The Fiscal Impact of Population Aging: Accounting for the Role of Demography

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Abstract
In the coming years many countries will reform public systems because of fiscal pressures and in response to the needs of rapidly growing elderly populations. Many countries, particularly in the developing world, will be developing new programs that again respond to the needs of growing elderly populations and, hopefully, do so in a way that recognizes demographic realities. The goal of this paper is to clearly identify the linkages between population aging, the public sector, and public policy. The analysis is carried out using the National Transfer Account (NTA) framework and data. We rely on a simulation model that allows us to consider the long-term implications of changes in population age structure for public spending, revenue, deficits, and debt. We consider a group of countries that vary widely in their public sectors, their levels of development, and their position in the demographic transition. A companion paper will extend the analysis to consider the implications for the private, as well as, the public sectors.
I. Introduction
Population aging has important implications for the public sector. Many countries will reform existing systems because of fiscal pressures and in response to the needs of rapidly growing elderly populations. Many countries, particularly in the developing world, will be developing new programs that again respond to the needs of growing elderly populations and, hopefully, do so in a way that recognizes demographic realities.

The goal of this paper is to clearly identify the linkages between population aging, the public sector, and public policy. The intent is not to recommend a particular course of action, but rather to identify options and quantify the implications of alternative courses of action. The emphasis is on a broad understanding of the generational features of the public sector, its role in reallocating resources from working-age adults to children and the elderly. This necessarily involves particular public sector programs – public pensions, publicly funded health care, and public education. Our purpose is not, however, to examine these programs in detail.

The analysis is carried out using the National Transfer Account (NTA) framework and data. We rely on a simulation model that allows us to consider the long-term implications of changes in population age structure for public spending, revenue, deficits, and debt.

We consider a group of countries that vary widely in their public sectors, their levels of development, and their position in the demographic transition. The emphasis is on developing countries, but the experience of and prospects for high-income countries are highly relevant.

This paper emphasizes the experience of India. Two case studies are being prepared – one on Brazil and a second on Thailand. Future work will emphasize the comparative aspects. The next section of the paper provides an overview of the NTA framework and the public sector drawing on NTA data. Section III provides a brief overview of the model and data. Detailed information is provided in appendices. Section IV presents key results drawing on prospects for India. Section V discusses important extensions and section VI concludes.

II. Overview: NTA framework and the public sector
The public sector model emphasized here conforms to the National Transfer Accounts framework. Here we provide a brief overview of NTA and the public sector through the NTA lens.

Lifecycle and age reallocation
There are extended periods at the beginning and the end of life when consumption exceeds labor income. This is true of all contemporary economies for which National Transfer Accounts have been constructed. National Transfer Accounts documents the economic lifecycle in each country, the left-hand-side of equation (1), and the economic mechanisms and institutions involved in reallocating resources across age, the right-hand-side of the equation. This feature of the generational economy is captured in the basic flow identity that guides NTA, which holds for an individual, a generation, or a national economy:
\[ C(x) - YL(x) = T(x) + RA(x) \]  \hspace{1cm} (1)

where \( C \) is consumption, \( YL \) labor income, \( T \) net transfers, and \( RA \) asset-based reallocations all classified by age. Consumption is the value of all goods and services whether public or privately produced. Labor income is the pre-tax value of labor including earnings, two thirds of self-employment income, and an estimate of the value of the labor of unpaid family workers. The gap between consumption and labor income, called the lifecycle deficit, measures the extent to which each age group is producing enough through its labor to meet its material needs.

The per capita lifecycle profile, consumption and labor income by age, for China 2002 is shown in Figure 1. All values in the figure are expressed relative to the per capita labor income of persons 30-49, called YoLYs (for years of labor income). This is the standard way used to facilitate comparative analysis in NTA. The age pattern of labor income rises starting at around age 10, reaches a peak at about age 40 and then declines. The most distinctive feature of consumption is the high peak in the late teens which reflects high levels of public and private spending on education in China. Adult consumption (after the early 20s) is relatively flat with a slight decline at older ages. Adults between the ages of 23 and 59 have life cycle surpluses while those 22 and younger or 60 and older have lifecycle deficits.

![Figure 1. Per capita annual consumption and labor income by age, China, 2002. All values are expressed in YoLYs, the simple average of annual per capita labor income of persons 30 to 49. Source: See data appendix. File: Figures\NTA profiles.v3.xlsx](image-url)

The lifecycle deficit is balanced by transfers and asset-based reallocations. Public and private transfers are distinguished. Public transfers consist of transfer inflows and outflows distinguished by their purpose – education, health, pensions, and other public transfers. Private transfers are predominantly
familial transfers and include estimates of transfers within households, intra-household transfers, as well as, transfers between households, non-profit institutions, and the rest of the world.

Asset-based reallocations are equal to the excess of asset income over saving. Inflows are generated by asset income or dis-saving generating resources in excess of labor income. The resources could be used, for example, to fund consumption in excess of labor income – for example – during retirement. Or if parents’ labor income is insufficient to fund their own consumption plus net transfers, asset-based reallocations fill in. Asset-based reallocations can also generate outflows – saving or interest expense, for example. Both public and private asset-based reallocations are estimated.

The reallocation system for China is shown in Figure 2 with public transfers, private transfers, and asset-based reallocations (public and private combined) distinguished. Transfers are net values, meaning that they are equal to transfers received by members of the age group less transfers paid by the age group. The life cycle deficit for children is funded almost entirely by transfers. Private transfers are much more important public transfers for children. Transfers are also very important for the elderly, with public transfers very large for younger elderly and private transfers large for older elderly. Assets are generating net positive inflows for the young elderly, i.e., their saving is exceeded by their asset income. Saving exceeds asset income for the old elderly but the values are small.

Figure 2. Per capita public transfers, private transfers, and asset-based reallocations by age, China, 2002. All values expressed in YoLYs, the simple average of annual per capita labor income of persons 30 to 49. Source: See data appendix. File: Figures\NTA profiles.v3.xlsx

Public sector
In NTA the public sector consists of the public transfer system and the public asset-based reallocation system. The public transfer system consists of age-specific inflows (public transfers received by
individuals and outflows (taxes paid by individuals to the government) for each single-year age group and the rest of the world. Net public transfers by age are equal to public transfer inflows less public transfer outflows. Public transfers are defined in a very broad way including all public cash transfers and all in-kind transfers equivalent to public consumption. Four uses or functions for public transfers are distinguished in NTA: education, health, pensions, and all other uses.

The methods for estimating public transfer inflows and outflows and their age profiles are described in United Nations Population Division (2013). By way of illustration the public transfer inflows and outflows for China 2002 are presented in Figure 3. Public transfer inflows benefit children (education) and the elderly (health care and pensions) more than prime age adults. Public transfer outflows, however, flow mostly from prime age adults. Thus, net public transfers are positive for children and the elderly and negative for prime age adults. Per capita net public transfers for the elderly exceed net public transfers to children in China. This pattern is typical for high-income countries which have relatively comprehensive health care and pension coverage. The situation is more mixed in middle-income countries where net public transfers to the elderly may or may not be substantial.

By definition transfers always involve counterparties – a transfer inflow must be balanced by a transfer outflow. Net public transfers aggregated over all ages plus net public transfers to the rest of the world must equal zero. Public transfer inflows are observable while public transfer outflows are constructed under the constraint that total outflows and inflows are equal for all uses, education, health, pensions, and other. If taxes exceed public transfer outflows, the surplus is referred to as a public transfer surplus. If taxes are insufficient, the gap between taxes and outflows is referred to as the public transfer deficit.

Figure 3. Per capita public transfer inflows and outflows by age, China, 2002. All values expressed in YoLYs, the simple average of annual per capita labor income of persons 30 to 49. Source: See data appendix. File: Figures\NTA profiles.v3.xlsx
The public transfer deficit/surplus must be balanced by the public asset-based reallocation system. More specifically, the public transfer deficit is equal to public asset-based reallocations, i.e., asset income less saving. If taxes are insufficient to fund transfers, the deficit must be funded by relying on public asset income, if sufficient, and public borrowing to the extent that public asset income is insufficient. If taxes exceed public transfer outflows, the surplus can be devoted to paying interest expense on public debt (a negative asset income), if any, and to public saving if revenues are sufficient.

**Overview of net public transfers to children and the elderly**

Net public transfers to children and the elderly provide a useful way of summarizing and comparing public transfer systems around the world. Net public transfers by age are available for 29 countries in Africa, Asia, Latin America, and the West (Europe and the US). Net public transfers to children are summarized by the simple average of the annual single-year age specific flows over the 0 to 19 age range measured in YoLYs. For all countries combined, children on average received net transfers equal to 0.15 YoLYS while net public transfers to the elderly were substantially higher, averaging 0.24 YoLYS.

Net public transfers vary widely across countries (Figure 4). For children, net public transfers vary from below 0.05 YoLYS in India, Nigeria, and Senegal to a high of between 0.25 and 0.30 YoLYS in Japan, Finland, Hungary, Sweden, and the United States. In a few countries, Nigeria, Senegal, South Africa, and the Philippines, net public transfers to the elderly are negative – they pay more in taxes than they receive in benefits. Net public transfers to older adults are very high in Brazil at about 0.8 YoLYS followed by Hungary with 0.57 YoLYS.

![Figure 4. Per capita net public transfers to children (0-19) and elderly (60-79) for a recent year in 29 countries. Dashed lines represent transfers to elderly equal to 1 time or 2 times transfers to children.](image-url)
All values calculated as average of age-specific per capita flows over the indicated age interval. Source: See data appendix. File: Figures\tg children and elderly.xlsx

There are strong regional patterns evident in Figure 4. In Africa, net public expenditures are low and biased towards children. In all cases, net public transfers to children exceed net public transfers to the elderly. Asia is very diverse in many ways including the size of the public sector, but net public transfers to children and the elderly are more nearly equal than in other regions. Latin America is least diverse in its net public transfers and exhibits a stronger bias towards the elderly than the countries in other regions. The West is relatively diverse in the size of the public sector, but relatively consistent in the bias, with net public transfers to the elderly running at roughly two YoLYs for every one YoLY of net public transfers to children.

Average values by region are shown in Table 1 reinforcing the observations made above about regional patterns. An additional issue of considerable interest is whether there is a tradeoff between spending on children and spending on the elderly, the Preston hypothesis (Preston 1984). The data are consistent with Miller (2011) in showing no apparent tradeoff between net public transfers to children and the elderly. For the sample as a whole and for each regional grouping except Latin America, net transfers to the young and to the elderly are positively correlated. Even the negative correlation for Latin America is due entirely to Brazil. The correlation turns moderately positive (0.37) if Brazil is excluded from the calculation.

Table 1. Net public transfers to children, 0-19, and older adults, 60-79 for 29 countries, by regional grouping.

<table>
<thead>
<tr>
<th>Regional group</th>
<th>All</th>
<th>Africa</th>
<th>Asia</th>
<th>Latin America</th>
<th>Western</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young (0-19)</td>
<td>0.15</td>
<td>0.09</td>
<td>0.14</td>
<td>0.12</td>
<td>0.21</td>
</tr>
<tr>
<td>Older (60-79)</td>
<td>0.24</td>
<td>-0.02</td>
<td>0.11</td>
<td>0.40</td>
<td>0.36</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.47</td>
<td>0.69</td>
<td>0.80</td>
<td>-0.38</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Note: All values are expressed in YoLYs, the simple average of single year of age values normalized by labor income of those 30-49. Source: See data appendix. File: Figures\tg children and elderly.xlsx

Net public transfers to the elderly are influenced by the choice of the upper age group 80, particularly in countries where spending rises rapidly with age (Japan, Sweden, and the United States). The United States would be most affected by including the 80 to 90 age group. Average net public transfers would be twice as high, but rising only to 0.28 YoLYs. Using the alternative measure would bring the US more in line with other Western countries. In Figure 4, it is the low elderly spending outlier among Western countries.

The regional difference reported above may reflect inherent and possibly unchanging region-specific factors, but other factors undoubtedly are playing a role. Based on such limited data it would be difficult to say which factors are playing a role, but clearly development in general leads to greater net
public transfers to children and the elderly. This is the case even after controlling directly for income by expressing all transfers in terms of YoLYs.

Plotting net public transfers to children and the elderly against YoLYs is instructive (Figure 5). The upper panel shows net public spending on children 0-19 and the lower panel shows net public spending on elderly 60-90 plotted against YoLYs, the average of labor income of persons 30-49 expressed in 2010 US dollars. Note that net public transfers are expressed in YoLY so the positive slope for both children and the elderly indicates that per capita net public transfers to children and to the elderly both rise more rapidly than the labor income of prime age adults.

