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Mortality, fertility, education and capital accumulation in a simple OLG economy

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Abstract We develop a simple OLG model to analytically show that aging leads to increased educational efforts through a general equilibrium effect. The mechanism is that scarcity of raw labor increases the return of human relative to physical capital. While a reduction in the birth rate is shown to unambiguously increase educational efforts, increases in the survival rate have ambiguous effects. Falling birth rates also increase capital per worker, but the effects of rising survival rates are again ambiguous. We conclude that our model is a useful laboratory to highlight potentially offsetting effects in models with endogenous education and overlapping generations.

Keywords Aging · Education · Overlapping generations

JEL Classification J11 · J24 · O11

1 Introduction

Important aspects of economic history are the decline in mortality, the associated increase in life expectancy, and a notable rise in investment into human

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capital. Life expectancy at birth in the UK was about 40 years in 1850, was 65 years in 1950, and rose by ten more years until the year 2000 (Cutler et al. 2006). The share of children aged 10–14 attending primary schools rose from 10% in 1820 to 80% in 1930 (Flora et al. 1983). Universal schooling was reached soon thereafter. The same development took place for secondary and tertiary education. Net enrollment rates for secondary schooling increased from 67% in 1970 to 95% in 2000 (The World Bank 2004). As can be seen, these processes of rising life expectancy, falling birth rates, and rising investment into human capital are still going on in modern economies. The combined effect is that the population structure of developed countries is changing rapidly with a rising share of elderly people. This rise of the old-age dependency ratio and the associated rise in social security contributions have shifted the "aging problem" into the focus of the academic literature, as well as public policy.

In this paper, we develop an analytically tractable, two-generations OLG model in the spirit of Diamond (1965) in order to study the effects of demographic change on educational investment decisions and capital accumulation. We augment the simple textbook model with endogenous human capital formation. Population dynamics—the exogenous driving force of our model—are modeled by considering uncertain survival to old age and birth rates separately. Additionally, we look at the effects of changing lifetime labor supply. The strength of our setup is that we can analyze the general equilibrium effects of population dynamics using closed-form solutions. The contribution of our paper is that, using this rich setup, we are able to show that changes in life expectancy, population growth, and lifetime labor supply have, in general, ambiguous effects on the capital stock and education. We demonstrate that it is key to consider the interactions between annuity markets, the pension system, and productivity of education for understanding the qualitative and quantitative effects of variations in the population structure on changes in physical and human capital accumulation.

The relationship between mortality and investment into human capital has been investigated in a number of theoretical and empirical studies. Empirical studies find that falling mortality and the associated rise in life expectancy increase investment into human capital. Using data for post-war India, Ram and Schultz (1979) find that improvements in mortality played a major role in the rise of educational attainment. Eckstein et al. (1999) provide evidence for Sweden that the fall in child mortality was the most important factor for the demographic transition and the rising educational attainment. On the other hand, Mincer (1995) and Foster and Rosenzweig (1996) present empirical evidence that rising education premia have a positive effect on schooling.

Theoretical work dealing with the ageing–education nexus by Boucekkine et al. (2002), de la Croix and Licandro (1999), Echevarria and Iza (2006), and Heijdra and Romp (2009) use variations of a Blanchard (1985) type of perpetual youth setup. By employing this model family, the authors obtain closed-form solutions and derive a number of insights by relying entirely on analytical results. These papers assume that the production processes use only labor (human capital) as an input or they consider only small open economies. Thus, the general equilibrium feedback effect of population dynamics on relative prices is ruled out by construction. A general conclusion of this literature is that increasing life expectancy increases investment into human and physical capital.

