Bequest Session

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Wittgenstein Centre
FOR DEMOGRAPHY AND GLOBAL HUMAN CAPITAL
A COLLABORATION OF IIASA, VID/ÖAW, WU

agenta
Ageing Europe – An Application of National Transfer Accounts for Explaining and Projecting Trends in Public Finances

IIASA International Institute for Applied Systems Analysis www.iiasa.ac.at

Vienna Institute of Demography ÖAW Austrian Academy of Sciences

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Goals

1. Previous model for estimating bequest
   - Drawback #1: The bequest model was deterministic
   - Drawback #2: Unrealistic profiles early in life

2. Proposing a new model for estimating bequest
   - The new model should be consistent with economic theory and rigorous with the demographic setup
     - Dynamic General Stochastic Economic (DGSE) model populated by overlapping generations
     - Stochasticity comes from the risk of mortality rather than through productivity or income shocks

3. Pending research questions
   - Assessment of the role of bequests vs inter-vivos intergenerational transfers as sources of wealth
   - Relation to annuitization of wealth

Model background

- Time is discrete
- Individuals are assumed to receive a stream of income over their lifecycle $\{y_x\}_{x=0}^{\omega}$ and to make decisions about consumption/saving
- Individuals face mortality risk
- Let π_x be the conditional probability of surviving to age x and $\ell_x = \prod_{u=0}^{x-1} \pi_u$ be the probability of surviving from birth to age x
- Let θ_x be a random variable that denotes whether the parent of an individual of age x is alive (s) or dead (d), i.e. $\theta_x \in \{\mathrm{s}, \mathrm{d}\}$
- Let $\theta^x = (\theta_0, \theta_1, \ldots, \theta_x)$ represent the history of the variable θ up to age x

Transition probabilities: $\mathrm{P}\left(\theta_{x+1} | \theta^{x}\right)$

In a stable population the probability that the parent of an individual of age x dies is characterized by the following Markovian process:

$$\begin{pmatrix} \ell_{x+1}^{\theta} \\ 1-\ell_{x+1}^{\theta} \end{pmatrix} = \begin{pmatrix} \pi_x^{\theta} & 0 \\ 1-\pi_x^{\theta} & 1 \end{pmatrix} \cdot \begin{pmatrix} \ell_x^{\theta} \\ 1-\ell_x^{\theta} \end{pmatrix}$$

$$\text{with } \ell_0^{\theta}=1 \text{ and } \pi_x^{\theta}=\frac{\sum_{u=0}^{\omega-x} e^{-nu} f_u \ell_{u+x+1}}{\sum_{u=0}^{\omega-x} e^{-nu} f_u \ell_{u+x}},$$

where

ℓ_a^{θ} is the survival probability of the parent of an individual at age a ℓ_a is the survival probability to age a

ω is the maximum longevity

n is the population growth rate

f_a is the fertility rate at age a

Average bequest received

Per capita bequest received

$$b_x = \ell_x^{\theta}(1-\pi_x^{\theta})\,\mathrm{E}[\mathcal{B}_x]$$

In a stable population the average bequest received at age u is given by

$$\mathrm{E}\left[\mathcal{B}_u\right] = \sum_{x=0}^{\omega-u} \mathrm{P}(\mathbb{A}_u = x)\,\mathrm{E}\left(\mathcal{B}|\mathbb{A}_u = x\right)$$

with

$$\mathrm{P}(\mathbb{A}_u = x) = \frac{e^{-nx} f_x d_{x+u}}{\sum_{x=0}^{\Omega-u} e^{-nx} f_x d_{u+x}},$$

$$\mathrm{E}\left(\mathcal{B}|\mathbb{A}_u = x\right) = \sum_{h=0}^{\infty} \frac{\mathrm{E}\left[a(\theta^{x+u})\right]}{1+h}\,\mathrm{P}\left(\mathcal{H}_{x+u} = h\right),$$

where

- $\mathbb{A}_u$ is the random variable ‘Age of death of a parent of an individual of age u’
- $d_x = \ell_x - \ell_{x+1}$ is the fraction of deaths between ages x and $x+1$
- $\mathcal{H}_x$ is the number of additional heirs at age x
- $\mathrm{E}\left[a(\theta^x)\right] = \int a(\theta^x) d\,\mathrm{P}(\theta^x)$ is the mean financial wealth at age x

