effective workers to effective consumers, L/N.

I want to build on this in three ways:

1. If labor, L, grows more rapidly, then capital per worker may fall, reducing productivity growth
2. If share of elderly increases, won't they bring more capital and boost productivity?
3. If older populations bring more capital, won't that raise wages and reduce the rate of interest?

From UN Manual, p.10

$$\frac{C}{N} = \frac{(1-s)Y}{L}\frac{L}{N}$$

$$gr\left[\frac{C}{N}\right] = gr\left[\frac{(1-s)Y}{L}\right] + gr\left[\frac{L}{N}\right]$$

Growth rate of consumption per effective consumer = Growth rate of productivity + Growth rate of support ratio

My approach: The economy

- In a closed economy with no technological progress

$$Y = F\left(L, K\right)$$

- At baseline, each variable is the sum of population age distribution times an NTA age profile: here, K, and L. Later also C, and s
- Assume that as we move away from baseline the shapes of the age profiles remain the same but their level may change
- Example:
  - $y_l(x)$ is labor earnings at age x at baseline as usual in NTA
  - Let $\tilde{y}_l\left(x\right)$ be the age shape of the profile, indicating relative efficiency at age x (time, effort, ability) by age. But not in monetary units
  - Let w be the wage earned by one unit of $\tilde{y}_l$. It is the "level" of earnings
  - w will be determined in the "model" and

$$y_l\left(x\right) = w\,\tilde{y}_l\left(x\right)$$

Building up the macro variables

- Total amount of labor *in efficiency units* is called L:

  $L = \int_0^{\omega} \tilde{y}_l(x) P(x) dx$

- Total labor income is Y_l

  $Y_l = wL$

- Similarly, k(x) is amount of capital owned at age x
- NTA observes asset income, the flow, by age. I assume $y_A(x) = \tilde{r}k(x)$, where I assume $\tilde{r}$ is .05. That is, $k(x) = y_A(x)/.05$
- Total amount of capital in economy, K is:

  $$K = \int_0^{\omega} k(x) P(x) dx$$

- Thereafter, r is determined in the "model" and $y_K(x) = r\,\tilde{y}_K(x)$

The population age distribution

- Initial population at baseline is P(x)
- Suppose it changes at each age by amount $\delta u\left(x\right)$
  - δ describes the size of the change
  - $u\left(x\right)$ describes the direction or age distribution of the change, + or -.
  - The new population is given by:

    $$P\left(x,\delta\right)=P\left(x\right)+\delta u\left(x\right)$$

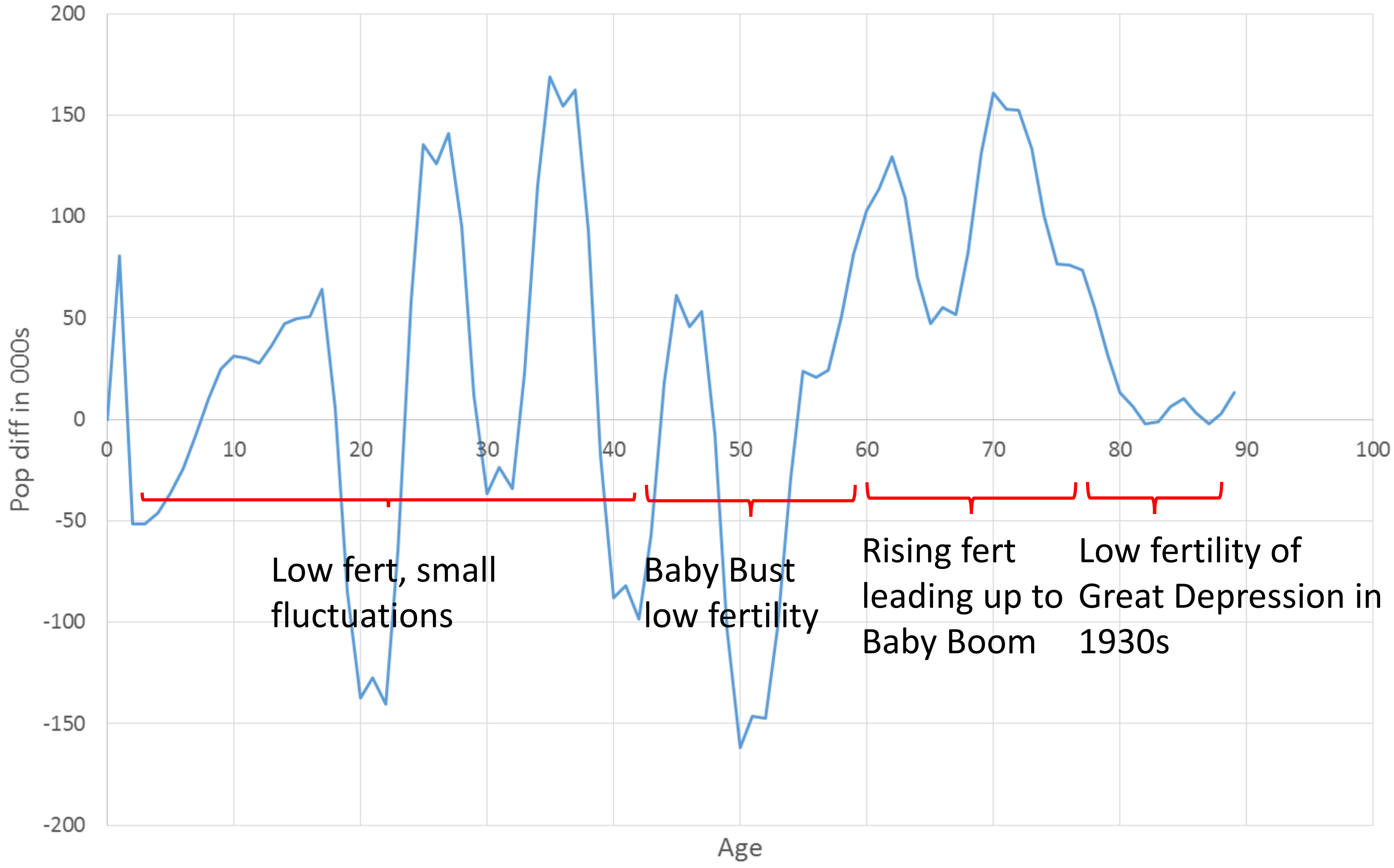
- Now we can describe changes in age distribution by the single parameter δ, which will be useful