The papers by Hu (1999) and Kalemli-Ozcan et al. (2000) are closest in spirit to our work. They also employ a perpetual youth setup but overcome the limitations of the above-mentioned papers by developing tractable general equilibrium models. Our contributions to their work are threefold. First, we do not only study the effects of changes in mortality but also the effects of changing fertility on investment in education and human, as well as physical, capital accumulation. Second, we also analyze how changes in the lifetime working horizon affect educational decisions and capital accumulation. This additional channel in our model stands in for a lifelong learning motive and is increasingly important in aging societies that reform their PAYG financed pension systems by increasing retirement ages. Third, by using an OLG rather than a perpetual youth model, we reconfirm some of the findings of the above-mentioned authors: Rising survival rates *may* lead to increasing educational efforts and capital accumulation. However, we emphasize that there are potentially important offsetting effects. The lower degree of analytical tractability of our OLG model—in comparison to the perpetual youth model—buys us the possibility to include and to understand several interaction effects and to show how these may change results. For example, using an equilibrium relationship of their model, Kalemli-Ozcan et al. (2000) argue that the interest rate varies positively with mortality, "as would be expected from the simple intuition that shorter lives lead to lower wealth accumulation" (p. 11). We show that this positive effect is smaller when annuity markets are larger and that, by interpreting an equilibrium condition only, Kalemli-Ozcan et al. (2000) ignore two important and potentially offsetting effects: increasing mortality (1) decreases the workforce and (2) may decrease educational efforts, and both effects ceteris paribus lead to a negative variation of mortality and the interest rate.

Finally, Zhang et al. (2001) add to this literature by modeling endogenous fertility and child education employing a two-generations OLG setup as we do but using a dynastic framework. These differences in the two approaches make their work less suitable as a benchmark for comparison. Furthermore, as a consequence of the endogenous nature of fertility decisions, these authors cannot study the impact of changing fertility and mortality in isolation as we do.

The remainder of this paper is structured as follows: Section 2 introduces the model. The results of the comparative static analysis are derived in Section 3. In the same section, we also show the results of our calibration exercise, where we perform an extensive sensitivity analysis. Some concluding remarks are in Section 4. Separate appendices contain proofs and additional results.

2 The model

We develop a simple OLG model with endogenous education decisions and a PAYG financed social security system. The setup is as follows: agents live for two periods whereby survival to the second period is uncertain. In the first period, agents choose time investment into education, saving, and consumption. In the second period, they consume their entire wealth and work only an exogenously given fraction ω of their time. The rest of their time $(1-\omega)$, they are retired and receive a lump-sum pension, p_{t+1}. We make this assumption for analytical tractability; it allows us to analyze the effects of different social security regimes in a model of human capital accumulation à la Ben-Porath (1967) within a two-generations model. In this setup, the parameter ω reflects a motive for life-long learning that can be affected by policy, e.g., by increasing the retirement age.

2.1 Demographics

Each period, there are $N_{t,0}$ young households and $N_{t,1}$ old households. Let γ_t^N be the birth rate so that $N_{t,0} = \gamma_t^N N_{t-1,0}$, and let s_t be the survival rate; hence, $N_{t,1} = s_t N_{t-1,0}$. Using these definitions, the old-age dependency ratio (oadr$_t$)—the fraction of the old to the young—in the economy is given by

$$\text{oadr}_t = \frac{N_{t,1}}{N_{t,0}} = \frac{s_t}{\gamma_t^N}. \tag{1}$$

2.2 Markets for annuities

We assume the existence of (imperfect) annuity markets for insurance against survival risk. Let $a_{t,0}$ be savings of the period t young. Period $t+1$ asset holdings are consequently given by

$$a_{t,0} + \lambda a_{t,0} \frac{1-s_{t+1}}{s_{t+1}} = a_{t,0} \frac{\zeta_{t+1}}{s_{t+1}}, \tag{2}$$

where

$$\zeta_{t+1} \equiv s_{t+1} + \lambda(1-s_{t+1}) \tag{3}$$

is an annuity factor introduced here for convenience, and $0 \leq \lambda \leq 1$ is the degree of annuitization, also see Hansen and Imrohoroğlu (2008). Notice that, in the case of no annuitization, we have $\lambda = 0$ and $\zeta_{t+1} = s_{t+1}$, and for complete (perfect) annuity markets, we have $\lambda = 1$ and $\zeta_{t+1} = 1$. Full annuitization implies that the assets of the deceased agents are distributed uniformly among the surviving old agents, which is an insurance against longevity (Yaari 1965).