Household problem

A household head of age $x > x_0$ maximizes the following conditional expected utility with respect to consumption (c):

$$V\left[a\left(\theta^x\right)\right] = U\left[c(\theta^x)\right] + \beta\pi_x \sum_{\theta_{x+1}\in\{\mathrm{s},\mathrm{d}\}} V\left[a\left(\theta^{x+1}\right)\right] \mathrm{P}\left(\theta_{x+1}|\theta^x\right),$$

s.t. the budget constraint

$$\begin{cases} a(\theta^{x+1}) = R\left[a(\theta^x) + y_x + \tau_x - c(\theta^x)\right] + R\,\mathrm{E}(\mathcal{B}_x) & \text{If } (\theta_{x+1}, \theta_x) = (\mathrm{d}, \mathrm{s}), \\ a(\theta^{x+1}) = R\left[a(\theta^x) + y_x + \tau_x - c(\theta^x)\right] & \text{Otherwise.} \end{cases}$$

$a(\cdot) \geq 0$ is the financial wealth (borrowing constraint)
$R > 1$ is the capitalized interest rate
$\mathrm{E}[\mathcal{B}_x]$ is the average bequest received at age x
y_x is the endowment at age x
τ_x is the transfer at age x

Optimal consumption/saving decision

The optimal consumption path is characterized by the following Euler equation

$$\begin{cases} U_c\left[c(\theta^x)\right] = R\beta\pi_x\left(\pi_x^\theta U_c\left[c(\mathrm{s},\theta^x)\right] + (1-\pi_x^\theta)U_c\left[c(\mathrm{d},\theta^x)\right]\right) & \text{If } \theta_x = \mathrm{s}, \\ U_c\left[c(\theta^x)\right] = R\beta\pi_x U_c\left[c(\mathrm{d},\theta^x)\right] & \text{If } \theta_x = \mathrm{d}, \end{cases}$$

Note that there exists saving for precautionary motive when $\theta_x = \mathrm{s}$ (Jensen's inequality)

Assumptions (target: maximum bequest-output ratios)

1. Stable populations that differ by their fertility and mortality schedules (data collected from UNPD, *World Population Prospects: The 2012 Revision*)
2. The consumption of children is supported by parents
3. The labor income profile is that of a developed country (*to maximize savings for retirement motive*)
4. The financial wealth of an individual without offspring is taxed at 100% and distributed (τ_x) according to the expected bequest profile
5. Two strong demographic assumptions: At the aggregate level the total number of offspring at age x of an individual at age u is assumed to be given by the sum of N_{u-x} independent Bernoulli random variables, where N_{u-x} is assumed to be distributed according to a Poisson of parameter f_{u-x}
6. Fixed interest rate $r = 5\%$ and no productivity growth $g = 0\%$

Underlying demographic data

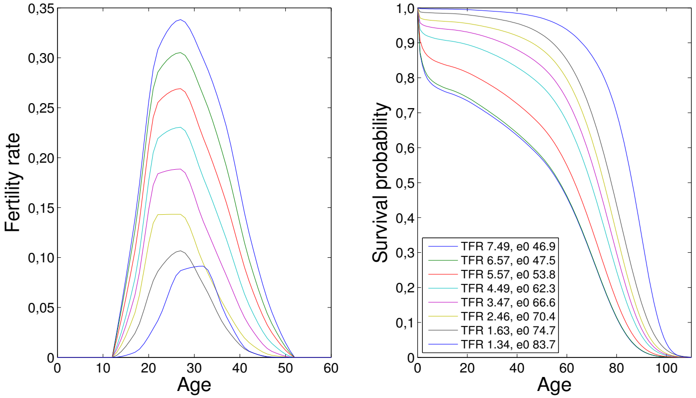


Figure: Fertility (f_x) and survival (ℓ_x) profiles. Source: UNPD, *World Population Prospects: The 2012 Revision*.