Net public transfers to children rise less as development proceeds. In the six poorest countries with YoLYs below $5000, a child on average received net transfers equal to only 8 percent of a YoLY. In the seven countries with YoLYs between 10 and 20 thousand US dollars, net public transfers to children were 17 percent of a YoLY. In nine richest countries with YoLYs over 20,000 US dollars, net transfers were somewhat higher at 21 percent of a YoLY.

Net public transfers to the elderly are very low in the six poorest countries in our sample, averaging only 2 to 3 percent of a YoLY depending on whether we measure net public transfers using the 60 to 79 or the 60 to 90 age span. In the six countries with YoLY of 5 to 10 thousand dollars a year, net public transfers averaged 33 percent or 36 percent of a YoLY depending on the measure used. Net public transfers to the elderly relative to prime adult labor income were actually lower in the 10 to 20 thousand YoLY groups at either 24 or 28 percent, while in the nine high income countries net public transfers were 0.33 YoLY using the 60-79 age group and 0.44 YoLY using the 60-90 age group.
The relationship between development and net public transfers to children and the elderly is very noisy. This is true in particular of transfers to the elderly. It may seem surprising that countries with relatively low levels of income were none the less providing substantial per capita benefits to the elderly. Keep in mind, however, that in these countries the fiscal costs of these programs were relatively modest because the populations were so young. As these countries develop, the demographics and the level of development work against each other. They will have more income but older populations.

In contrast development and demographics are reinforcing when it comes to public transfers to children. Countries are wealthier and can presumably afford to spend more on children. Fertility decline has led to lower child dependency so that funding any per capita transfer is less burdensome for taxpayers.

In some countries, private transfers to children and/or the elderly are substantial, particularly in East Asia but to varying degrees elsewhere. This accounts for some of the dispersion about the fitted lines in Figure 5.

If the practice in higher income countries is a useful guide, net public transfers to children and the elderly will rise in low income countries as they develop. Further analysis of net transfer to children indicates that the rise is a consequence both of higher incomes and lower fertility (the quantity-quality tradeoff). (I have the regression analysis that Jennifer did for me quite a while ago.)

The cross-sectional data shed only a little light on net public transfers to the elderly. It seems likely that net public transfers will increase over time in countries with initially low levels. It also seems likely that
they will decline in countries with moderate levels of income as population aging proceeds. We see this already in reforms that have been instituted in some Latin American countries.

If we consider the nine countries that are most developed we see very diverse approaches to public support for the elderly. Net public transfers for 60 to 90 average only 0.27 YoLYs in Spain and the US while in the six countries with the highest net public transfers (Slovenia, Japan, Finland, Germany, Sweden and Austria), net public transfers average 0.52 YoLYs. UK falls between these two extremes. These two groups offer alternative visions about policy towards old-age support that we will use to model the likely course of public spending.

III. Public sector projection model and data

Model
A detailed discussion of the model is presented in the appendix. Here we will present the broad outlines of the model.

The purpose of the model is to assess how changes in population age structure will influence the public sector given exogenously specified policies that govern taxes and spending, drawing on annual population projections by single year of age.

Aggregate economic growth depends on growth in the population at each age and changes in the productivity of workers at each age. Labor productivity growth is introduced using an exogenous, time varying rate of productivity growth that leads to increases in a countries YoLY. Except as noted below, the age profile of labor income shifts upward at the same rate as the YoLY. Aggregate labor income is then determined by the population at each age and labor income at each age. Aggregate asset income grows at the same rate as aggregate labor income.

Several other macroeconomic factors influence the results. All values are expressed in nominal terms with prices changing at an exogenously specified rate. The rate of interest is assumed to be equal to the nominal rate of GDP growth plus a country-specific premium. The public interest rate is a constant multiple of the general interest rate.

The public sector is modelled relying heavily on public sector age profiles normalized on the countries’ base year YoLY. Unless otherwise stated, it should be understood that “age profiles” refers to flows normalized on YoLYs. If the normalized age profile is fixed, then the raw flows at each age increase with productivity growth.

Per capita taxes paid at each age are determined by the tax profile and productivity growth. Aggregate taxes are determined, in turn, by the population at each age and per capita taxes paid at each.

In similar fashion, all public spending, that is public transfer inflows, is determined by three factors: productivity growth, the relevant public transfer age profile, and population at each age. Net public transfers to the rest of the world are assumed to be constant as a proportion of public transfer inflows.
Age profiles of education, health, pensions, and all other public transfer inflows are employed to project public spending by purpose or function.

Following standard methodology, the gap between tax revenues and public transfer inflows is equal to the public transfer deficit.

Public asset-based reallocations, public asset income less public saving, equal the public transfer deficit by definition. Public asset income is determined by the public interest rate and public assets (or debt). Public assets at the end of the year equal public assets at the end of the preceding year plus public saving, equal in turn to public tax revenues plus public asset income less public transfer outflows.

**Policy scenarios**
The implications of population aging, reform to taxes and public spending, and changes in other variables of interest are assessed using scenarios implemented by varying normalized age profiles of public transfer inflows and taxes.

**Status quo scenario**
The status quo scenario holds normalized public sector age profiles fixed at their base year values. This scenario is useful, for example, for assessing the implications of changes in population age structure in the absence of changes in public policy.

**Target scenarios**
The model is designed to allow for a transition from the age profiles in the base year to exogenously specified alternative age profiles. The years at which the transition begins and is completed are exogenously specified (or triggered by per capita GDP). The transition path is linear.

In the applications described below, the target scenarios are used to model the transition of the public sector in lower income countries. Two targets are used – one based on the age profiles of high income countries with high net public transfers to the elderly and one based on the age profiles of high income countries with low net public transfers to the elderly as described above (see Figure 5 and discussion).

**Adjusting public transfers to longer survival**
Over time, the elderly are becoming healthier with declining age-specific rates of disability and mortality. One policy under consideration in many countries is to extend the age at retirement and to adjust taxes and transfers for pensions and sometimes health care, accordingly.

This scenario is incorporated into the model using age specific survival rates as a proxy for age specific health. As age specific survival rates improve for older adults, it is assumed that their age specific labor income and taxes rise, and their public transfer inflows decline.

This scenario is appropriate only for countries with mature, well-developed old-age support systems. These are mostly, but not exclusively, high income countries. Survival adjusted age profiles could be used as target scenarios for lower income countries.
**Constraints on size of government and public debt**
The size of government can be constrained by imposing an upper limit on taxes and/or public transfer inflows as a share of GDP. This constraint is incorporated by rescaling the tax and/or public transfer inflow age profiles. In other words the value at each age is reduced by the same percentage.

The public debt constraint also imposes an upper limit on public debt as a percentage of GDP. The debt constraint is soft and forward-looking, however. If the debt to GDP ratio is projected to exceed the cap at the end of an exogenously specified planning horizon, taxes are scaled upward by an amount sufficient to satisfy the constraint at the end of the planning horizon.

**Data**
Detailed information about data, methods and sources are provided in the data appendix.


Aggregate economic flows have been compiled for a base year (usually 2010) from a variety of sources relying heavily on UN SNA estimates. Age profiles of all economic flows for each country are based on the most recently available National Transfer Accounts data. The age profiles have been adjusted to insure consistency with the base year aggregate flows.

Future values of real labor productivity growth and the rate of inflation are based on a variety of sources.

**IV. Public sector over the demographic transition: the India case**
The implications of population aging for the public sector are explored using the projection model for the case of India. The India experience is instructive for several reasons. First, it is still relatively early in its demographic transition, still experiencing the demographic dividend and with population aging still to come. Using projections from 2010 to 2100 allows us to project changes over a large portion of the demographic transition. Second, India’s public sector, as it relates to generational issues, is relatively under-developed. As in many other developing countries, net transfers to the elderly in India are modest at present. Thus, the implications of population aging for the public sector will depend crucially on the policies towards old-age support pursued in India. Third is that India will soon be the largest developing country in the world and its success is of particular import.

**Three public sector scenarios**
The discussion is based on a standard set of output generated by the model. For each scenario six graphs are generated as shown in Figure 6. Labels along the top of the profile indicate the country for the simulation, the type of scenario, “fixed age profiles” in this case, and the size of government and public debt constraints – “none” in this case.
Figure 6. Status quo scenario, India. In Panel A the constraint conditions are equal to 1 when the constraint is imposed. In this scenario, the values are zero, because the simulation is unconstrained. Definitions of terms: TGI and TGO: public transfer inflows and outflows; TGD: public transfer deficit, YAG: public asset income; SF: public saving; AG: public assets. File: Figures\INDIA2004_COMBO_fixed_noconstr.jpg

Panel A indicates the year in which a constraint is violated if any with the indicator equal to 1 if the constraint is violated. No constraints are imposed and, hence, none are violated in this scenario.

Panel B reports the public transfer deficit, public transfer inflows, and tax revenues as a share of GDP. In India changes in population age structure lead to a decline in public transfers as a share of GDP, in the medium term, and a permanent decline relative to tax revenues. As a consequence the public transfer deficit in the base year turns into a surplus (a negative value) around 2020.

This reflects the changes in population age structure that India is experiencing interacting with the fixed profiles of public transfer inflows (tgi) and taxes shown in panels C and D.

The implications for public assets and assets-based flows are shown in panel E. Initially India has net public debt (-AG is greater than zero) as shown by the red line in Panel E. As a consequence, public asset income is negative (interest on public debt) as is public saving. The deficit is small, however, at about 2% of GDP. As a consequence, public debt is increasing but not as rapidly as GDP. Hence, public debt as a share of GDP is declining as shown in Panel B. Around 2040, net public debt reaches zero (YAG and – AG in Panel E) and India begins to accumulate public assets that reach almost 200 percent of GDP by 2100 (-AG in Panel E).

The final panel, Panel F, reports the mean ages of public transfer inflows, taxes, consumption, and labor income. These results are used to assess the intergenerational role of the public sector. These results are discussed below.
The next set of results is based on a targeted scenario using the simple average of the age-specific values for Spain and the United States as the target (Figure 7). We call this the “capitalist” target, because the elderly rely more heavily on their asset income to fund their retirement and less heavily on public pensions. Panels C and D show the changes in the age profiles of public transfer inflows and taxes. Public transfer inflows increase to young children but actually decline, relative to YoLYs, to teens. Public education is less important and private education more important in the capitalist profile. Public transfers to prime age adults also decline, while transfers to the elderly rise very sharply. Taxes become more concentrated on the working ages with lower taxes for children and the elderly.

![Figure 7. Capitalist target scenario, India. Target reached when projected per capita income reaches upper-income level in 2074. File: Figures\INDIA2004_COMBO_capitalist_noconstr.jpg](Figures\INDIA2004_COMBO_capitalist_noconstr.jpg)

These changes are readily accommodated given the population age structure of India over the simulation period. The changes in taxes more than compensate for the changes in spending, so that net debt declines even more rapidly than under the status quo or fixed profile scenario. This kind of outcome can occur when programs are adopted that involve upward transfers, i.e., transfers from younger to older age groups. In these programs there is a delay between collection of taxes and payment of benefits. During the start-up phase this can produce surpluses and the accumulation of public assets.

At the end of the simulation, the transfer deficit is approaching zero. India’s population will still be aging at this point and eventually public finances could begin to deteriorate.

The final scenario presented is also a targeted profile based on the six high-income countries identified in Figure 5. This is called the “Social Welfare” target and envisions a much greater expansion of public transfers to the elderly. In this case, we impose a constraint on the size of government that neither taxes nor public transfer inflows can exceed 30% of GDP that is binding around 2070. (The 30% ceiling
was selected to illustrate the effect of the constraint, not because it has any inherent merit.) A public
debt constraint of 90% of GDP is not binding.

Figure 8. Social welfare target scenario, India. Constraints: public transfer inflows capped at 30% of
GDP; net public debt capped at 90% of GDP. File: Figures\Components_INdia social welfare with
constraints 03-Jan-2015.bmp

The social welfare target scenario pushes public transfer inflows and taxes higher as compared with the
capitalist target scenario. As with all other scenarios debt declines as a share of GDP, because initially
public dis-saving is much lower than economic growth and later because the transfer deficit turns into a
surplus. The surplus is smaller in this scenario and, hence, public assets do not rise as sharply as under
the capitalist scenario. Still at the end of the simulation, the transfer system has a surplus, public saving
is very high at about 8% of GDP, and public assets are growing.