Without annuity markets, there is no "insurance effect," but agents receive a lump-sum payment tr_{t+1} from the government. To keep the analysis analytically tractable, we assume that, in the case of incomplete annuitization, the government distributes the accidental bequests to the old.[1] Accidental bequests are then redistributed to households as lump-sum transfers and given by

$$tr_{t+1} = (1-\lambda)\frac{a_{t,0}(1+r_{t+1})(1-s_{t+1})N_{t,0}}{N_{t+1,1}}, \tag{4}$$

and, using the fact that

$$N_{t+1,1} = N_{t,o}s_{t+1},$$

we have

$$tr_{t+1} = (1-\lambda)\frac{a_{t,0}(1+r_{t+1})(1-s_{t+1})}{s_{t+1}}. \tag{5}$$

2.3 Household optimization

Households maximize expected lifetime utility

$$\max_{c_{t,0},c_{t+1,1}} \log c_{t,0} + \beta s_{t+1} \log c_{t+1,1}, \tag{6}$$

subject to the constraints

$$c_{t,0} + a_{t,0} = (1-e_t)h_0 w_t(1-\tau_t) \tag{7a}$$

$$c_{t+1,1} = \frac{(1+r_{t+1})\zeta_{t+1}}{s_{t+1}}a_{t,0} + \omega h_{t+1,1}w_{t+1}(1-\tau_{t+1}) + (1-\omega)p_{t+1} + tr_{t+1}, \tag{7b}$$

where β is the raw time discount factor, e_t is investment into education when young, h_0 is the stock of human capital given at birth (taken as exogenous and constant over cohorts), w_t is the wage rate per unit of human capital, r_{t+1} is the return on financial assets, τ_t denotes the social security contribution rate, p_{t+1} are lump-sum pension payments, and tr_{t+1} are the distributed accidental bequests.

Due to the representative agent setup, two interpretations of ω are conceivable. In the first interpretation, ω is the fraction of time the representative

[1]To see why this assumption is useful, assume that bequests are distributed to the young. Then, transfers are given by

$$tr_t = (1-\lambda)\frac{a_{t-1,0}(1+r_t)(1-s_t)N_{t-1,0}}{N_{t,0}} = (1-\lambda)a_{t-1,0}(1+r_t)\frac{1-s_t}{\gamma_t^N}.$$

As $a_{t-1,0}$ shows up in the above equation, the analysis would involve a second-order difference equation for k_t, which would tremendously reduce analytical tractability. Assuming that bequests are distributed to the young *and* old will obviously cause the same problem.

agent of age 1 works. In the second, it is the fraction in the population of age 1 that works. Either way, ω works like a policy variable, and a change in ω could be interpreted, e.g., as a change in retirement legislation or labor market incentives affecting participation rates.

The present value budget constraint is accordingly given by

$$c_{t,0}+s_{t+1}\frac{c_{t+1,1}}{\zeta_{t+1}(1+r_{t+1})}=(1-e_t)h_0w_t(1-\tau_t) + s_{t+1}\frac{\omega h_{t+1,1}w_{t+1}(1-\tau_{t+1})+(1-\omega)p_{t+1}+tr_{t+1}}{\zeta_{t+1}(1+r_{t+1})}. \quad (8)$$