Example: Let u(x) be a one-year change in population by age, from 2015 to 2016

$$u(x) = Pop_{2016}(x) - Pop_{2015}(x)$$

- Consider $P\left(x,\delta\right) = P_{2015}\left(x\right) + \delta u\left(x\right)$
- When $\delta = 0$, we get $P\left(x,0\right) = Pop_{2015}\left(x\right)$
- When $\delta = 1$ we get $P\left(x,1\right) = Pop_{2016}\left(x\right)$
- When δ is between 0 and 1, the age distribution is interpolated between 2015 and 2016

Example for u(x) = Pop(2016)-Pop(2015) in US (United Nations data)



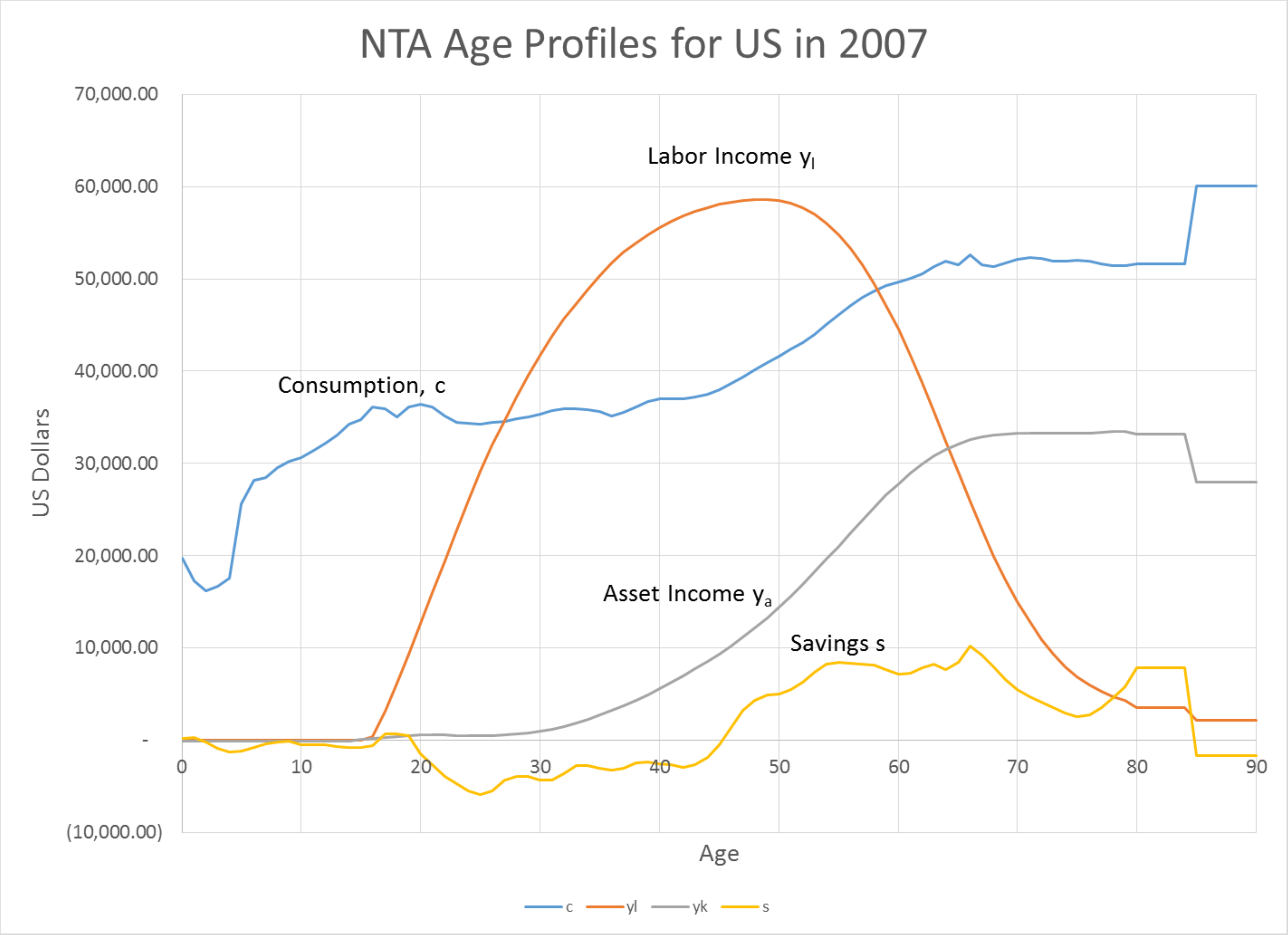
Evaluating *L'* and *K'*

- Differentiate L with respect to amount of change in pop age distr in direction u(x); denote this derivative with ‘:

$$\frac{dL}{d\delta} = L' = d\int_0^{\omega}\left[P(x)+\delta u(x)\right]\tilde{y}_L(x)\Big/ d\delta$$

$$L' = \int_0^{\omega} u(x)\,\tilde{y}_L(x)\,dx$$

- This derivative is easily evaluated based on
  - The NTA labor income age profile, and
  - The definition that has been chosen for u(x) for this analysis, such as single year change
- Similar for K, C, s, and any other variable of interest


NTA Age Profiles for US in 2007
70,000.00
60,000.00
50,000.00
40,000.00
30,000.00
20,000.00
10,000.00
-
(10,000.00)
US Dollars
0
10
20
30
40
50
60
70
80
90
Age
Labor Income y_l
Consumption, c
Asset Income y_a
Savings s
c
yl
yk
s

How does a change in pop age distribution affect total output, Y?

- Differentiate Y with respect to δ

$$Y = F\left(L, K\right)$$

$$\frac{dY}{d\delta} = F_L \frac{dL}{d\delta} + F_K \frac{dK}{d\delta}$$

$$Y' = F_L L' + F_K K'$$

- F_L and F_K are the wage rate, w, and rate of return to capital, r (different than $\tilde{r}$)
- Assume Cobb-Douglas production function with constant returns to scale: $Y = L^{\alpha} K^{1-\alpha}$

Under Cobb-Douglas

$$Y' = \frac{\alpha Y}{L} L' + \frac{(1-\alpha)Y}{K} K'$$

$$\frac{Y'}{Y} = \alpha \frac{L'}{L} + (1-\alpha)\frac{K'}{K}$$

- The changing age distribution affects both labor and capital
- The proportional change in output, Y, is the weighted sum of the proportional changes in labor and capital
- Typically α is around 2/3; can estimate within NTA as Y_l/Y

Does per capita income go up or down?

- The proportional population growth due to changes u(x) is:

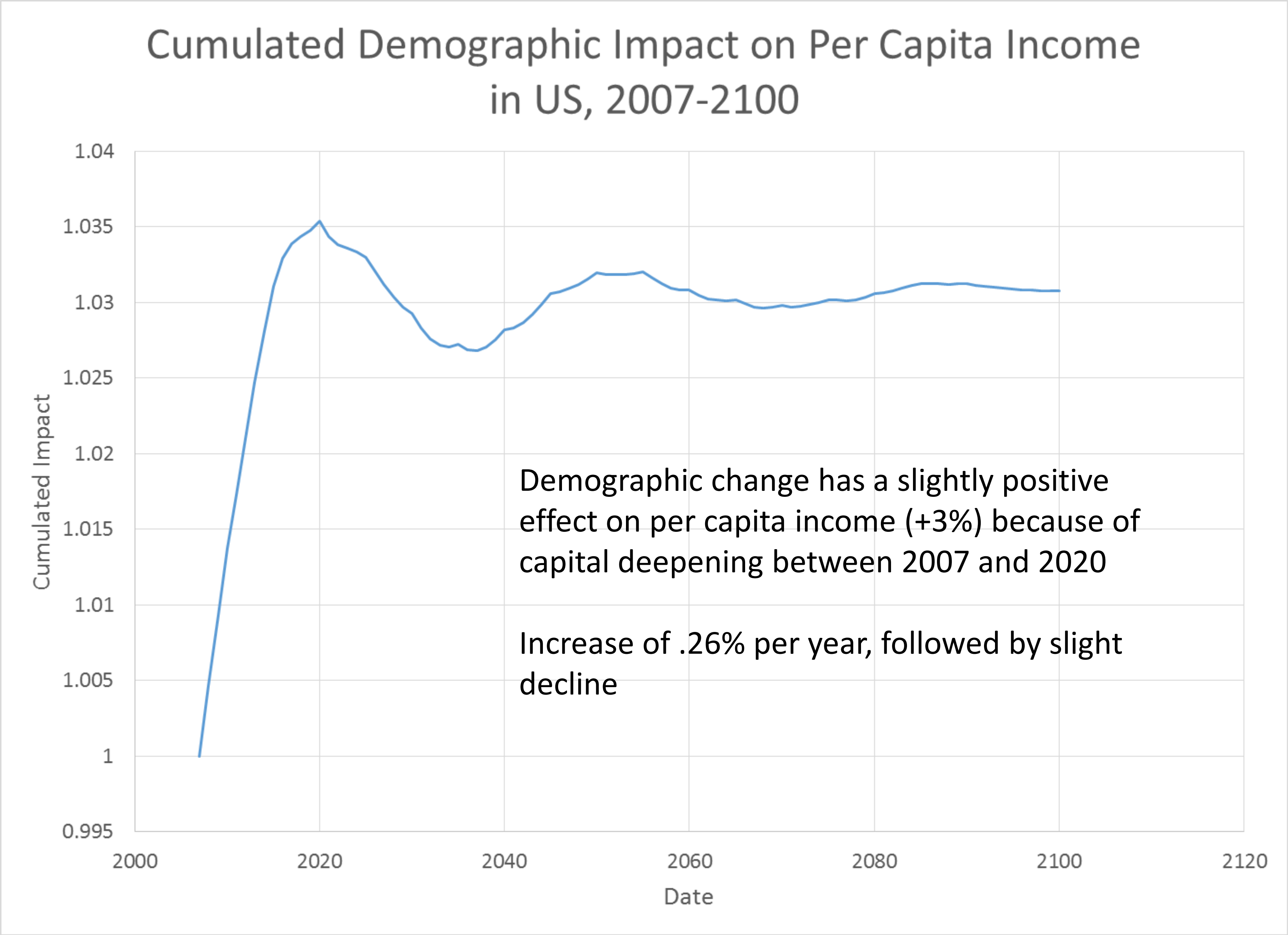
$$P'/P = \int_0^{\omega} u(x)\,dx \Big/ P$$