High mortality prevents splitting the wealth among too many heirs

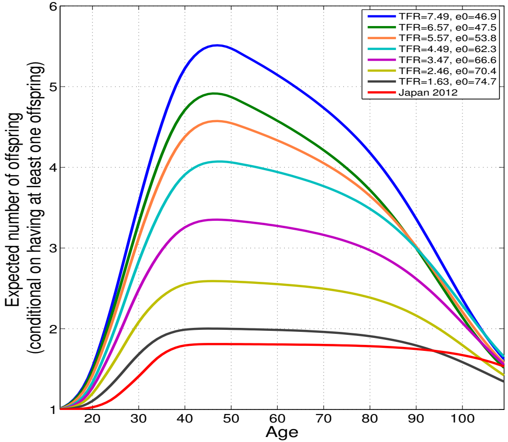


Figure: Expected number of offspring of a parent at age x (conditional on being one of the offspring)

The per capita bequest inflow shifts to the right the higher the proportion of bequest given to spouses

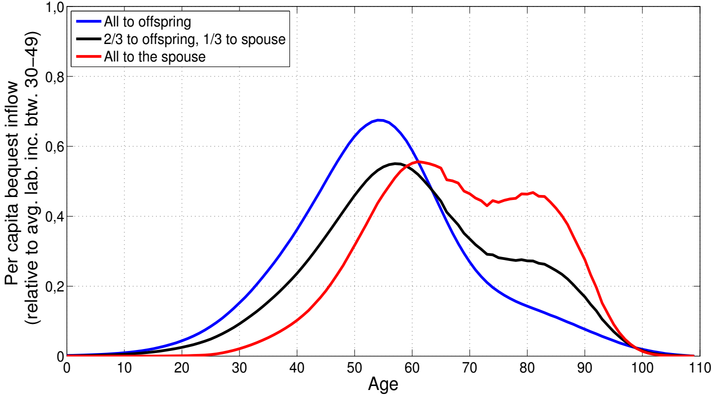


Figure: Per capita bequest inflow over the lifecycle

Per capita bequest inflows and outflows could be as large as the labor income earned in advanced countries

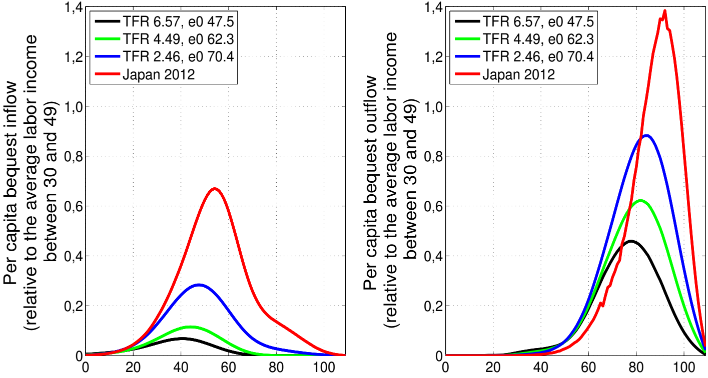


Figure: Per capita bequest inflows across the lifecycle by fraction of bequest given to offspring

The bequest received increases more than proportionally with declines in TFR due to uncertainty