The size of government constraint binds in roughly 2070 and we see a downward shift in the transfer
inflow profile.

**Public transfer inflows by purpose**
The effects of changes in population age structure on pensions, health, education and other public
transfer inflows can be seen in Figure 9 which reports simulation results for India using fixed age profiles
for each purpose. The normalized age profiles by purpose are shown in panels B to E (labels missing, but
going counter-clockwise as before). Education is concentrated among children (panel B). Pensions flow
to all adults perhaps because of survivor benefits and are modest (panel C). Health benefits also are
highest at older ages and similar in magnitude to pension benefits (panel E). Other public transfer inflows are somewhat higher for adults than children and do not vary much with adult age (panel D).

![Graphs showing public transfers inflows and outflows](Figures\Components_INdia_Fixed profiles 03-Jan-2015.bmp)

Panel A displays public transfer inflows, total and by purpose, as a percentage of GDP starting from the base year 2010. As shown in panel B of Figure 6, public transfer inflows as a percentage of GDP declines from above 15 percent in 2010 to a low of less than 14 percent in the early 2040s and then rises again to about 15 percent of GDP at the end of the simulation. The decline is driven by education spending which is expected given the decline in child dependency in India that is on-going.

Reinforcing the decline in education is a decline in other public transfer inflows as a share of GDP. This occurs because favorable changes in population age structure leading to more rapid growth in GDP per capita.

Public spending on pensions and health care are rising as a share of GDP over the course of the simulation, but the values are small. The rise in overall public transfers as a share of GDP is as much a consequence of the rise in other public transfer inflows. The only explanation for this change is that after mid-century, labor income is growing more slowly than productivity because of changes in the age structure of the labor force.

The trend in other public transfer inflows is, in part, an artifact of the model specification – the decision to normalize flows on the average labor income of prime age adults. This isn’t obviously superior to other approaches, such as, normalizing on average labor income of all adults. The changes in the India simulation are quite small given the long time period used in the projection.
Public transfer outflows, shown in panel F, are not informative. They differ from public transfer inflows only by net transfers to the rest of the world. Given the naïve approach to modelling net transfers to ROW, the trends in outflows and inflows are identical. Panel F should be dropped and can be replaced with something else.

V. Extensions and further results

Public policy and redefining old age
Not only are people living longer, but they are also healthier. As a consequence, reforms could be introduced extending the normal work life and delaying access to pension and health care benefits. The US is unusual in that only those 65 and older qualify for Medicare, the principal publicly-funded health care system. Other countries might pursue policies that would reduce health care spending commensurate with improvements in health. These policies would result in a delay in the age at which labor income and taxes decline and reduced public transfer inflows at older ages. The potential from this kind of reform has been explored in the past using NTA data by assessing the delay in retirement needed to offset completely the effects of population aging arising both from declining fertility and from longer life (Committee on the Long-Run Macro-Economic Effects of the Aging US Population of the National Academy of Sciences 2012). The approach followed here is to base reform on the observed relationship between the age-specific survival rate, a proxy for health, and age-specific flows. The relationship between survival and annual flows is quantified in one of two ways. For a mature economy we use the observed relationship between the flows and survival rates for the country in question. For the social welfare and capitalist targets we use the average values for the two country groups. For each of the base year flows for Japan and the US, presented here, the mapping between the age specific flows and the age specific survival rates is used to project flows for older adults. As the projected survival rate at any age x increases over time, labor income and taxes at that age adjust upward while public transfer inflows adjust downward.

The normalized age profiles projected for the US using this procedure are shown in Figure 10 (the right-hand-side graphs).
Figure 10. Survival rate driven age of profiles of public transfer inflows, taxes, and labor income, United States. File: ..\Simulations\US Japan surv driven\Profiles USA_scenario 3 No constraints 09-Sep-2014.bmp

The shift over the 90 year simulation, from the solid red to the solid blue line, is quite substantial. The rightward shift is greater at younger ages than at older ages, but we are seeing something like a ten year shift in the profile, a bit less at the older ages. As I recall, this is about the increase in retirement age that Ron Lee suggested was needed to offset the public sector effects of aging in the US. This is certainly a major change but it does amount to about one year per decade.

The effects of such a policy have been assessed by running simulations for the US and for Japan. In Figure 11 I present the US results with the profiles shifting only due to survival rates. No constraints are imposed on the size of government or public debt. These are a little tricky to interpret because the US was running a huge deficit in the base year. In the absence of any reform (holding profiles fixed) public debt would have increased to almost 10 times GDP. Partly this is aging but also it is just the continuation of the deficit spending that characterized the great recession. We will want to purge this from our long-run simulations before the results are finalized.
In any event, the survival related reform has a powerful effect. Despite aging in the US, tax revenues do not decline. It is interesting that they do not increase more than they do, but the policy is driving labor income and, hence, GDP higher as well. Tax revenues are growing more rapidly but so is GDP. We see a substantial change in public transfer inflows relative to GDP. In the absence of reform, public transfer inflows reach about 43 percent of GDP. With the reform, public transfer inflows drop below 30 percent of GDP, less than the 2010 value.

The results for Japan are very similar and are not shown here. Taxes as a share of GDP decline very moderately, but public transfer inflows drop very substantially from just over 40 percent of GDP in 2010 to a little over 30 percent of GDP in 2100.

This method is not appropriate to countries that don’t have mature old-age support systems. One possible way to use this approach would be to use survival-based reform targets. This could be done using social welfare and capitalist targets, such as those employed above, but adjusted for the survival rate of the developing country.

**Costs versus benefits**

The descriptive measures presented above (and the discussion) emphasize the public sector costs of population aging. These are transfers, however, and represent costs to taxpayers and benefits to their recipients. Somehow it does not come across that way. This may be just a matter of exposition, but we should consider measures that emphasize the benefits of public programs in a more compelling manner. Some possible measures: a) human capital spending per child; b) public transfers as a share of consumption for children and the elderly.
**Generational measures**

Public transfers systems have generational effects that are not adequately captured in the material presented above (and in many other studies). The focus above is the impact of population aging on the balance between public sector inflows and outflows. The public sector has an important generational role—shifting resources across age. Population aging requires a fundamental shift providing greater support for old-age populations commensurate with their greater numbers.

Generational effects are explored extensively in Lee’s 1994 work and further applications using NTA (Lee and Mason (2011), chapters 2 and 4, Lee and Mason (2010), and Lee (2000). That analysis makes use of transfer wealth and Lee arrows to summarize generational effects. Public transfer wealth, the present value of net public transfers, can be calculated by age (birth cohort) or at a point in time for the population. Transfer wealth can be calculated for the public transfer system in its entirety or by subsystems with different purposes (education, health, pensions, and other).

The mean ages of flows are calculated and included in some of the figures presented above and reproduced in Figure 12 for the fixed age profile scenario and the social welfare reform scenario. We see from either scenario that the mean ages of consumption and labor income increase under the influence of changing population age structure. Mean age of consumption increases much more than the mean age of labor income which will lead to a rise in the demand for all life cycle wealth, including public transfer wealth. If we look at the mean age of TGI and Tax for the fixed age profile scenario (left panel), we see that public transfers are strongly downward throughout the simulation with the direction of public transfers barely influenced by population aging. We see an entirely different picture with the social welfare reform scenario (right panel). In this case the mean age of taxes rises more slowly as the tax burden is shifted onto working-age adults and we see a very rapid increase in the mean age of public transfer inflows as reform introduces health and pension benefits for older Indians. The direction of transfers reverses at mid-century and is strongly upward by the end of the simulation.
Figure 12. Mean ages of selected flows, India, fixed age profiles (left panel) and social welfare type reform (right panel). Source: See Figures 6 and 8.

The mean ages of taxes and public transfer inflows describe generational features of the public transfer system in the current period. In contrast public transfer wealth is based on expectations about future flows (as projected). Both seem potentially quite useful.

For any transfer system we can calculate the mean age of the inflows and the outflows and we know the magnitude of the flow (per capita flow or flow relative to total consumption or GDP, for example). We can calculate an indicator of the intergenerational impact of public transfers in their entirety or particular components.

\[
IGIndex(t) = vtg(\text{ATGI}(t) − \text{ATGO}(t))
\]

\[vtg = TGI(t)/C(t)\]

\[\text{ATGI}(t) = \sum_{x=0}^{\alpha} xTGI(x,t)/TGI(t)\]

\[\text{ATGO}(t) = \sum_{x=0}^{\alpha} xTGO(x,t)/TGO(t)\]

(2)

The first term on the right-hand-side measures the size of transfer system as total public inflows relative to total consumption. The mean ages measure the direction and the extent (distance across age) of the intergenerational shift in resources. In the steady state, golden rule growth case the IGIndex is equal to public transfer wealth relative to aggregate consumption.

For any current period, assuming that net transfers to ROW are zero, we can show that:

\[AC − AYL = vtg(\text{ATGI} − \text{ATGO}) + vff (\text{ATFI} − \text{ATFO}) + vra(\text{ARA} − \text{AYL})\]

(3)

The difference between the average age of consumption and labor income, the left-hand-side of the equation, is an aggregate summary measure of the economic lifecycle. It tells us at a point in time the number of years of age by which each dollar consumed, on average, precedes or lags each dollar generated through labor. In a young population characterized by high rates of child dependency, the lifecycle problem is that members of the population are, on average, consuming before they are producing. In an old population, the lifecycle problem is that members of the population, on average, are consuming after they are producing.

The right-hand-side of the equation summarizes the contribution of the components of the age-reallocation system to solving the lifecycle problem. The contribution of each component depends on two terms. The first measures the size of the flows for each system as a share of total consumption. The second term measures the extent to which the system shifts resources across age as the difference between the average age at which the inflow is received and the outflow is generated.
Here our interest is in the contribution of the public transfer system to funding the total shift in resources over the lifecycle. In 2010, the mean age of consumption precedes the mean age of labor income by almost seven years. Over time, the lifecycle gap declines. In fifty years, the mean ages of consumption and labor income are equal – the economic lifecycle is balanced, but only temporarily. As aging sets in the mean age of consumption rises relative to the mean age of labor income. By 2100, the mean age of consumption exceeds the mean age of labor income by 2.7 years (Figure 13).

Three public sector scenarios are compared in Figure 13: the status quo scenario based on fixed normalized age profiles of taxes and public transfers, the social welfare scenario and the capitalist scenario. None of the scenarios is subject to constraints on the size of government or public debt.

Essentially, the status quo transfer system does not respond to the changing lifecycle needs of an aging society. The public transfer system contributes about a two years shift in consumption relative to labor income currently and in the future. The current public sector generates resources for children and youth, but not the elderly.

An important point here is that under either reform scenario, the public sector is playing a much more important role in meeting the lifecycle needs of the elderly than meeting the lifecycle needs of children. The public sector is contributing about one third of the resources needed to deal with lifecycle needs of a young population. This is not unusual as familial transfers are generally more important source of support for children than for the elderly. Given either of the high-income country models, the public sector plays a very important role in dealing with lifecycle needs.

Figure 13. Summary of economic lifecycle and contribution of the public transfer system, India, 2010 to 2100. Three scenarios: fixed age profile scenario, unconstrained capitalist scenario, and unconstrained social welfare scenario.
The two reform scenarios are very responsive to population aging. The capitalist profile reorients the public transfer system from one that provides resources sufficient to fund a two year downward shift in the lifecycle to one that funds nearly a two year upward shift in resources. The social welfare scenario is shifting more resources to old age than are needed given current consumption and labor income profiles.

The intergenerational effects of the public sector can be further decomposed to distinguish education, health, pension, and other public programs. Namely,

\[
vtg(\text{ATGI} - \text{ATGO}) = vtge(\text{ATGEO}) + vtgh(\text{ATGHI} - \text{ATGHO}) + vtgp(\text{ATGPI} - \text{ATGPO}) + vtgx(\text{ATGXI} - \text{ATGXO})
\]

where vtg with an e, h, p, or x appended denotes the aggregate inflows for each sector as a share of total consumption, the mean ages for inflows and outflows for education is represented by ATGEO and ATGEO, and the mean ages for other sectors are similarly represented using H, P, and X to represent health, pensions and other public transfers.

VI. Comparative Results from 11 countries

The public sector model has been applied to eleven countries that were selected to represent different regions, levels of development, and demographic conditions. Middle income countries are heavily represented. Large countries were selected from Africa (Nigeria and South Africa), Asia (China, Thailand, and India), and Latin America (Brazil and Mexico). Japan, Germany, and the United States are included to represent high-income countries which do vary in the pace of aging and their policies. Hungary is included as a transition economy.