- The change in per capita income is given by:

$$y'/y = Y'/Y - P'/P$$

- If u(x) is concentrated in childhood, then there will be little or no effect on Y while P will rise considerably, so per capita income will fall
- If u(x) is concentrated in old age there will be little impact on Y through labor, but the increased capital holdings the elderly own will still boost Y somewhat, softening the negative impact on y

Can continue recursively beyond the first year, as we do with the support ratio


Cumulated Demographic Impact on Per Capita Income in US, 2007-2100
1.04
1.035
1.03
1.025
1.02
1.015
1.01
1.005
1
0.995
Cumulated Impact
2000
2020
2040
2060
2080
2100
2120
Date
Demographic change has a slightly positive effect on per capita income (+3%) because of capital deepening between 2007 and 2020
Increase of .26% per year, followed by slight decline

How does changing age distribution affect the wage and interest rate?

- Wage and interest rates are:

$$w = F_L = \alpha Y/L$$

$$r = F_K = (1-\alpha)Y/K$$

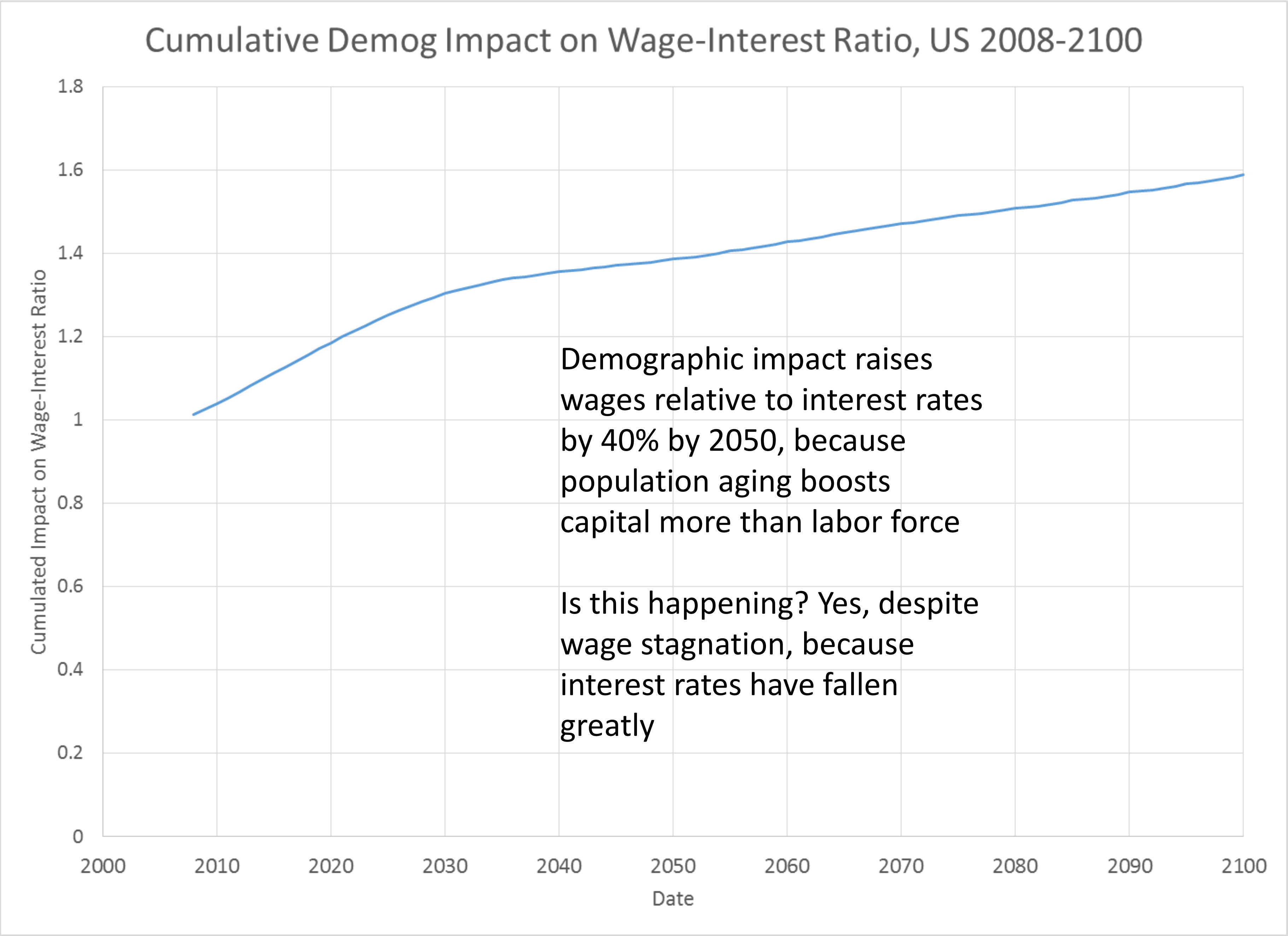
- Consider their ratio, w/r:

$$\frac{w}{r} = \frac{\alpha Y/L}{(1-\alpha)Y/K}$$

- Differentiate its log

$$d\ln\left(\frac{w}{r}\right)\Big/ d\delta = K'/K - L'/L$$

- When changing population age distribution makes labor grow more rapidly than capital, then the wage falls relative to the interest rate


Cumulative Demog Impact on Wage-Interest Ratio, US 2008-2100
Cumulated Impact on Wage-Interest Ratio
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
2000
2010
2020
2030
2040
2050
2060
2070
2080
2090
2100
Date
Demographic impact raises wages relative to interest rates by 40% by 2050, because population aging boosts capital more than labor force
Is this happening? Yes, despite wage stagnation, because interest rates have fallen greatly

Now consider aggregate consumption

- Given age specific saving rates we can build up aggregate saving, s
  $C = (1-s)Y$
- Changing population age distribution will change s as well as Y

$$C' = -s'Y + (1-s)Y'$$

$$C' = -s'Y + (1-s)\frac{\alpha Y}{L}L' + \frac{(1-\alpha)Y}{K}K'$$

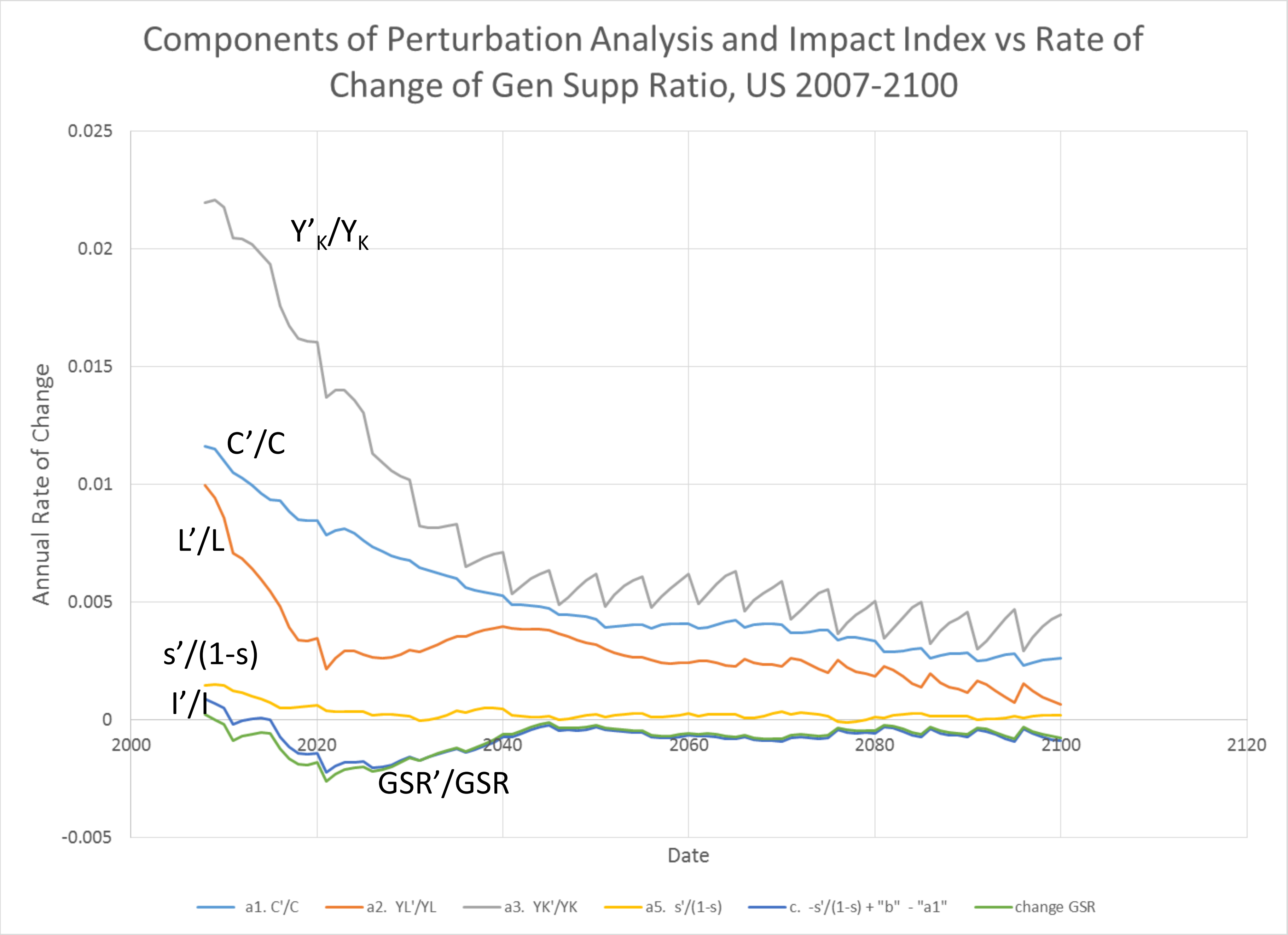
$$\frac{C'}{C} = -\frac{s'}{1-s} + \alpha\frac{L'}{L} + (1-\alpha)\frac{K'}{K}$$