The education technology is

$$h_{t+1,1} = (1 + g(e_t))h_0, \quad (9)$$

with g being a function mapping educational investment into the formation of human capital. We choose g such that it is increasing, is concave in e, and fulfills the lower Inada condition. These are standard assumptions about the education function (see Willis 1986).[2] Later, we specify a parametric form for $g(e_t)$ to obtain a closed-form solution. Solving the maximization problem gives the Euler equation

$$c_{t+1,1} = \beta\zeta_{t+1}(1 + r_{t+1})c_{t,0}. \quad (10)$$

Solving for the optimal educational investment gives

$$g'(e_t) = \frac{\zeta_{t+1}(1 + r_{t+1})}{s_{t+1}}\frac{w_t(1 - \tau_t)}{\omega w_{t+1}(1 - \tau_{t+1})}. \quad (11)$$

This condition says that an individual invests into schooling until the marginal return of schooling equals the return on net wages relative to the effective interest rate. Following Bouzahzah et al. (2002), we define the education function $g(e_t)$ in Eq. 9 as

$$g(e_t) = \xi e_t^{\psi}, \qquad \text{where } 0 < \psi < 1,\ \xi > 0. \quad (12)$$

Optimal education is then given by

$$e_t = \left[\omega\xi\psi\frac{w_{t+1}(1 - \tau_{t+1})}{w_t(1 - \tau_t)}\frac{s_{t+1}}{\zeta_{t+1}(1 + r_{t+1})}\right]^{\frac{1}{1-\psi}}. \quad (13)$$

It can be seen that educational decisions depend positively on the ratio of net wage growth to the return on capital holdings. This is the key general equilibrium effect we are interested in. The scarcity of raw labor resulting from demographic change will lead to rising wages and falling interest rates.

[2] For analytical reasons, we assume zero depreciation of human capital, and we do not make h an argument of g as in the standard Ben-Porath (1967) technology. This parametric restriction is also superimposed in some empirical studies, see the review in Browning et al. (1999).

According to Eq. 13, this will induce general equilibrium feedback effects by leading to increases in education and, thereby, to an increase in the second period human capital.

In addition to these general equilibrium effects, Eq. 13 shows direct effects on educational efforts through the educational productivity, ξ and ψ; the fraction of time working in the second period, ω; and the probability of survival if there is some annuitization, i.e., if $\lambda > 0$. The direct effect of survival on educational decisions has in part been labeled as an effect due to an extension of the adult planning horizon, e.g., by Heijdra and Romp (2009). This is a misleading interpretation because the direct effect of survival is, in fact, a result of the induced adjustment of the rate of return to physical capital if there is some annuitization.[3] In the absence of annuitization, there is no adjustment of the rate of return to physical capital to the survival rate, and changes in the survival rate have a direct effect only on the inter-temporal allocation of consumption (via the changing effective discount rate $s_{t+1}\beta$). In our model, the "pure" effect of extending the planing horizon is represented by an increase in ω.

Finally, households' optimal consumption follows from using Eq. 10 in Eq. 8 as

$$c_{t,0} = \frac{1}{1+\beta s_{t+1}} \Bigg((1-e_t) h_0 w_t (1-\tau_t) + s_{t+1} \frac{\omega h_{t+1,1} w_{t+1}(1-\tau_{t+1}) + (1-\omega) p_{t+1} + tr_{t+1}}{\zeta_{t+1}(1+r_{t+1})} \Bigg)$$

and using the above in Eq. 7a gives savings as

$$a_{t,0} = \frac{1}{1+\beta s_{t+1}} \Bigg(\beta s_{t+1} (1-e_t) h_0 w_t (1-\tau_t) - s_{t+1} \frac{\omega h_{t+1,1} w_{t+1}(1-\tau_{t+1}) + (1-\omega) p_{t+1} + tr_{t+1}}{\zeta_{t+1}(1+r_{t+1})} \Bigg). \tag{14}$$