Demographic conditions vary considerably across these countries (PRB 2014). Nigeria is early in its demographic transition. Mortality conditions have improved to some extent, although life expectancy at birth is still only 52 years. Its total fertility rate is somewhat below its peak, but remains quite high at 5.5 births per woman. As a consequence, Nigeria’s population is very young. The demographic transition is much further along in the other countries studied here. Life expectancy at birth is 60 in South Africa and 66 in India. Fertility has dropped to near or below replacement level, 2.1 births per woman, in all. In Brazil, India, and South Africa the total fertility rate is slightly above replacement level. Below replacement fertility has taken hold in the other developing countries, Brazil, Thailand, and China. The decline in fertility is leading to a rise in the share of the working-age population and the demographic dividend. With a delay lower fertility combined with longer life expectancy will lead to aging and a decline in the share of the working age population.

Four of the countries are further along in their demographic transitions. Japan, Germany, and Hungary all have very low fertility rates – 1.4 births per woman in the case of Japan and Germany and 1.3 births per woman in Hungary. As a consequence, they can expect to experience very rapid population aging. The US fertility rate is higher and for this reason among others population aging will be more gradual.
Age structure and the economies

Population aging has important implications for the economy because of the mis-match between consumption and labor income over the lifecycle. Those at the beginning and the end of life are consuming much more than they produce through their labor while “prime-age” adults are producing more than they are consuming. The lifecycle and changes in population age structure over the demographic transition lead to two important changes in the economy. First, the number of workers rises and falls relative to the number of workers. This phenomenon supported more rapid economic growth in both developed and developing countries in the past. More recently in most developed countries, however, the workforce is declining relative to the number of consumers, thereby slowing economic growth.

Second, the direction of intergenerational flows is changing. In the past and still today, the predominant direction of economic resources flows is downward from adults to children. These economies were youth oriented with large public and private intergenerational transfers to children. The direction of flows is reversing, however, as populations age. In many advanced countries, the direction of economic flows is now upward. As time progresses, more countries will certainly switch from youth to elderly oriented economies.

Changes in the relative number of consumers and producers are captured by the support ratio, plotted for each country in Figure 14. The values for each country are expressed relative to its support ratio in 2015, which is set to 1. Four developing countries, Nigeria, India, Mexico and South Africa, are experiencing the first demographic dividend – their support ratio is rising. In six other countries the support ratio is projected to decline. For the next two decades the most rapid decline in the support ratio occurs in China and Germany. Japan and Thailand are also experiencing very rapid decline in their support ratios. More moderate decline is projected for Hungary and the US. Currently Brazil’s support ratio is changing little, but decline is on the horizon.

Given output per worker, the projected trend in the support ratio would produce an increase in GDP per effective consumer of 20 percent in Nigeria over a fifty year period. This modest gain would be greater if fertility declined more rapidly than anticipated in medium scenario produced by the UN. At the other extreme, in Thailand, GDP per effective consumer would decline by 30 percent from 2015 levels given output per effective worker. A critical issue is whether or not the demographic changes are likely to lead to changes in productivity (GDP per effective worker), yielding what we term a second demographic dividend.
Figure 14. Support ratio, eleven countries, 2015 – 2065. Source: Calculated by authors. File: active\world bank\Draft paper1\Figures\comparative figures.xlsx

The results in Figure 14 cannot be used to make compare the levels of the support ratio across countries. All values in 2015 have been normalized to equal 1. The value of 1.2 for Nigeria versus 0.7 for Thailand in 2065 tells us nothing about the age structure of Nigeria versus Thailand in that year. The numbers only tell us that the changes between 2015 and 2065 were favorable in Nigeria and unfavorable in Thailand.

The direction of economic flows is important because of the implications for public and private transfers, saving and assets, and financial markets. An economy that is heavily youth oriented will require large age reallocations or resource shifts from older to younger ages. In practice, this is accomplished through public and private transfer systems that are subject to stress and strain. To a lesser degree, resources for youth can be generated relying on credit markets – student loans, for example. An economy that is elderly oriented will require reallocations are resource shifts from younger to older ages. Again public and private transfers often play an important role in generating resources for older residents. Working age adults can also accumulate assets, through funded pensions, personal saving, acquisition of a home, etc., to support themselves in old age. As economies become more elderly oriented, there are important implications for public and private old age transfer systems and also for the demand for assets and for financial markets.

The direction of economic flows in an economy is assessed by comparing the age at which people produce resources through their labor with the age at which people consume those resources. The age at which resources are produced is summarized by the mean age of labor income, calculated using the age of workers weighted by their labor income. We also refer to this as the mean age of effective
workers. The mean age of consumption, also referred to as the mean age of effective consumers, is calculated in similar fashion weighting those at each age by the amount consumed.

The resource shift is downward in seven countries led by Nigeria with a downward shift of almost 13 years of age. There the average age of effective consumers (not shown) was only 24 as compared with an average age of effective workers of 37. South Africa, Mexico, and India were also strongly youth oriented, with large intergenerational shifts from adults to children. The economies of four countries are elderly oriented with resources flowing upward. The resource shift is greatest, about 4 years of age, in Japan and Germany. The shift is much smaller in Hungary and in China generational balance almost prevails.

<table>
<thead>
<tr>
<th>Country</th>
<th>Resource Shift (year of ages)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nigeria</td>
<td>-15.0</td>
</tr>
<tr>
<td>S Africa</td>
<td>-10.0</td>
</tr>
<tr>
<td>Mexico</td>
<td>-5.0</td>
</tr>
<tr>
<td>India</td>
<td>0.0</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.0</td>
</tr>
<tr>
<td>Thailand</td>
<td>0.0</td>
</tr>
<tr>
<td>China</td>
<td>0.0</td>
</tr>
<tr>
<td>Hungary</td>
<td>0.0</td>
</tr>
<tr>
<td>US</td>
<td>0.0</td>
</tr>
<tr>
<td>Germany</td>
<td>0.0</td>
</tr>
<tr>
<td>Japan</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td><strong>Upward</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Downward</strong></td>
</tr>
</tbody>
</table>

Figure 15. Resource shift in years of age, 11 countries, 2015 (upper panel) and 2065 (lower panel). Shift calculated as the mean age of consumption less the mean age of labor income. Projection to 2065
The impact of aging on the direction of economic flows is very apparent when we compare the resource shifts projected for 2065 with the values for 2015. These changes are due entirely to changing population age structure as the age profiles of consumption and labor income are fixed (in relative terms). Only two economies, those of Nigeria and South Africa, are projected to have a youth orientation in fifty years. India’s economy is generationally balanced in 2065. The remaining 8 have elderly oriented economies with varying degrees of upward intergenerational flows. The largest age shifts are in Japan and Germany, where effective consumers are ten years older than effective workers. But very large upward flows are projected for many other countries with 7 years shifts found in China, Thailand, and Brazil.

Although the gaps between the mean age of consumption and labor income are largest in Japan and Germany, this was also the case in 2015. A large increase is projected between 2015 and 2065 – about 6 years in each country – but more rapid adjustment will be required in other countries. The gap between the mean ages is projected to increase by 11.5 years in Thailand, 10.5 years in Brazil, 8.4 years in Mexico, and 7.0 years in China. These economies may experience the most profound economic effects from changing population age structure.

Public sector
Analyses of public sector and age structure often emphasize budgetary pressures that arise with population aging. This is an important topic that we return to below, but our treatment of the public sector begins with a focus on benefits, public transfer inflows, provided by the public sector and how they are likely to be influenced with changing age structure. We show that given current age profiles of public transfer inflows, the public sectors in some countries would play a very important role in reallocating resources over the lifecycle as their populations age. In other countries this is not the case. The public sectors in some countries are very small and not designed to support the needs of aging societies.

The public sectors in the eleven study countries vary substantially in their size and their purpose. Both influence their effect on intergenerational flows. Size is measured in two ways in Table 1 which presents status quo projected values for 2015. Public transfer inflows as a share of GDP measures the claim on national resources while public transfers as a share of consumption captures the potential impact of the public sector on consumption patterns. The two measures are correlated but can differ substantially because of large variation across countries in consumption as a share of GDP. Public transfers play the largest potential role in affecting consumption in Hungary, followed by Germany, Japan, and Brazil. In these four countries, public transfer inflows exceed one-half of total consumption. Public transfers have the smallest potential impact in Nigeria followed by Mexico and India. In these countries, public transfer inflows are less than one-quarter of total consumption. The public sector has a moderate potential impact on consumption, with public transfer inflows varying from one-third to one-half of total consumption, in the other four countries: Thailand, the US, South Africa, and China.
The broad purpose of public transfer systems, as assessed here, is whether the programs effect large downward or upward transfers. This is determined by comparing the mean age of public transfer outflows with the mean age of public transfer inflows. In Japan, for example, public transfers outflows originate with taxpayers who are 50.1 years of age, but are destined to beneficiaries who are 54.8 years of age. (Note that the values are calculated by weighting each age group by the size of the transfer inflow or outflow.) On average, then, the public transfer systems shifts resources upward by 4.7 years. The public transfer system in Japan, Germany, Hungary and China all have public sectors that are oriented toward the elderly – they shift resources upward from younger people to older people.

Table 1. Public sector features projected to 2015.

<table>
<thead>
<tr>
<th></th>
<th>Public transfer inflows as a share of GDP</th>
<th>Public transfer inflows as a share of Consumption</th>
<th>Mean ages</th>
<th>Public transfer age shift</th>
<th>Public sector orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan</td>
<td>44.0</td>
<td>56.7</td>
<td>54.8</td>
<td>50.1</td>
<td>4.7</td>
</tr>
<tr>
<td>Germany</td>
<td>41.5</td>
<td>60.8</td>
<td>53.2</td>
<td>49.0</td>
<td>4.2</td>
</tr>
<tr>
<td>US</td>
<td>33.2</td>
<td>38.3</td>
<td>45.6</td>
<td>49.5</td>
<td>-3.9</td>
</tr>
<tr>
<td>Hungary</td>
<td>41.0</td>
<td>68.5</td>
<td>47.4</td>
<td>41.3</td>
<td>6.0</td>
</tr>
<tr>
<td>China</td>
<td>17.1</td>
<td>44.7</td>
<td>43.1</td>
<td>40.7</td>
<td>2.3</td>
</tr>
<tr>
<td>Thailand</td>
<td>19.6</td>
<td>32.9</td>
<td>36.8</td>
<td>42.7</td>
<td>-5.9</td>
</tr>
<tr>
<td>Brazil</td>
<td>37.9</td>
<td>56.3</td>
<td>37.6</td>
<td>41.2</td>
<td>-3.6</td>
</tr>
<tr>
<td>India</td>
<td>15.3</td>
<td>24.8</td>
<td>30.0</td>
<td>38.8</td>
<td>-8.8</td>
</tr>
<tr>
<td>Mexico</td>
<td>16.8</td>
<td>23.4</td>
<td>34.3</td>
<td>37.6</td>
<td>-3.3</td>
</tr>
<tr>
<td>S Africa</td>
<td>30.4</td>
<td>43.3</td>
<td>30.9</td>
<td>43.1</td>
<td>-12.3</td>
</tr>
<tr>
<td>Nigeria</td>
<td>13.2</td>
<td>16.9</td>
<td>20.8</td>
<td>35.8</td>
<td>-15.0</td>
</tr>
</tbody>
</table>

Note. All values based on fixed age profiles of public transfer inflows and outflows and UN population projections (medium fertility scenario). Source: Calculated by authors.

File: comparative figures.xlsx

The public transfer system in the other seven countries are oriented towards children and youth. In Nigeria, public transfer outflows originate on average from those who are about 36 years old and are destined for those who, on average, are about 21 years old. The downward resource shift is a very large 15 years. The downward shifts are also large in South Africa, India, and Thailand. The US system stands out among high income countries for being mildly youth oriented.

The effect of changes in population age structure on the size and purpose of the public sector varies considerably depending on the country. First, we consider the effect on the size of public transfer inflows of population age structure between 2015 and 2065 (Figure 16). Changes in age structure have little effect in three countries. In India and Nigeria, public transfer inflows as a percentage of consumption decline slightly as a consequence of changes in population age structure. South Africa experiences little change over this fifty year period. This is a consequence of the public sector focus in
these countries – more on children and less on the elderly. Decline in the number of children reduced the demand for public spending while aging has relatively little effect.