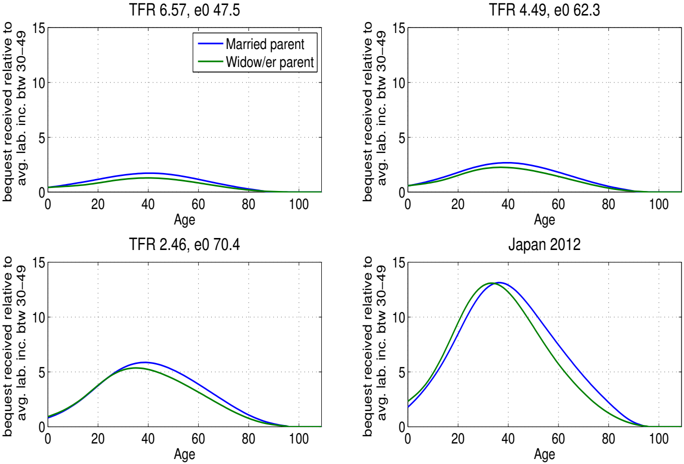


Figure: Bequest received at death of the parent, by marital status, $E[\mathcal{B}_x]$

Strong effect of mortality decline on the bequest-output ratio

Table: Stochastic Model: Maximum bequest-to-output ratios for $r = 5\%$, $g = 0\%$, $\alpha = 100\%$ under a stable-population structure (Results in %)

Life expectancy	Total fertility rate (TFR) 7.49	6.57	5.57	4.49	3.47	2.46	1.63	Japan
46.9	**4.05**	4.63	5.87	8.20	12.12	21.36	48.32	89.07
47.5	4.01	**4.57**	5.84	8.08	11.88	21.04	47.10	86.23
53.8	3.67	4.25	**5.37**	7.42	10.92	19.24	41.44	72.54
62.3	3.29	3.87	4.91	**6.82**	10.02	17.27	36.71	59.62
66.6	3.07	3.63	4.62	6.47	**9.67**	16.57	34.56	55.22
70.4	2.90	3.45	4.44	6.29	9.31	**16.26**	33.01	51.40
74.7	2.73	3.29	4.27	6.11	9.22	16.32	**32.84**	50.55
Japan	2.05	2.53	3.40	5.04	7.90	14.62	30.58	**42.97**

[1] We have used a CES production function $Y_t = \left(aK_t^{\frac{\sigma}{\sigma-1}} + (1-a)H_t^{\frac{\sigma}{\sigma-1}}\right)^{\frac{\sigma-1}{\sigma}}$ with $\sigma = 1.2$ and $a = .25$, so that higher capital/output ratios lead to higher capital shares $\alpha_t = a\left[\frac{K_t}{Y_t}\right]^{1-\frac{1}{\sigma}}$. [2] We have assumed the following instantaneous utility function at any age x, ($U(c_x) = \eta_x \frac{(c_x/\eta_x)^{1-\sigma}-1}{1-\sigma}$ with $\sigma = 2$.)

Strong effect of the mortality decline on the bequest-output ratio

Table: Deterministic Model: MAXIMUM BEQUEST-TO-OUTPUT RATIOS FOR $r = 5\%$, $g = 0\%$, $\alpha = 100\%$ UNDER A STABLE-POPULATION STRUCTURE (RESULTS IN %)

Life expectancy	Total fertility rate (TFR) 7.49	6.57	5.57	4.49	3.47	2.46	1.63	Japan
46.9	**4.94**	5.75	7.13	9.44	12.98	19.62	31.47	43.02
47.5	4.86	**5.66**	7.02	9.30	12.78	19.33	31.05	42.60
53.8	4.41	5.17	**6.47**	8.72	12.23	19.05	31.53	41.94
62.3	3.85	4.58	5.84	**8.06**	11.64	18.88	32.70	42.37
66.6	3.57	4.28	5.50	7.69	**11.27**	18.68	33.14	42.80
70.4	3.31	4.01	5.23	7.41	11.04	**18.71**	34.00	43.92
74.7	3.09	3.79	5.00	7.20	10.95	19.08	**35.87**	46.75
Japan	2.30	2.89	3.93	5.91	9.46	17.72	36.45	**49.01**

[1] We have used a CES production function $Y_t = \left(aK_t^{\frac{\sigma}{\sigma-1}} + (1-a)H_t^{\frac{\sigma}{\sigma-1}}\right)^{\frac{\sigma-1}{\sigma}}$ with $\sigma = 1.2$ and $a = .25$, so that higher capital/output ratios lead to higher capital shares $\alpha_t = a\left[\frac{K_t}{Y_t}\right]^{1-\frac{1}{\sigma}}$. [2] We have assumed the following instantaneous utility function at any age x, ($U(c_x) = \eta_x \frac{(c_x/\eta_x)^{1-\sigma}-1}{1-\sigma}$ with $\sigma = 2$.)