$$\frac{C'}{C} - \frac{Y'}{Y} = -\frac{s'}{1-s}$$

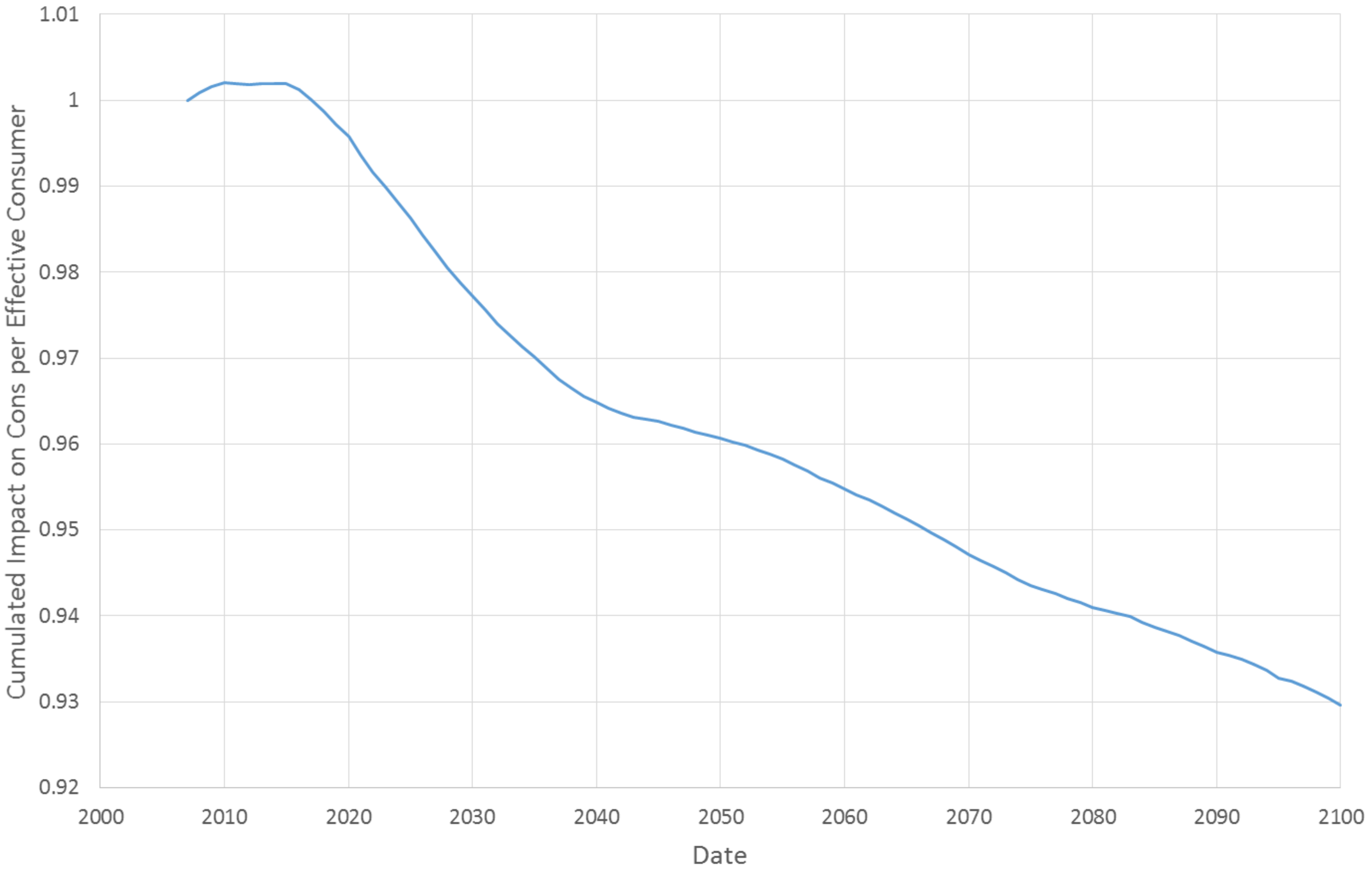
- Aggregate consumption will rise ***relative to Y*** (and the saving rate will fall) if s' is negative. If s' is positive then consumption will fall relative to Y