2.4 Firms

Firms produce output using a standard Cobb–Douglas production function

$$Y_t = K_t^{\alpha} (A_t L_t)^{1-\alpha}. \tag{15}$$

A_t is the firm's technology level, which is determined by

$$A_{t+1} = A_t \gamma^A, \tag{16}$$

where γ^A is the exogenous gross growth rate. L_t is effective labor input, which is the sum of human capital weighted labor supply of the young and of the old

[3] This has already been shown by Hu (1999).

and accordingly given by

$$L_t = (1 - e_t)h_0 N_{t,0} + \omega h_{t,1} N_{t,1}. \tag{17}$$

Competitive markets ensure that factors get paid their marginal products. We assume that capital depreciates fully after one period so that

$$1 + r_t = \alpha k_t^{\alpha-1} \tag{18a}$$

$$w_t = (1 - \alpha) A_t k_t^{\alpha}, \tag{18b}$$

where $k_t \equiv \frac{K_t}{A_t L_t}$.

2.5 Government

The role of the government is twofold. First, the government taxes accidental bequests in the case of incomplete annuitization at a confiscatory rate and redistributes them as lump-sum payments to the old. Second, the government runs a PAYG financed social security system with a balanced budget in all periods requiring that total contributions by workers equal total pension payments.[4] By Eq. 17, we then have

$$w_t \tau_t \left((1 - e_t)h_0 N_{t,0} + \omega h_{t,1} N_{t,1} \right) = (1 - \omega) p_t N_{t,1}. \tag{19}$$

Notice that the above, using Eq. 1, implies that

$$(1 - \omega) p_t = w_t \tau_t \left((1 - e_t)h_0 \frac{\gamma_t^N}{s_t} + \omega h_{t,1} \right). \tag{20}$$

Changes in the population structure require adjustments of the social security policy. Let ϱ_t denote the replacement rate, i.e., the ratio of pension income to average net wage income. Then pension income can be expressed as

$$p_t = \varrho_t \frac{(1 - \tau_t) w_t \left((1 - e_t)h_0 N_{t,0} + \omega h_{t,1} N_{t,1} \right)}{N_{t,0} + \omega N_{t,1}}.$$

Using the above definition in Eq. 19 and simplifying then links contribution and replacement rates by

$$\tau_t = \frac{(1 - \omega)\varrho_t}{\gamma_t^N / s_t + \omega + (1 - \omega)\varrho_t}. \tag{21}$$

It can be readily observed that τ_t increases in the fraction of pensioners, $1 - \omega$, the generosity of the pension system, ϱ_t, and in the old-age dependency ratio, s_t / γ_t^N. Using this setup, fixing $\tau_t = \bar{\tau}$ corresponds to a fixed contribution rate system and holding $\varrho_t = \bar{\varrho}$ corresponds to a fixed replacement rate system.[5]

[4] While we explicitly model this inter-generational transfer system as a pension system, it may also be interpreted as a metaphor for a more general intergenerational transfer system, e.g., a health care system.

[5] Notice that these definitions are not the same as what is referred to as defined contribution and defined benefit systems in the literature.

2.6 Equilibrium

In equilibrium, all markets clear, households maximize utility, and firms make zero profits. Market clearing on the capital market requires that

$$K_{t+1} = a_{t,0} N_{t,0}. \tag{22}$$

Using Eq. 1 in Eq. 17, aggregate labor supply can be rewritten as

$$L_t = N_{t,0} h_0 \left((1 - e_t) + \omega \frac{s_t}{\gamma_t^N} (1 + g(e_{t-1})) \right). \tag{23}$$

Collecting elements, the following proposition gives the law of motion of the aggregate economy.