![Figure 16. Public transfer inflows as a percentage of total consumption, 11 countries, 2015 and 2065. Source: Calculated by authors. File: active\world bank\Draft paper1\Figures\comparative figures.xlsx](active\world bank\Draft paper1\Figures\comparative figures.xlsx)

In other countries, given current age patterns of public transfer inflows, changes in population age structure would produce an increase in public transfer inflows relative to consumption. The rise is greatest in China with an increase of 10 percentage points followed by Germany with a rise of 8 percentage points. In Mexico, Hungary, Brazil, and the US, a rise of between 4 and 6 percentage points is projected.

Population aging has a very modest effect on the generational orientation of the public transfer systems in Nigeria and India (Figure 17). In 2065 after 50 years of changing age structure, the public sectors in both countries are shifting resources heavily downward. The downward shift in South Africa, which drops from about 12 years to about 7 years, is still strongly downward in 2065. In four countries, Mexico, Brazil, Thailand, and the United States, public transfers have reversed direction from downward to upward. In the remaining four countries, the population aging has strongly reinforced the elderly orientation already present in 2015.

Note that the direction of public transfers depends on the age structure of effective taxpayers which, in turn, depends on the population age structure and features of the tax system. A proportional tax on labor income, for example, would produce a mean age of public transfer outflows that was identical to the mean age of labor income. Income or property tax will typically result in a higher mean age of outflows higher. A progressive tax system will have the same effect to the extent that income rises with age.
Figure 17. Resource shift in years of age from public transfers, 2015 and 2065. Source: Calculated by authors. File: active\world bank\Draft paper1\Figures\comparative figures.xlsx

Public Revenue, budget balance, and debt
Changes in population age structure influence tax revenues to the extent that taxes vary over the lifecycle. Tax revenues as a share of GDP (Figure 18) vary with age structure depending on the differential effect of age structure on GDP and tax revenues. Rapid growth in the working age population, for example, leads to more rapid growth in earnings taxes and more rapid growth in GDP with no obvious effect on earnings taxes relative to GDP.

In two countries, Nigeria and Hungary, changes in population age structure have no effect on projected tax revenues as a share of GDP. In the other nine countries, changes in age structure over the next 50 years lead to a rise in tax revenues relative to GDP. The changes are fairly substantial with tax revenues as a share of GDP increasing by six percentage points in Brazil and five percentage points in Germany and China. More moderate increases, ranging from one percentage in India to three percentage points in Japan, Thailand, and South Africa, are projected elsewhere. The changes are concentrated between 2015 and 2050 with little change after that except in Brazil.
Will the increase in revenues be sufficient to cover the cost of public programs? The gap between tax revenues and public spending is measured by the transfer deficit expressed as a proportion of GDP in Figure 19. The transfer deficit is projected to increase in the eight countries for which the fiscal support ratio is declining as shown above. The impact of aging given the constant age profile scenario would be especially severe with the transfer deficit approaching one-quarter of GDP in 2065. This partly reflects the impact of aging but also the high initial transfer deficit. The increase in Germany of 14 percentage points is almost as great followed by Hungary (11 percentage points), Brazil (7.5 percentage points), and China, Thailand, and the US (6 percentage points).

In three countries, India, South Africa and Nigeria, the transfer deficit is declining as a share of GDP. The decline is greatest in South Africa (5 percentage points) while the decline is by about 2.5 percentage points in India and South Africa.
Figure 19. Public transfer deficit as a share of GDP, 2015 to 2065, 11 countries. Source: Calculated by authors. File: Figures\comparative figures.xlsx

Transfer deficits as large as those projected for advanced economies and some middle income economies are completely unsustainable as can be seen from the debt projections shown in Figure 20. By 2050 debt would exceed 400 percent of GDP in Japan, Germany, Hungary and the United States. By 2065 debt in Nigeria and Brazil would approach 200 percent of GDP. In India, Mexico, and South Africa public assets would be positive. In rapidly aging China and Thailand, net debt would be equal to about 50% of GDP.
In the final results presented here we compare outcomes for the 11 countries in 2065 for three scenarios: The fixed profile scenario, the capitalist scenario, and the social welfare scenario (Table 2). The capitalist and social welfare scenarios are both subject to constraints on the size of government and debt. The size of government constraint is that taxes and public transfer inflows do not exceed 35 percent of GDP in the capitalist scenario or 45 percent of GDP in the social welfare scenario. The net debt constraint is 90 percent of GDP for both scenarios. The capitalist and social welfare scenarios assume that the target profiles are met when the country achieves high-income status.

Three sets of results are presented. The first panel reports public transfer inflows as a share of total consumption. The second panel reports the mean age of public transfer inflows less the mean age of public transfer outflows. The final panel reports the intergenerational index – the product of the values in the first two panels.

Rather than discuss the values in detail we will make only a few broad observations. First, size of government and debt constraints play an important role in the outcomes. For the social welfare scenario one or more constraints is binding in every country except India and South Africa. The debt constraint binds in Mexico after 2070 and the size of government constraint binds in Thailand shortly after 2050. Nigeria is an unusual case because tax revenues are so low that the debt constraint binds between 2025 and 2050. If oil revenues were to recover a very different scenario could play out, but in these simulations prospective debt problems are resolved by substantially raising taxes.

The impact of the size of government constraint on public transfer inflows as a share of consumption varies because of differences across countries in consumption as a share of GDP. Most notably, consumption in China is very low relative to GDP and, hence, constraining public transfer inflows to be no more than 35 or 45 percent of GDP is consistent with public transfer inflows that are very substantial relative to consumption.
Table 2. Effect of alternative scenarios.

<table>
<thead>
<tr>
<th>Country</th>
<th>2015 fixed</th>
<th>2065 fixed</th>
<th>2065 capitalist constrained</th>
<th>2065 social welfare constrained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public transfer inflows as a share of consumption</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>0.57</td>
<td>0.62</td>
<td>0.35</td>
<td>0.45</td>
</tr>
<tr>
<td>Germany</td>
<td>0.61</td>
<td>0.69</td>
<td>0.40</td>
<td>0.51</td>
</tr>
<tr>
<td>US</td>
<td>0.38</td>
<td>0.42</td>
<td>0.30</td>
<td>0.46</td>
</tr>
<tr>
<td>Hungary</td>
<td>0.68</td>
<td>0.72</td>
<td>0.43</td>
<td>0.63</td>
</tr>
<tr>
<td>China</td>
<td>0.45</td>
<td>0.54</td>
<td>0.60</td>
<td>0.87</td>
</tr>
<tr>
<td>Thailand</td>
<td>0.33</td>
<td>0.34</td>
<td>0.36</td>
<td>0.53</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.56</td>
<td>0.62</td>
<td>0.37</td>
<td>0.54</td>
</tr>
<tr>
<td>India</td>
<td>0.25</td>
<td>0.23</td>
<td>0.26</td>
<td>0.38</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.23</td>
<td>0.28</td>
<td>0.31</td>
<td>0.53</td>
</tr>
<tr>
<td>S Africa</td>
<td>0.43</td>
<td>0.43</td>
<td>0.33</td>
<td>0.55</td>
</tr>
<tr>
<td>Nigeria</td>
<td>0.17</td>
<td>0.16</td>
<td>0.33</td>
<td>0.49</td>
</tr>
<tr>
<td>Mean age of public transfer inflows less outflows</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>4.68</td>
<td>9.70</td>
<td>11.82</td>
<td>14.14</td>
</tr>
<tr>
<td>Germany</td>
<td>4.07</td>
<td>8.42</td>
<td>9.70</td>
<td>12.06</td>
</tr>
<tr>
<td>US</td>
<td>-3.94</td>
<td>1.05</td>
<td>5.27</td>
<td>8.06</td>
</tr>
<tr>
<td>Hungary</td>
<td>6.02</td>
<td>10.37</td>
<td>6.87</td>
<td>9.43</td>
</tr>
<tr>
<td>China</td>
<td>2.32</td>
<td>7.21</td>
<td>6.29</td>
<td>8.76</td>
</tr>
<tr>
<td>Thailand</td>
<td>-5.88</td>
<td>3.31</td>
<td>9.00</td>
<td>11.23</td>
</tr>
<tr>
<td>Brazil</td>
<td>-3.56</td>
<td>4.10</td>
<td>7.67</td>
<td>10.10</td>
</tr>
<tr>
<td>India</td>
<td>-8.77</td>
<td>-7.26</td>
<td>-1.46</td>
<td>1.50</td>
</tr>
<tr>
<td>Mexico</td>
<td>-3.31</td>
<td>8.64</td>
<td>6.87</td>
<td>9.35</td>
</tr>
<tr>
<td>S Africa</td>
<td>-12.25</td>
<td>-6.86</td>
<td>-0.06</td>
<td>2.80</td>
</tr>
<tr>
<td>Nigeria</td>
<td>-15.03</td>
<td>-14.67</td>
<td>-10.71</td>
<td>-7.42</td>
</tr>
<tr>
<td>Intergenerational index</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>2.65</td>
<td>6.03</td>
<td>4.11</td>
<td>6.32</td>
</tr>
<tr>
<td>Germany</td>
<td>2.47</td>
<td>5.79</td>
<td>3.85</td>
<td>6.15</td>
</tr>
<tr>
<td>US</td>
<td>-1.51</td>
<td>0.44</td>
<td>1.58</td>
<td>3.70</td>
</tr>
<tr>
<td>Hungary</td>
<td>4.12</td>
<td>7.52</td>
<td>2.95</td>
<td>5.96</td>
</tr>
<tr>
<td>China</td>
<td>1.04</td>
<td>3.92</td>
<td>3.81</td>
<td>7.65</td>
</tr>
<tr>
<td>Thailand</td>
<td>-1.93</td>
<td>1.13</td>
<td>3.28</td>
<td>6.00</td>
</tr>
<tr>
<td>Brazil</td>
<td>-2.00</td>
<td>2.53</td>
<td>2.82</td>
<td>5.45</td>
</tr>
<tr>
<td>India</td>
<td>-2.17</td>
<td>-1.68</td>
<td>-0.38</td>
<td>0.58</td>
</tr>
<tr>
<td>Mexico</td>
<td>-0.78</td>
<td>2.40</td>
<td>2.14</td>
<td>4.94</td>
</tr>
<tr>
<td>S Africa</td>
<td>-5.31</td>
<td>-2.97</td>
<td>-0.02</td>
<td>1.54</td>
</tr>
<tr>
<td>Nigeria</td>
<td>-2.54</td>
<td>-2.33</td>
<td>-3.50</td>
<td>-3.67</td>
</tr>
</tbody>
</table>
Note. For capitalist scenario taxes and transfer inflows are constrained to 35% of GDP. The size constraint for the social welfare scenario is 45% of GDP. Both scenarios are constrained to have net debt equal to 90% of GDP.
Source: Calculated by authors.

The mean age of public transfer inflows less outflows can be heavily influenced by the reform scenarios. The constraints are not as important here as they are met through rescaling spending and taxes at all ages rather than by reforming particular programs. The reforms exert influence both by affecting the inflows and outflows. The US is a case in point. The capitalist scenario leads to a substantial increase in the extent to which public transfers are flowing in an upward direction – 5.27 years versus 1.05. Most of this change reflects changes on the tax side rather than on the spending side.

The reform scenarios do not have much effect on the intergenerational shift in resources in China but we see very large changes in other middle and low income countries. For example, the capitalist reform leads to an increase in the upward flow of resources by 5.5 years in Thailand, 5.8 years in India, and 6.8 years in South Africa.

VII. Conclusions
Population aging has important implications for public finances. First, population aging will force difficult decisions about the overall size of government and the taxes required to fund operations. Moderately high fertility creates a population age structure that is relatively favorable for public finances, but as development and the demographic transition proceed, moderately high fertility appears to be an unlikely outcome. Moreover, public finances is only one of many important considerations that should guide public policy towards fertility and reproductive health (Lee, Mason et al. 2014).

Second, population aging will require a substantial restructuring of what government does. Broadly speaking, aggregate public resources will and should be used to address important needs of the elderly. This simply reflects the large increase in the number of elderly relative to those in other age groups. The changes in age pattern of spending are also reflected in the growth in programs that deliver those resources – predominantly public pensions and publicly funded health care systems.