Strong effect of savings for retirement motive on the wealth-output ratio

Table: Deterministic Model: MAXIMUM WEALTH-TO-OUTPUT RATIOS FOR $r = 5\%$, $g = 0\%$, $\alpha = 100\%$ UNDER A STABLE-POPULATION STRUCTURE (RESULTS IN %)

Life	Total fertility rate (TFR)							
expectancy	7.49	6.57	5.57	4.49	3.47	2.46	1.63	Japan
46.9	**2.33**	2.53	2.91	3.53	4.37	5.78	8.16	11.52
47.5	2.30	**2.49**	2.87	3.48	4.31	5.71	8.06	11.44
53.8	2.39	2.59	**2.99**	3.66	4.58	6.19	8.91	12.00
62.3	2.56	2.79	3.23	**3.99**	5.07	7.01	10.35	13.27
66.6	2.63	2.88	3.34	4.15	**5.30**	7.42	11.10	14.06
70.4	2.74	3.00	3.50	4.36	5.60	**7.93**	12.01	15.10
74.7	2.89	3.18	3.73	4.67	6.07	8.72	**13.46**	16.92
Japan	3.35	3.72	4.40	5.59	7.39	10.93	17.54	**22.16**

[1] We have used a CES production function $Y_t = \left(aK_t^{\frac{\sigma}{\sigma-1}} + (1-a)H_t^{\frac{\sigma}{\sigma-1}}\right)^{\frac{\sigma-1}{\sigma}}$ with $\sigma = 1.2$ and $a = .25$, so that higher capital/output ratios lead to higher capital shares $\alpha_t = a\left[\frac{K_t}{Y_t}\right]^{1-\frac{1}{\sigma}}$. [2] We have assumed the following instantaneous utility function at any age x, ($U(c_x) = \eta_x \frac{(c_x/\eta_x)^{1-\sigma}-1}{1-\sigma}$ with $\sigma = 2$.)

Discussion and future work

Pros

- The assumptions of the model allow us to widely use available demographic data
- We can model several alternatives of transfers between parents and children and between spouses

Cons

- It is very difficult to theoretically justify the introduction of an annuity market
- Saving for precautionary motive does not lead to higher wealth unless uncertainty is very high (high mortality and wealth)

Future work

- Introduction of a housing market (additional savings still needed)
- Assessment of the contribution of savings for precautionary motive to total savings

- **Bequest wealth** at age x for the cohort born in year s:

$$w_{x,s} = \sum_{z=x}^{\omega} \left(\text{bequest}_{z,s}^{\text{inflow}} - \text{bequest}_{z,s}^{\text{outflow}} \right) \left(\prod_{u=x}^{z} \frac{\pi_{u,s}}{1+r} \right),$$

where $\pi_{x,s}$ is the conditional probability of surviving from age x to $x+1$ for the cohort born in year s

- **Aggregate wealth** in year t:

$$W_t = \sum_{x} w_{x,t-x} N_{x,t-x}$$

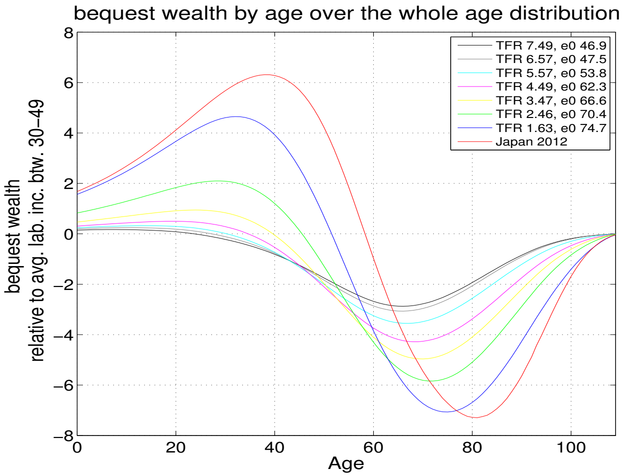


Figure: Bequest wealth

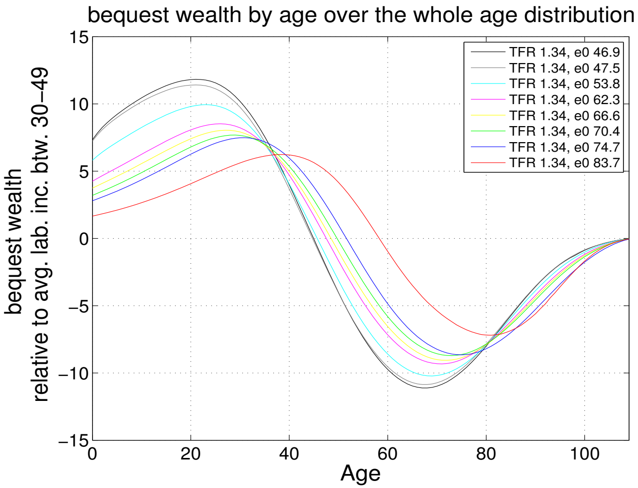


Figure: Bequest wealth (fixed fertility)

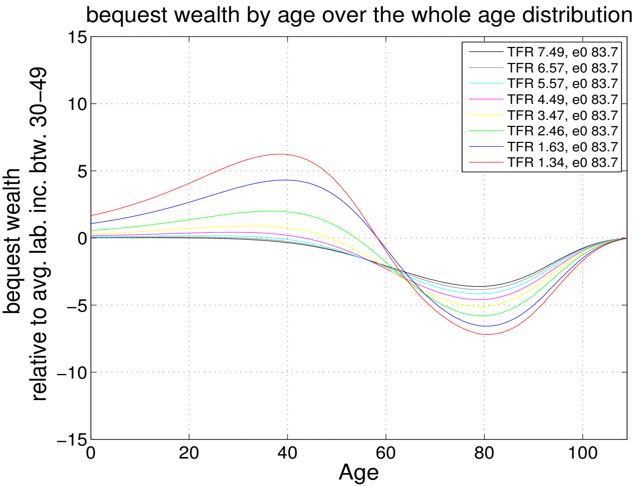
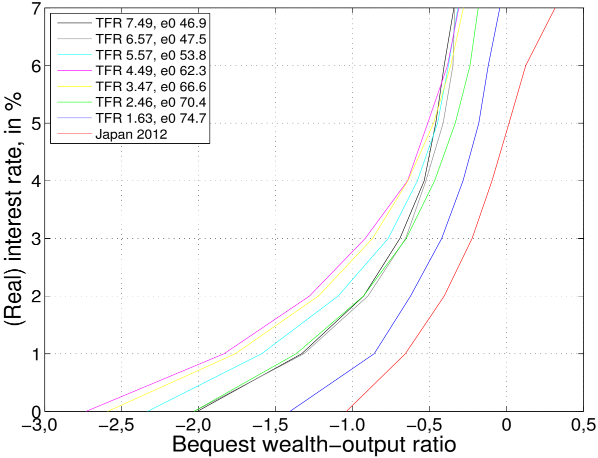


Figure: Bequest wealth (fixed mortality)


TFR 7.49, e0 46.9
TFR 6.57, e0 47.5
TFR 5.57, e0 53.8
TFR 4.49, e0 62.3
TFR 3.47, e0 66.6
TFR 2.46, e0 70.4
TFR 1.63, e0 74.7
Japan 2012
(Real) interest rate, in %
0
1
2
3
4
5
6
7
−3,0
−2,5
−2,0
−1,5
−1.0
−0,5
0
0,5
Bequest wealth−output ratio