Proposition 1 *For given k_0, the aggregate dynamics of the economy are described by the system of first-order difference equations in $\{k_t, e_t\}$ given by*

$$k_{t+1} = \frac{\varphi_t \alpha (1-\alpha)(1-\tau_t)}{\phi_t} k_t^\alpha \tag{24a}$$

$$e_t = \left(\frac{s_{t+1}}{\zeta_{t+1}} \omega \xi \psi \frac{\gamma^A (1 - \tau_{t+1}) k_{t+1}}{\alpha (1-\tau_t) k_t^\alpha} \right)^{\frac{1}{1-\psi}}, \tag{24b}$$

where

$$\begin{aligned}\phi_t \equiv \gamma^A \Bigg(&\left(\alpha (2 + \hat{\rho}_{t+1}) + \varphi_t \frac{(1-\alpha)\tau_{t+1}}{\zeta_{t+1}} (1 + \hat{\rho}_{t+1}) \right) \frac{1 - e_{t+1}}{1 - e_t} \gamma_{t+1}^N \\ &+ \omega s_{t+1} \left(\alpha (2 + \hat{\rho}_{t+1}) + \varphi_t \frac{1-\alpha}{\zeta_{t+1}} (1 + \hat{\rho}_{t+1}) \right) \frac{1 + g(e_t)}{1 - e_t} \Bigg)\end{aligned} \tag{25a}$$

$$\varphi_t \equiv \frac{(2 + \hat{\rho}_{t+1}) \zeta_{t+1}}{(2 + \hat{\rho}_{t+1}) \zeta_{t+1} + (1 - s_{t+1})(1 - \lambda))} \tag{25b}$$

and $\hat{\rho}_{t+1} = \frac{1}{s_{t+1}\beta} - 1$.

Proof Relegated to Appendix A. □

Proposition 2 *If there is an equilibrium, education e_t is always interior on the interval* (0, 1). *Further, education always converges to its steady state value.*

Proof Relegated to Appendix A. □

2.7 Steady state analysis

Definition 1 Along the balanced growth path (steady state) of the economy, all variables grow at constant rates so that $k = k_{t+1} = k_t$ and $e = e_{t+1} = e_t \; \forall \, t$.

Proposition 3 *For $0 < \alpha < 1$ and $0 \leq \tau < 1$, the unique steady state of the economy is given by*

$$k = \left(\frac{\varphi\alpha(1-\alpha)(1-\tau)}{\phi}\right)^{\frac{1}{1-\alpha}} \tag{26a}$$

$$e = \left(\omega\xi\psi\frac{\gamma^A}{\alpha}\right)^{\frac{1}{1-\psi}}\left(\frac{s}{\zeta}\right)^{\frac{1}{1-\psi}} k^{\frac{1-\alpha}{1-\psi}} \tag{26b}$$

where

$$\phi \equiv \gamma^A\left(\left(\alpha(2+\hat{\rho}) + \varphi\frac{(1-\alpha)\tau}{\zeta}(1+\hat{\rho})\right)\gamma^N + \omega s\left(\alpha(2+\hat{\rho}) + \varphi\frac{1-\alpha}{\zeta}(1+\hat{\rho})\right)\frac{1+g(e)}{1-e}\right), \tag{27a}$$

$$\varphi \equiv \frac{(2+\hat{\rho})\zeta}{(2+\hat{\rho})\zeta + (1-s)(1-\lambda)} \tag{27b}$$

and $\hat{\rho} = \frac{1}{s\beta} - 1$.

Proof Relegated to Appendix A. □

3 Comparative statics

In this section, we use our framework to study the effects of demographic change on the economy by conducting a comparative statics analysis in steady state. In this respect, our model is a useful laboratory to provide intuition for the results of much of the quantitative work, e.g., by Fougère and Mérette (1999), Sadahiro and Shimasawa (2002), Bouzahzah et al. (2002), and Ludwig et al. (2008). To this end, we analyze—by looking at partial derivatives—the effects of changing fertility, mortality, and working time on the capital stock and education. We first do so in a social security scenario with constant contribution rates and then consider the opposite extreme by holding replacement rates constant. While we can uniquely determine the signs of many partially derivatives, we fail to do so in some cases. In these cases, our closed-form solutions help us to understand the various offsetting effects at work and to detect the sources of indeterminacy. Finally, we use a calibrated version of our model to illustrate how the signs of partial derivatives depend on the parametrization of the model in the ambiguous cases.