Restructuring also means changes in the resources available for children. Inevitably, total public resources devoted to children as a group will not grow as rapidly as public resources devoted to the elderly. Again this reflects a simple but important reality that there are fewer children relative to the number of elderly. In many countries, the number of children is declining in absolute terms. This does not necessarily mean that resources devoted to children will decline, just that they will decline relative to the resources available to the elderly. And it certainly does not mean the resources per child will decline. This is an issue that should be addressed in more detail in the future research.
High income countries have mature, established public systems of old-age support. These were established under highly favorable demographic circumstances – many taxpayers, not so many elderly. These programs are not sustainable in their current form. Adjustments are inevitable, but the form that those adjustments take is a very open question.

Some middle-income countries, Brazil being a prime example, also have mature, established public old-age support systems. For these countries, reform to current systems will be critical. Other countries, like India, have a relatively clean slate when it comes to old-age support systems. This is advantageous in some respects because fulfilling long-term, but perhaps impossible, commitments is not an issue.
Appendix A: Model documentation

Variables, notation, and data
We use standard NTA variable notation (see UN manual). Notation for other variables is provided in the section of the paper which describes how they are computed. Upper case letters are used to denote aggregate values for an age group and lower case letters for per capita values. The age index is \( x \), or \( x \) and \( y \) as needed, and the year index is \( t \). The base year is indicated by \( t=b \). Variables with no age argument represent values computed across all ages. For example, \( Y_l(x,t) \) is the total labor income of persons age \( x \) in year \( t \); \( y_l(x,t) \) is per capita labor income of persons age \( x \) in year \( t \). \( Y_l(t) \) and \( y_l(t) \) are aggregate labor income and per capita labor income, respectively.

All values are nominal unless otherwise indicated. Stock values are as of the end of the year. Present value calculations involving flows assume that the flows are realized at the end of the year in which they occur.

We make use of a growth index denoted by a leading “\( g \)”. The growth index is defined as:

\[
g_Z(t) = \frac{Z(t)}{Z(t-1)} \\
g_Z(x,t) = \frac{Z(x,t)}{Z(x,t-1)}.
\]

The baseline year for the model is a recent year for which aggregate data, but not necessarily NTA data, are available. Depending on the country, the baseline year is 2009, 2010, or 2011. For many countries the most recent NTA estimates are not available for the baseline year, but have been constructed for an earlier year we call the profile year. The NTA estimates are updated to the baseline year relying on the model. More detailed information about the procedures is provided in Appendix A.

Demographic and Macroeconomic Block
The variables that are introduced or constructed within this block are described in Table 1.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Variable name</th>
<th>Calculation method</th>
</tr>
</thead>
<tbody>
<tr>
<td>N(x,t)</td>
<td>Population by age</td>
<td>UN World Population Prospects, medium fertility scenario (1).</td>
</tr>
<tr>
<td>y_l(x,t)</td>
<td>Per capita labor income by age</td>
<td>( y_l(x,t) ) shifts due to productivity growth. YoLY(t) is simple average of single-year values. Normalized value is labor income divided by YoLY.</td>
</tr>
<tr>
<td>YoLY(t)</td>
<td>Average labor income of 30-49-year-olds</td>
<td></td>
</tr>
<tr>
<td>y_l_n(x)</td>
<td>Normalized per capita labor income</td>
<td></td>
</tr>
<tr>
<td>Y_l(x,t)</td>
<td>Total labor income by age</td>
<td></td>
</tr>
<tr>
<td>Y_l(t)</td>
<td>Aggregate labor income</td>
<td></td>
</tr>
<tr>
<td>ya(x,t)</td>
<td>Per capita asset income by age</td>
<td>Per capita profile fixed in relative</td>
</tr>
<tr>
<td>YA(x,t)</td>
<td>Total asset income by age</td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Notes</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>-------</td>
</tr>
<tr>
<td>YA(t)</td>
<td>Aggregate asset income</td>
<td>terms, but scaled to match total asset income. Total asset income assumed to grow at same rate as total labor income.</td>
</tr>
<tr>
<td>GDP(t)</td>
<td>Gross domestic product</td>
<td>Grows at the same rate as labor income.</td>
</tr>
<tr>
<td>r(t)</td>
<td>Nominal interest rate</td>
<td>Equal to the nominal rate of GDP growth plus a country-specific risk premium (risk) that is exogenously specified.</td>
</tr>
</tbody>
</table>

(1) See data appendix for sources and technical information.

**Exogenous inflation and productivity growth**

The rate of inflation is incorporate using an exogenously determined price index:

\[
gP(t) = \frac{P(t)}{P(t-1)}
\]  

(6)

The index of nominal productivity growth is incorporated using:

\[
gyl(t) = gryl(t)gP(t)
\]  

(7)

Where \(gryl(t)\) is the exogenous specified index of real productivity growth.

**Population**

Population of age \(x\) in year \(t\) is exogenously determined based on the medium fertility scenario from the UN World population prospects. The index of population growth is calculated as:

\[
gN(x, t) = \frac{N(x, t)}{N(x, t-1)}
\]  

(8)

**Labor income**

Age-specific per capita nominal labor income grows at the real rate of nominal productivity growth:

\[
yl(x, t) = yl(x, t-1)gyl(t).
\]  

(9)

Total labor income by age and aggregate labor income are calculated as:

\[
Yl(x, t) = N(x, t)yl(x, t)
\]

\[
Y(t) = \sum_x Yl(x, t)
\]  

(10)

All aggregate values are calculated in similar fashion and this will not be repeated.
**Asset income**

Asset income is assumed to grow at the same rate as aggregate maintaining a constant share of total primary income (asset income plus labor income). The age profile of asset income is scaled, given the population age distribution, to realize the aggregate asset income. The index used to scale the per capita asset income age profile is \( g_{ya}(t) \).

\[
Y_A(t) = g_{YL}(t)Y_A(t-1) \\
g_{ya}(t) = \frac{Y_A(t)}{\sum_x y_a(x,t-1)N(x,t)}
\]  

(11)

where \( g_{YL}(t) \) is the growth index for total labor income. Per capita asset income is calculated as:

\[
y_a(x,t) = y_a(x,t-1)g_{ya}(t).
\]

(12)

This approach is appealing in that it implies that labor income and asset income are constant shares of primary income. This is a property of some well-known production functions that capital income and labor income are constant shares of GDP. The approach is broadly consistent with NTA evidence that the per capita asset income age profile are lower relative to the labor income profile in old, high-income countries because asset income is more heavily concentrated at older ages than labor income.

**Gross Domestic Product (GDP) and Primary Income (not sure this is every used)**

Nominal gross domestic product is assumed to growth at the same rate as nominal labor income, \( g_{YL}(t) \):

\[
GDP(t) = g_{YL}(t)GDP(t-1)
\]

(13)

Primary income is equal to labor income plus asset income by definition:

\[
y(x,t) = y_l(x,t) + y_a(x,t) \\
Y(x,t) = Y_L(x,t) + Y_A(x,t)
\]

(14)

**Nominal interest rate**

The nominal interest rate is calculated as the nominal rate of GDP growth plus an exogenously specified country-specific risk premium, \( risk \).

\[
r(t) = g_{GDP}(t)(1+risk) - 1
\]

(15)

**Public Sector**

In the NTA framework flows are modelled from the perspective of individuals or the age groups to which individuals belong. Public flows are defined as those that originate in age groups and are destined for age groups mediated by or dictated by national, state, or local governments. Public flows are classified as falling within two systems that conform to the economic mechanisms by which resources are reallocated across age: public transfers and public asset-based reallocations. The broad outlines of the public sector are represented in Figure 2.
The public transfer system consists of public transfer inflows to and outflows from those at each age. Including net transfers to the rest of the world, inflows and outflows must be equal. Tax revenues are used to fund public transfer outflows. If tax revenues are more than sufficient, the result is a public transfer surplus. If tax revenues are insufficient, they generate a public transfer deficit.

Governments balance a public transfer surplus or deficit by relying on public asset-based reallocations. A transfer deficit, for example, can be funded by relying on public asset income and, if needed, public borrowing (negative saving). If public asset income exceeds the public transfer deficit, the remainder is saved. If governments have a public transfer surplus and public asset income, the surplus tax revenues and asset income are saved. Public asset income is often negative as many governments must pay interest on their public debt. In this case, public saving is realized in the public transfer surplus is more than sufficient to cover negative public asset income. If it is insufficient, public dis-saving will occur.

Figure 2. Population age structure and the public sector: public transfer system in lower left and public asset system to right. File: NTA Model.pptx
The public transfer systems and the public asset systems both influence the wealth of the economy and of each age group. Transfer wealth, measured by the present value of expected net public transfers for each age group, is created by redistributing resources from one age group to another and from future generations to those currently alive. Public assets are another form of wealth (or debt) held by current generations. Through public saving the public asset system generates larger assets or through dis-saving smaller assets.

Population and public policy influence the public sector through two channels: tax revenues and public transfer inflows. Consider what we refer to as the status quo scenario in which the per capita age-specific transfer inflows and tax revenues normalized on YoLYs, shown for Germany in Figure 3, are held constant at their baseline values. Increases in labor productivity lead to proportionately higher age-specific taxes and benefits. Given population age structure, transfer inflows and tax revenues are constant as a share of GDP. The transfer deficit would also be a constant share of GDP. If these circumstances prevailed, the asset system would also reach an equilibrium over time with asset income, saving, and assets constant relative to GDP.

Figure 3. Per capita public transfer inflows and taxes by age, Germany 2003. All values expressed in YoLYs (years of labor income). Source: Calculation by authors based on NTA data. File: Figure 3 tax and tgi profile.xlsx

The influence of population age structure, under the status quo scenario, can easily be seen by comparing the normalized per capita age profiles of public transfer outflows and tax revenues in Germany. Taxes are low and inflows are high for children while taxes are high and inflows are low for working-age adults. Given these profiles, shifts in the population out of young ages and into working ages will lead to a fiscal dividend: a rise in tax revenues relative to inflows. The changing balance between transfer inflows and tax revenues will produce a higher saving rate, the accumulation of public assets (or a reduction in public debt), and higher public asset income (or a reduction in public interest expense).
Shifts in population from the working-age to old age will have the opposite effect leading to deterioration in public finances. Tax revenues would decline relative to inflows. The shifting balance between the two will lead to lower public saving, a decline in public assets (or rise in public debt), and a decline in public asset income (or a rise in public interest expense).

Public policy is incorporated into the mode by shifting the age profiles of public transfer inflows and taxes.

Public sector model
The economic flows and stocks that comprise the public sector model are described in detail in this section. The subsequent section completes the model by describing policy scenarios that determine how the per capita age profiles of taxes and public transfer inflows change over time. The variables in the public sector model are described in Table 2.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Variable name</th>
<th>Calculation method</th>
</tr>
</thead>
<tbody>
<tr>
<td>tgi(x,t)</td>
<td>Public transfer inflows: per capita, normalized, and aggregate.</td>
<td>Per capita inflows shift with YoLY(t) and policy.</td>
</tr>
<tr>
<td>tgi_n(x,t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TGI(x,t), TGI(t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TGtoROW(t)</td>
<td>Net public transfers to the rest of the world.</td>
<td>Held constant relative to TGI(t)</td>
</tr>
<tr>
<td>tgo(x,t)</td>
<td>Public transfer outflows: per capita, normalized, and aggregate.</td>
<td>Aggregate values are calculated as TGI(t) + TGtoROW(t). Per capita values scaled to match TGI(t) using population, N(x,t).</td>
</tr>
<tr>
<td>tgo_n(x,t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TGO(x,t), TGO(t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>tg(x,t)</td>
<td>Net public transfers, per capita and aggregate.</td>
<td>Public transfer inflows less public transfer outflows.</td>
</tr>
<tr>
<td>TG(x,t), TG(t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>tax(x,t)</td>
<td>Taxes paid: per capita, normalized, and aggregate.</td>
<td>Normalized per capita values based on NTA estimates. Per capita values rescaled using YoLY(t).</td>
</tr>
<tr>
<td>tax_n(x,t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TAX(x,t), TAX(t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>tgd(x,t)</td>
<td>Public transfer deficit</td>
<td>Public transfer outflows less taxes.</td>
</tr>
<tr>
<td>TGD(x,t), TGD(t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rg(t)</td>
<td>Nominal interest rate on public assets/debt</td>
<td>Proportional to general interest rate r(t).</td>
</tr>
<tr>
<td>AG(t)</td>
<td>Public assets</td>
<td>Public assets in previous year plus public saving</td>
</tr>
<tr>
<td>yag(x,t)</td>
<td>Public asset income: per capita and aggregate.</td>
<td>Public assets equal product of nominal rg(t) and AG(t-1). yag(x,t) is calculated by rescaling to match YAG(t) given N(x,t).</td>
</tr>
<tr>
<td>YAG(x,t), YAG(t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SG(t)</td>
<td>Public saving in year t</td>
<td>Taxes plus public asset income less public transfer outflows.</td>
</tr>
<tr>
<td>wtg(x,t)</td>
<td>Public transfer wealth</td>
<td>Present value of net public transfers.</td>
</tr>
<tr>
<td>WTG(x,t), WTG(t)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Public wealth

\[ WG(t) \]

Public assets plus public transfer wealth.

\[ TGEI(x,t), TGEO(x,t), TGE(x,t) \]

Public transfers by purpose: education, health, pensions, other cash, and other in-kind.

\[ TGI, TGO, TG \]

Calculated using same methods as used for TGI, TGO, and TG.

\[ CG(x,t), CGE(x,t), CGH(x,t), CGX(x,t) \]

Public consumption.