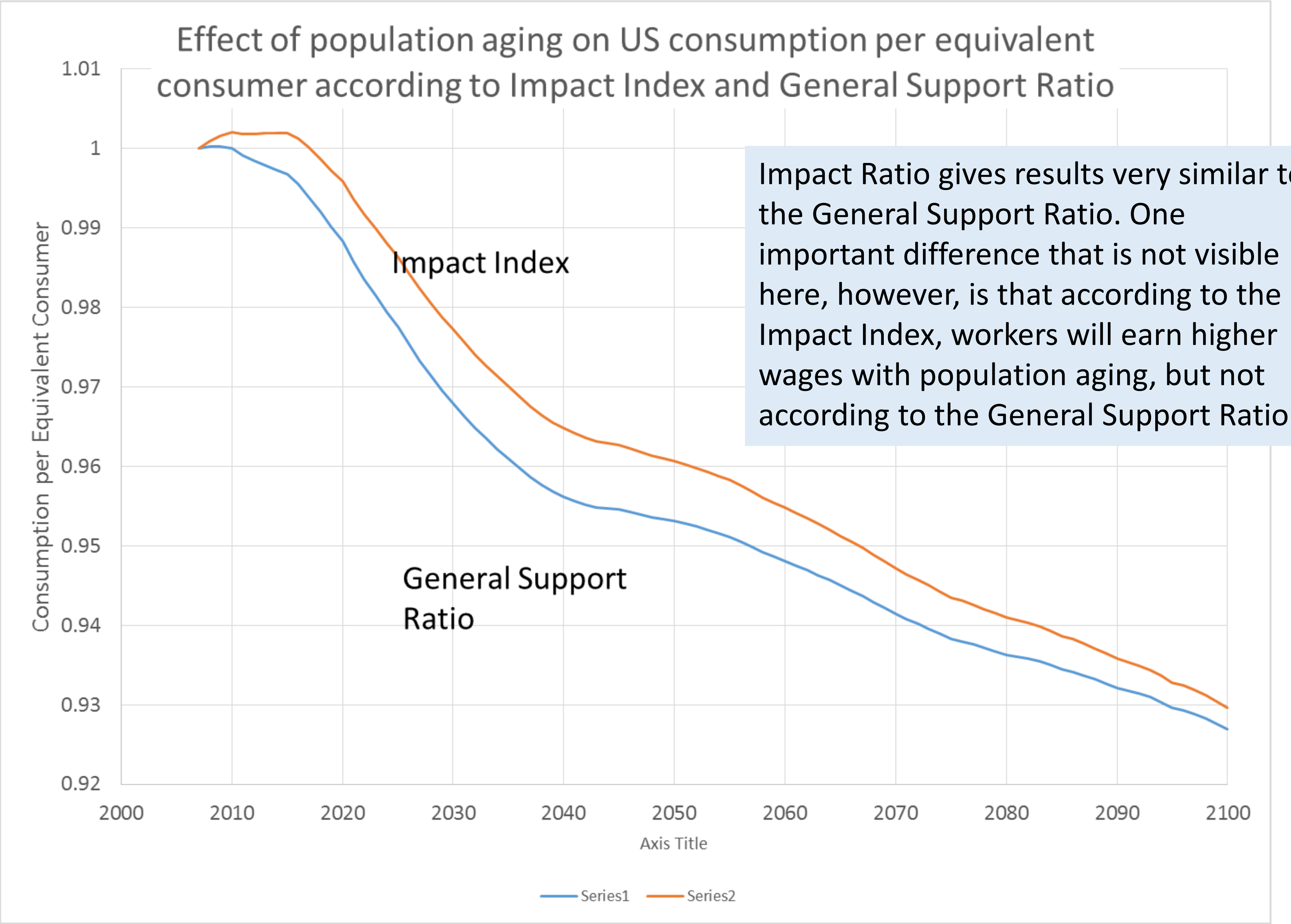
Our usual NTA measure of wellbeing is Consumption per Effective Consumer, C/N

- Effective consumers, N, is population-weighted sum of c(x)
- Proportional change in C/N is:
- $$\frac{C'}{C}-\frac{N'}{N}=-\frac{s'}{1-s}+\alpha\frac{L'}{L}+\left(1-\alpha\right)\frac{K'}{K}-\frac{N'}{N}$$ this is the rate of change of the "Impact Index"
- This is closely related to the Support Ratio and the General Support Ratio
  - Rate of change of Support Ratio is: $\frac{L'}{L}-\frac{N'}{N}$
  - This overstates the role of labor force growth while ignoring the roles of capital and saving