3.1 Analytical results

We drop the time indices to indicate steady state values. To begin with, we provide analytical results followed by an interpretation.

Proposition 4 *In the steady state of the economy, we have*

1. *for $\tau = \bar{\tau}$ that*

$$\left.\frac{\partial k}{\partial \gamma^N}\right|_{\tau=\bar{\tau}} < 0 \ \ and \ \ \left.\frac{\partial e}{\partial \gamma^N}\right|_{\tau=\bar{\tau}} < 0, \tag{28a}$$

$$\left.\frac{\partial k}{\partial s}\right|_{\tau=\bar{\tau}} \gtrless 0 \ \ and \ \ \left.\frac{\partial e}{\partial s}\right|_{\tau=\bar{\tau}} \gtrless 0. \tag{28b}$$

$$\left.\frac{\partial k}{\partial \omega}\right|_{\tau=\bar{\tau}} < 0 \ \ and \ \ \left.\frac{\partial e}{\partial \omega}\right|_{\tau=\bar{\tau}} \gtrless 0, \tag{28c}$$

2. *For the relationship between the cases $\tau = \bar{\tau}$ and $\varrho = \bar{\varrho}$, we have that*

$$\left.\frac{\partial k}{\partial \gamma^N}\right|_{\varrho=\bar{\varrho}} > \left.\frac{\partial k}{\partial \gamma^N}\right|_{\tau=\bar{\tau}} \ \ and \ \ \left.\frac{\partial e}{\partial \gamma^N}\right|_{\varrho=\bar{\varrho}} > \left.\frac{\partial e}{\partial \gamma^N}\right|_{\tau=\bar{\tau}} \tag{29a}$$

$$\left.\frac{\partial k}{\partial s}\right|_{\varrho=\bar{\varrho}} < \left.\frac{\partial k}{\partial s}\right|_{\tau=\bar{\tau}} \ \ and \ \ \left.\frac{\partial e}{\partial s}\right|_{\varrho=\bar{\varrho}} < \left.\frac{\partial e}{\partial s}\right|_{\tau=\bar{\tau}}, \tag{29b}$$

$$\left.\frac{\partial k}{\partial \omega}\right|_{\varrho=\bar{\varrho}} > \left.\frac{\partial k}{\partial \omega}\right|_{\tau=\bar{\tau}} \ \ and \ \ \left.\frac{\partial e}{\partial \omega}\right|_{\varrho=\bar{\varrho}} > \left.\frac{\partial e}{\partial \omega}\right|_{\tau=\bar{\tau}}, \tag{29c}$$

Proof Relegated to Appendix A. □

Interpretation of the partial derivatives of the capital stock and education in Eq. 28a is rather straightforward. First, observe from Eq. 26b that there is no direct effect of the birth rate, γ^N, on the education decision, e. Second, an increase of the birth rate increases the effective supply of labor in the economy, which decreases k, cf. Eqs. 26a and 27a. Therefore, a change in the birth rate affects the relative prices of physical and human capital through its effect on k. An increase of k increases the wage rate, w, and decreases the return on physical capital, r. While the growth rate of wages ($\frac{w_{t+1}}{w_t}$) is unchanged in our steady state comparison, the return on physical capital decreases. Consequently, optimal education goes up, cf. Eq. 13.