Public consumption by purpose.

Public consumption is equal to public in-kind transfer inflows.

### Public interest rates

The public interest rate \( rg(t) \) is assumed to be constant at the average effective interest rate for recent years (see data appendix). Then it converges linearly to a constant multiple of \( r(t) \), i.e.,

\[ rg(t) = \beta r(t) \quad (16) \]

### Taxes

The normalized age profile for taxes varies depending across variants described below. For all variants, taxes by age and total tax revenues are given by:

\[
\begin{align*}
tax(x,t) &= tax \_ n(x,t)YoLY(t) \\
YoLY(t) &= \sum_{x=20}^{49} yl(x,t) / 20 \quad (17) \\
TAX(t) &= \sum_{x=0}^{49} tax(x,t)N(x,t)
\end{align*}
\]

### Public transfer inflows

The normalized age profile for public transfer inflows is calculated in different ways depending on the variants as described below. For all variants, public transfer inflows by age and total inflows are given by:

\[ tgi(x,t) = tgi \_ n(x,t)YoLY(t) \quad (18) \]

### Public transfers to the rest of the world

Net public transfers to the rest of world (\( TG\_ROW(t) \)) are assumed to be a constant fraction of public transfer inflows, i.e.,

\[ TGtoROW(t) = TGI(t)(TGtoROW(b)/TGI(b)) \quad (19) \]

### Public transfer outflows and the public transfer deficit

Total public transfer outflows are equal to total public transfer inflows plus net transfers to the rest of the world:
\[ TGO(t) = TGI(t) + TGloROW(t) \]  
(20)

By assumption, public transfer outflows have the same relative age profile as taxes. Thus,

\[
TGO(x,t) = TGO(t) \left( \frac{TAX(x,t)}{TAX(t)} \right) \\
tgo(x,t) = TGO(x,t)/N(x,t)
\]

(21)

The public transfer deficit is equal to public transfer inflows less taxes:

\[ TGD(t) = TGO(t) - TAX(t) \]  
(22)

**Public asset-based reallocations, public asset income, and public saving**

Public asset income is equal to the rate of return to public assets, \( rg(t) \), times the values of public assets at the end of the previous period:

\[ YAG(t) = rg(t)AG(t-1). \]  
(23)

Public saving is equal to:

\[ SG(t) = TAX(t) + YAG(t) - TGO(t) \]  
(24)

Note that YAG includes interest expense on public debt (as a negative value) and that TGO includes all public consumption and cash transfers to residents and to the rest of the world.

**Public transfers by purpose or sector**

Public transfer inflows and outflows can be disaggregated by purpose or sector. NTA distinguishes education \((e)\), health \((h)\), public pensions \((p)\), other cash \((xc)\) and other in-kind \((xk)\) transfers. The inflows and outflows are calculated using the same procedures as employed for public transfers for all sectors combined. The age profiles employed are based on NTA per capita profiles classified by purpose. Per capita public transfer inflows are calculated using:

\[
tgSi(x,t) = tgSi \_n(x,t)YoLY(t) \\
S = e, h, p, xc, xk
\]

(25)

Per capita public transfer outflows are calculated using the tax age profile for all sectors combined.

**Public consumption**

Public consumption is equal to public in-kind transfers by definition, i.e.,

\[
CGE(x,t) = TGEI(x,t) \\
CGH(x,t) = TGHI(x,t) \\
CGX(x,t) = TGXKI(x,t) \\
CG(x,t) = CGE(x,t) + CGH(x,t) + CGX(x,t)
\]

(26)

where TGXKI(x,t) is other in-kind public transfer inflows.
Public wealth
Public assets at the end of period $t$ are calculated by:

$$ AG(t) = AG(t-1) + SG(t) $$

(27)

Public assets by age are calculated using the age-distribution of taxes:

$$ AG(x,t) = AG(t) \frac{TAX(x,t)}{TAX(t)} $$

$$ ag(x,t) = AG(x,t)/N(x,t) $$

(28)

Transfer wealth is calculated as the present value of net public transfers:

$$ WTG(x,t) = \sum_{z=1}^{\omega-z} D(t+z)TG(x+z,t+z) $$

$$ D(t+z) = \frac{1}{t} \prod_{y=1}^{z} (1 + discount_{g}(t+y)) $$

$$ WTG(t) = \sum_{x} WTG(x,t) $$

(29)

where $\omega$ is the maximum years lived and $discount_{g}(t+y)$ is the discount rate for the public sector in year $t+y$, where:

$$ discount_{g}(t+y) = rg(t+y). $$

(30)

Public wealth is the sum of public assets and public transfer wealth:

$$ WG(x,t) = AG(x,t) + WTG(x,t) $$

$$ WG(t) = AG(t) + WTG(t) $$

(31)

Public policy

Variant 1: Status quo or fixed profiles
The fixed profile projections are based on the assumption that the normalized age profiles of taxes and public transfer inflows remain fixed at the base year values, i.e.,

$$ tgi_{n}(x,t) = tgi_{n}(x,b) $$

$$ tax_{n}(x,t) = tax_{n}(x,b) $$

(32)

The fixed profile projections illustrate the impact of population aging in the absence of any reform to taxes and public transfer systems. Imbalances between taxes and spending are captured entirely by public saving, public assets (debt) and, hence, public asset income (interest expense).
Variant 2: Target profiles

Variant 2 of the model provides a general specification that can be employed to consider the fiscal impact of population aging in the face of exogenously specified reform to taxes and public spending. Reform might occur for many reasons, but four are of potential interest. First, low- and middle-income countries may introduce or expand existing programs that provide age-related benefits, e.g., pensions, health care, and education. Second, countries with expansive public transfer systems may choose to retrench existing programs because of large budget deficits, large public debt, or for other reasons, e.g., concern that public transfer programs are adversely affecting work or saving incentives. Third, countries may be considering reform to their tax system that will affect the age groups on which taxes fall or other important features of the tax system. Fourth, countries may choose to introduce reforms in response to improvements in health at older ages, e.g., extending the retirement age.

Projections are based on a transition from current normalized age-profiles of transfer inflows and taxes to target profiles, $tgiT(x,t)$ and $taxT(x,t)$, to be realized in year $t+H(t)$, where $H(t)$ is the planning horizon. We assume that the normalized per capita public flows follow an adjustment path:

$$
tgi_n(x,t+1) = tgi_n(x,t) + \left( tgiT(x,t) - tgi_n(x,t) \right)/H(t)
$$

$$
taxes_n(x,t+1) = taxes_n(x,t) + \left( taxesT(x,t) - taxes_n(x,t) \right)/H(t)
$$

Given the very general nature of this specification, it is worthwhile to discuss a few special cases.

$H(t)=1$: Adjustment occurs immediately. The value in $t+1$ equals the target value in period $t$.

$H(t)=H$: If the target is fixed and $H(t)$ is fixed, then the value approaches the target asymptotically. This occurs because the “planner” is always looking forward the specified number of years. This is illustrated below using $H(t)=20$, $tgi(1)=.1$ and $tgiT=.4$ (Figure 4).

Linear path: A linear adjustment path that realizes the target in $H$ years can be realized by reducing the planning horizon $H(t)$ by one year each year. For example:

$$
H(t) = \bar{H} \text{ for } t = t1
$$

$$
H(t+1) = \max(H(t) - 1, 1) \text{ for } t > t1.
$$

This would describe a case where a policy was introduced in year $t1$ to change a value linearly over the subsequent $H$ years.
Figure 4. Illustration of two adjustment paths to target value.

The asymptotic adjustment and the linear adjustment cases are shown in the figure for adjusting the value of tgi for those of age x from 0.1 to 0.4 with a planning horizon of 20 years.

To insure consistent planning over time, we assume that H(t)-H(t+1) must be greater than or equal to 1.

**Fiscal constraints**

In countries with significant public transfer systems for the elderly, population aging may lead to fiscal problems requiring some combination of higher taxes, lower transfers, and higher public debt. Policy responses can be modelled directly using variant 2. For example, taxes could be raised and transfers reduced proportionately to maintain fiscal balance using the fiscal support ratio.

An alternative approach is to introduce constraints on public finances that must be met by adjusting public spending or revenue streams. Two kinds of constraints are used. One is a constraint on the size of government and the second is a constraint on the size of public debt.

**Size of government constraint**

The size of government constraint limits government’s share of GDP to an exogenously specified ceiling. For the size of government constraint we use a hard constraint that is satisfied in every period it is imposed. The size of government is imposed on both TGI and TAXES as a share of GDP. The variants described above are used to construct provisional projections. If the provisional value of transfer inflows and/or taxes relative to GDP in year t+1 exceeds the size of government constraint, the normalized per capita profiles of public transfer inflows and/or tax revenues are scaled downward to match the size of government constraint.
\[ tgi_{-n}(x,t+1) \leftarrow \min \left( 1, \frac{\text{limit1} \cdot GDP(t+1)}{TGI_p(t+1)} \right) tgi_{-n}(x,t+1) \]

\[ \text{taxes}_{-n}(x,t+1) \leftarrow \min \left( 1, \frac{\text{limit1} \cdot GDP(t+1)}{TAX_p(t+1)} \right) \text{tax}_{-n}(x,t+1) \]

(35)

Where limit1 is the size of government constraint and the subscript p denotes the provisional values of the public sector age profiles. In equation Error! Reference source not found., public transfer inflows are final values because they are not revised further. Taxes by age are provisional because they may be adjusted upward to satisfy public debt constraints.

**Public debt constraint**

A public debt constraint is imposed by exogenously specifying that public debt as a share of GDP cannot exceed a specified limit, limit2. However, we impose this as a soft constraint. If countries exceed the allowed debt, the planning horizon allows for a period during which debt can be brought into conformance. Taxes in period t+1 are raised, in percentage terms, by an amount sufficient to satisfy the public debt constraint if maintained over the subsequent H2 years.

If provisional debt as a share of GDP in year t+H2 falls within the allowable limit, then the normalized tax profile is set to the provisional profile completing the calculation of public sector values. If provisional debt exceeds the debt limit in period t+1, then taxes are scaled higher in period t+1.

Computing the tax increase in period t+1 requires provisional public sector values of public transfer inflows, taxes, asset income, and public debt. These are all calculated as described above. Excess provisional debt in year t+H2 is: \(-AG^p(t+H2) - \text{limit2} \cdot GDP(t+H2)\). If excess debt is positive, taxes are adjusted upward by the amount that would be sufficient if maintained over the period t+1 to t+H2 to eliminate the excess debt. The required scaling factor, k, must satisfy:

\[ -AG^p(t+H2) - \text{limit2} \cdot GDP(t+H2) = kTaxes_p(t+1)(1+r)^{H2-1} + kTaxes_p(t+2)(1+r)^{H2-2} + \ldots + kTaxes_p(t+H2) \]

\[ = k \sum_{z=1}^H Taxes_p(t+z)(1+r)^{H2-z} \]

(36)

The right-hand-side is the additional wealth realized if taxes were increased proportionately at all ages in all years t+1 to t+H2 by k.

Hence, the tax profiles are recalculated to meet the debt constraint by calculating k,

\[ k = \frac{-AG^p(t+H2) - \text{limit2} \cdot GDP(t+H2)}{\sum_{z=1}^H Taxes_p(t+z)(1+r)^{H2-z}} \]

(37)
If $k$ is greater than zero, the finalized tax profile is calculated by scaling up the provisional profile in year $t+1$:

$$\text{taxes}(x,t+1) \leftarrow (1 + k)\text{taxes}_n(x,t+1).$$

(38)

The upward revision of taxes in period $t+1$ will affect the calculation of YAG, SG, and AG in subsequent periods. Hence, these values are recalculated over the period $t+1$ to $t+H_2$. Once the values are updated, we calculate the projected value for $t+H_2+1$.