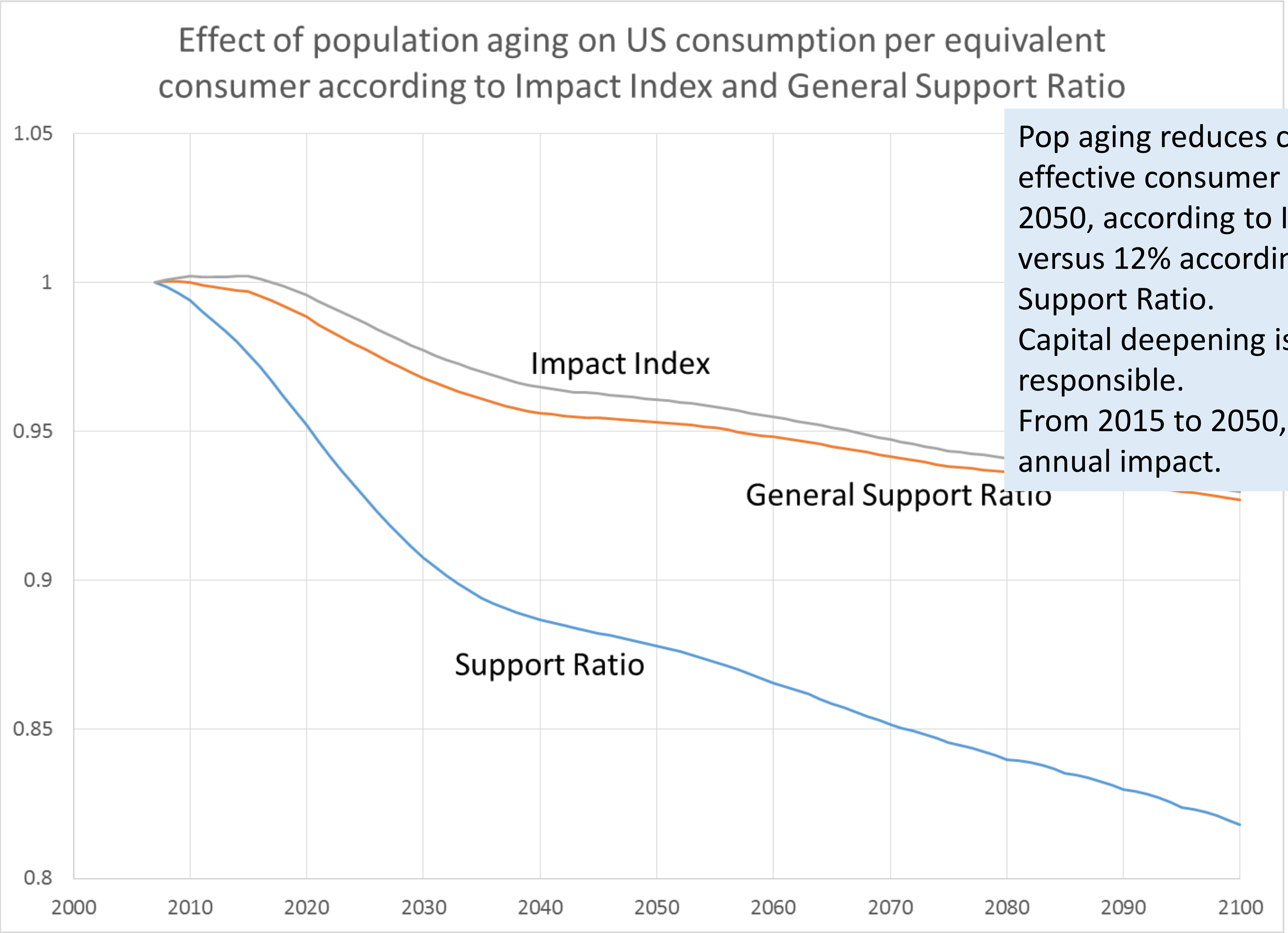

Components of Perturbation Analysis and Impact Index vs Rate of Change of Gen Supp Ratio, US 2007-2100
0.025
0.02
0.015
0.01
0.005
0
-0.005
Annual Rate of Change
2000
2020
2040
2060
2080
2100
2120
Date
Y'K/YK
C'/C
L'/L
s'/(1-s)
I'/I
GSR'/GSR
a1. C'/C
a2. YL'/YL
a3. YK'/YK
a5. s'/(1-s)
c. -s'/(1-s) + "b" - "a1"
change GSR

Cumulated Change in the Impact Index in the US, 2007 to 2100





Impact Ratio gives results very similar to the General Support Ratio. One important difference that is not visible here, however, is that according to the Impact Index, workers will earn higher wages with population aging, but not according to the General Support Ratio.



Pop aging reduces consumption per effective consumer by only 4% by 2050, according to Impact Index, versus 12% according to ordinary Support Ratio.
Capital deepening is mainly responsible.
From 2015 to 2050, -.12% average annual impact.

Limitations

- Assumes the economy is closed.
- Effect of savings rates on assets by age is not incorporated. Are they consistent?
- $Y_L(x)$ will likely change shape due to later retirement, changes in education, and other factors
- $y_A(x)$ profile reflect asset accumulation over the past 60 years or so, including long-ago bequests and saving
  - These profiles may change in the future due to expectations of longer retirements, longer spousal survival, lower fertility, and fewer heirs to share bequests, among other factors
- The production function makes lots of assumptions in itself, including constant factor shares. Perhaps should try more flexible production function (CES)

Conclusions

- NTA are accounting identities; deriving dynamic implications requires new assumptions and models
- Here give theoretical basis for some NTA-driven measures
- As with Support Ratio and General Support Ratio, the results can be applied over a long time horizon, but require assuming that profile shapes remain constant
- Perhaps this approach will help to bring NTA to bear on assessing
  - Macroeconomic impact of population aging—cons, factor prices, inc growth Demographic Dividend including partial Second Dividend effect of capital accumulation relative to labor
  - Consequences of an age fluctuation such as Baby Boom or Baby Bust