**Mortality-based targets**

Old-age support systems and retirement policy are not usually tied in any systematic way to improvements in health and disability at older ages. The model employed here is used to show how the public sector would change if work, taxes, and benefits later in life tracked a correlate of health status (death rates) rather than age. This variant of the model is not appropriate as a tool for forecasting public sector flows, because the important flows have not, in fact, tracked improvements in health (or the decline in death rates.) The analysis is useful, however, for assessing to what extent the impact of aging on public spending could be mitigated by tying work, taxes, and benefits to the health status of the population.

The use of mortality-based targets as envisioned here is most directly appropriate only for countries with fully developed old age support systems that provide public pensions and health care benefits intended to meet the needs of older populations who are disabled, experiencing cognitive decline, subject to higher rates of degenerative disease, and other features of senescence. Another use of this approach is to propose mortality-based targets for countries that may choose to develop more extensive old age support systems in the future. In countries with recently implemented old age support systems, benefits may decline at older ages because older individuals are not fully qualified for program benefits. The methods used here are not appropriate for that situation.

Three normalized age profiles are modelled using this approach: public transfer inflows, taxes, and labor income. Only flows to older individuals change over time. Cutoff ages for each flow are selected based on features of the age profile, but in no case are age profiles changed for ages less than 45. The local maximum is used for taxes and labor income, while the local minimum is used for public transfer inflows.

Denoting the death rate at age $x$ in year $t$ as $d(x,t)$, we use $tgi \_ n<d(x,t)>$ to represent a mapping from the normalized public transfer inflows to the age specific death rates based on the observed values in the base year. Similar notation is used for other flows, so that:

$$tgi \_ n(x,t) = tgi \_ n<d(x,t)>$$
$$\text{taxes} \_ n(x,t) = \text{taxes} \_ n<d(x,t)>$$
$$yI \_ n(x,t) = yI \_ n<d(x,t)>$$

(39)
Death rates are based on UN World Population Prospects 2012. By way of illustration the relationship between death rates and NTA flows for five European countries with relatively generous social welfare systems are presented in Figure 5.

![Figure 5. Annual flows (public transfer inflows, taxes, and labor income) for older adults normalized on labor income 30-49 plotted against the death rate. Average of values for five countries with relatively generous public support systems (Austria, Germany, Italy, Japan, and Sweden).](image)

The pattern shown in Figure 5 is similar in other advanced countries with a very rapid transition to higher benefits (tgi) and lower labor income (yl) and taxes (tax) as death rates reach a threshold value. This corresponds to the age of retirement and qualification for pensions and possibly health care benefits. There is no single age at retirement, of course, but a distribution that is captured by the pace of decline in labor income and taxes and the pace of increase in benefits. As mortality conditions improve, the threshold occurs at later ages and labor income and taxes paid increase at each age and the benefits received decline at each age.

**Appendix B: National Transfer Accounts (NTA) data**

The primary data employed for the fiscal model are National Transfer Accounts (NTA) estimates which are used to construct age profiles of labor income, public transfer inflows and outflows, and taxes. With a few exceptions, relevant aggregate data are available for more recent years than NTA data. Thus, a baseline year was selected and NTA age profiles were scaled to match the NTA aggregates in the baseline year. The countries, baseline year, and year of NTA profiles are reported in Table A.1.

<table>
<thead>
<tr>
<th>Country</th>
<th>Baseline year</th>
<th>Year of NTA profiles</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>
Brazil | 2009 | 1996 | Tax profiles based on 2003 estimates by Li and Chen; labor income and tgi profiles for 2010 based on Shen and Lee (forthcoming).
---|---|---|---
China | 2009 | 2003/2010 |
Germany | 2010 | 2003 |
Hungary | 2010 | 2005 |
India | 2010 | 2004 |
Japan | 2010 | 2004 |
Mexico | 2010 | 2004 |
Nigeria | 2010 |
South Africa | 2010 | 2000 |
Thailand | 2011 | 2011 |
United States | 2011 | 2011 |

### National Transfer Accounts

National transfer Account (NTA) age profiles are based on estimates reported in the NTA database (www.ntaccounts.org) for the most recently available year. In the case of China, updated estimates of labor income and public transfer spending on health, education, and pensions were available for 2010. These age profiles were used to obtain the baseline profile for public transfer inflows and its components.

National Transfer Account aggregates at baseline were compiled following the procedures outlined in the latest version of the United Nations (2013a) NTA Manual. Except for estimates for Nigeria, Thailand and the United States, which were compiled using System of National Account (SNA) estimates sourced directly from national statistical agencies, NTA country aggregate estimates were compiled using SNA figures submitted by country agencies to and collated by the United Nations (2013b) Department of Economic and Social Affairs’ Statistics Division. Researchers in Thailand Development Research Institute, for Thailand, and in University of California, Berkeley, for the United States, provided NTA estimates for their respective country. For other countries, in cases where necessary SNA details were not available from the source data, country-specific adjustments were utilized. For instance, for NTA country aggregate estimates compiled using the SNA compilation by the United Nations (2013b), within-government current transfers and current international cooperation were not available, and thus not netted out from SNA Other Current Transfers to estimate NTA inter-sectoral flows using the approximation method proposed in the United Nations (2013a) NTA Manual. Other country-specific adjustments used are described as follows.

1. **Brazil.** NTA Asset Income and Saving are gross of Consumption of Fixed Capital, which are not available in the SNA compilation by the United Nations (2013b).
2. **China.** Details of net taxes on products and production were allocated using the 2008-2010 average distribution of taxes by source provided in International Monetary Fund (2013) *Government Finance Statistics Yearbook, 2012*. Mixed Income by households was estimated by applying to the Gross Operating Surplus by households the income share of the informal sector relative to the total household income based on the 2000 Chinese Social Accounting Matrix estimate by Rada (2010). NTA Asset Income and Saving are gross of Consumption of Fixed Capital, which are not available in the SNA compilation by the United Nations (2013b).
3. **India.** Mixed Income by households was estimated by applying to the Gross Operating Surplus by households the income share of self-employed households to the total household income based on the 2003-2004 Social Accounting Matrix estimate by Ojha, et. al. (2009).
4. **Japan.** Details of subsidies on products and production were allocated proportionally using the distribution of direct taxes.

5. **Nigeria.** NTA estimates were compiled using SNA estimates published in the *Annual Abstract of Statistics, 2011* by the National Bureau of Statistics. Indirect taxes and Subsidies were allocated as taxes and subsidies, respectively, on products following figures in the United Nations (2013b) SNA compilation. Mixed Income by households was estimated by applying the 1970-2010 average share of the Nigerian informal sector to the total economy, which was estimated by Ogbuabor and Malaolu (2013). Sectoral distribution of (i) Operating Surplus, (ii) Property and Entrepreneurial Income, (iii) Other Current Transfers from the Rest of the World, and (iv) Consumption of Fixed Capital were estimated by applying corresponding shares of the public and private sectors based on the 2006 Nigerian Social Accounting Matrix estimate by Nwafor, Diao and Alpuerto (2010). Saving by sector was estimated by recompiling the SNA based on the estimated sectoral distribution of primary and secondary distribution of income to ensure internal consistency in both SNA and NTA.

6. **South Africa.** Mixed Income by households was estimated by applying the share of the informal sector to the total economy following the estimates by Davies and Thurlow (2010) for South Africa in 2002.

### Macroeconomic data

#### Productivity growth

Growth of real output per effective worker is assumed to grow at rates shown in Table A.2. Values for 2011 and 2012 are based on actual values while projected values are based to varying degrees on long-term growth estimates from other studies (Lee and Mason, forthcoming; Asian Development Bank, 2011; United States Department of Agriculture, 2014).

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>1.05</td>
<td>(0.48)</td>
<td>1.10</td>
<td>3.0</td>
<td>see note</td>
<td>1.50</td>
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<td>China</td>
<td>8.29</td>
<td>6.76</td>
<td>6.93</td>
<td>6.0</td>
<td>see note</td>
<td>1.50</td>
</tr>
<tr>
<td>Germany</td>
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<td>0.93</td>
<td>0.75</td>
<td>1.5</td>
<td>1.5</td>
<td>1.50</td>
</tr>
<tr>
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<td>1.96</td>
<td>(1.32)</td>
<td>1.60</td>
<td>2.0</td>
<td>see note</td>
<td>1.50</td>
</tr>
<tr>
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<td>2.78</td>
<td>3.13</td>
<td>5.0</td>
<td>see note</td>
<td>1.50</td>
</tr>
<tr>
<td>Japan</td>
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<td>2.05</td>
<td>2.13</td>
<td>1.5</td>
<td>1.5</td>
<td>1.50</td>
</tr>
<tr>
<td>Mexico</td>
<td>2.02</td>
<td>1.98</td>
<td>(0.90)</td>
<td>2.5</td>
<td>see note</td>
<td>1.50</td>
</tr>
<tr>
<td>Nigeria</td>
<td>1.96</td>
<td>3.99</td>
<td>4.50</td>
<td>2.5</td>
<td>see note</td>
<td>1.50</td>
</tr>
<tr>
<td>South Africa</td>
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<td>0.90</td>
<td>0.35</td>
<td>3.5</td>
<td>see note</td>
<td>1.50</td>
</tr>
<tr>
<td>Thailand</td>
<td>(0.42)</td>
<td>7.21</td>
<td>1.38</td>
<td>4.0</td>
<td>see note</td>
<td>1.50</td>
</tr>
<tr>
<td>United States</td>
<td>0.96</td>
<td>1.74</td>
<td>1.70</td>
<td>1.5</td>
<td>1.5</td>
<td>1.50</td>
</tr>
</tbody>
</table>

Note. If productivity growth exceeds 1.5 percent in 2030, it declines linearly to 1.5 percent between 2030 and 2050.

Inflation rates
Annual inflation rates up to 2030 are based on projections by the United States Department of Agriculture (2014). Country inflation rates below two percent in 2030 are assumed to remain constant until 2100. For economies with inflation rate higher than two percent in 2030, their inflation rates are assumed to decline by 0.05 percentage point annually until their inflation rate drops below two percent.

Interest Rates
Economy-wide interest rate forecasts are calculated as the sum of projected (i) productivity growth (Table A.1), (ii) growth in effective number of workers based on NTA country profiles and United Nation (2013c) population projection, (iii) inflation rate based on estimates by the United States Department of Agriculture (2014), and (iv) country risk premium estimated by Damodaran (2013). The projected interest rates are rescaled to match the public rate of return at baseline. Implied rate of return at baseline are estimated using Net Debt data from the International Monetary Fund (2014), and NTA Public Saving and Public Asset Income estimates.

Death Rates
Death rates of population aged 40 years and above by single-year age groups and by year were estimated using life table data from the United Nations (2013c) World Population Prospects: The 2012 Revision. Data on survivors $l(x)$ at exact age $x$ are available for five-year periods from 2010 to 2100 and for single-year age groups in five-year interval up to age 85. Period estimates of $l(x)$ were designated to the middle year of the period. The death rate $d(x, t)$ at exact age $x$ and period $t$ were calculated as

$$d(x, t) = 1 - \frac{s \overline{l(x + 5, t)}}{l(x, t)}$$

Death rate for a given country and year is modelled as log-linear piecewise regression function, which is then used to predict death rates in single-year age groups in each period. Specifically, the model used is

$$E(\ln d(x, t)) = \sum_i \left( a_i + b_i \text{age}_{ij} \right) I(\text{age}_{ij} \in \text{age}_i)$$  \hspace{1cm} (40)$$

where $E(.)$ is the expectation operator, and $I(\text{age}_{ij} \in \text{age}_i)$ is an indicator function, which takes the value of one if $(\text{age}_{ij} \in \text{age}_i)$ is true and zero if otherwise. The subscript $i$ refers to the collection of sets used to identify if $\text{age}_{ij}$ is part of the age group $\text{age}_i$, where $i = \{(40,55),(55,65),(65,70),(70,85)\}$. Death rates in intervening years are estimated by linearly interpolating parameter estimates between two reference periods, and using the interpolated parameters to predict death rates in single-year age groups.
References for data appendix
References


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1 The estimates for the oldest age groups tend to be noisy because of small sample sizes in the surveys on which estimates are based.