As stated in the proposition, the signs of the partial derivatives in Eq. 28b cannot be determined unambiguously. First, notice that there are various effects from increases of s on savings and, thus, k at work, cf. Eq. 27a: (1) An increase of s decreases the effective discount rate $\hat{\rho}$, which increases k. This is so because an increase of the survival rate increases savings via its effect on current period income, cf. the first term in the brackets of Eq. 14. (2) However, an increase in the survival rate also increases the value of second-period income as long as $\lambda > 0$ (so that $s_{t+1}/\zeta_{t+1} < 1$), which dampens the increase of savings. This dampening effect is stronger the larger the size of the annuity market is, i.e., the higher λ is.[6] (3) For $\lambda > 0$, there is a direct

[6] As can be immediately observed from Eq. 14, the overall effect of increasing survival on savings is unambiguously positive. However, it is larger for $\lambda = 0$ than for $\lambda = 1$.

effect of survival on education, cf. Eq. 26b, which varies positively with λ. This increases effective labor supply and thereby tends to decrease k. (4) As s increases, raw labor supply increases as long as $\omega > 0$. Observe that the last two effects are stronger when the average human capital productivity is high because ω interacts with ξ via the term $\frac{1+g(e)}{1-e}$ in Eq. 27a.

This discussion explains why the signs of the effects of s on k cannot be determined unambiguously. It can only be said that the capital stock is likely to increase if ω, λ, and ξ are sufficiently small. For too high values of these parameters, the reaction of effective labor supply is too strong and the capital stock k_t may decrease (so that r_{t+1} increases). Second, this ambiguity with respect to the effects of s on k translates into an ambiguous effect of s on e, cf. Eq. 26b. However, even if k varies negatively with s, education may still increase because of the direct effect of increasing survival on the education decision in the presence of annuity markets ($\lambda > 0$). Indeed, in all of our simulations of Subsection 3.3, schooling is found to increase if s rises, also in those cases in which k decreases when annuity markets are perfect. On the contrary, with missing annuity markets, we never find that k decreases in s so that there is also no ambiguity in the resulting educational adjustments.

The effect of a changing lifetime labor supply ω given in Eq. 28c is unambiguously negative for the capital stock but ambiguous for the optimal education decision. First, increasing ω increases total effective labor supply and, thus, decreases k. Second, an increase of ω has a direct effect on education, cf. Eq. 13. This leads to an additional increase of effective labor, which further decreases k. However, third, a decrease of k also exerts a dampening effect on education by increasing the return on physical capital. As this third effect is only a second-order general equilibrium feedback effect, it cannot offset the decrease of k which explains the unambiguous sign for the partial derivative of k. However, the direct effect of ω on k and the resulting general equilibrium price effect could potentially be strong enough to offset the direct effect of ω on education. This explains the ambiguous sign of the partial derivative of e. While this is so analytically, we show below, for a wide range of parameter constellations of our simulations, that education varies positively with ω.

The effect of an adjustment of the contribution rate τ is examined in the second part of Proposition 4. Recall that changing the contribution rate has only a direct effect on capital accumulation but does not distort education decisions in steady state. Thus, increasing the contribution rate only has an effect on steady state education to the extent that it crowds out savings in physical capital. The uniform conclusion is therefore that a rising (falling) contribution rate decreases (increases) the capital stock, thereby increasing (decreasing) the interest rate, and, thus, decreases (increases) the incentives to invest in education. A brief verbal summary of the results is that the effect of falling birth rates, rising survival rates, i.e., an aging of the population, or an extension of the lifetime labor supply has a larger effect (in absolute values) on the capital stock and on education if the contribution rate τ is held constant. The results do not say, however, that the signs do not change. Since we add one layer of complexity, it is even harder to pin down the direction of change.

3.2 Role of annuity markets

This subsection discusses the role of the degree of annuitization in more detail. We show in the Appendix that

Proposition 5 *In the steady state of the economy, we have*

1. *For $\tau = \bar{\tau}$, that*

$$\left.\frac{\partial k}{\partial \lambda}\right|_{\tau=\bar{\tau}} > 0 \ \ and \ \ \left.\frac{\partial e}{\partial \lambda}\right|_{\tau=\bar{\tau}} \gtrless 0, \tag{30}$$

2. *For the relationship between the cases $\tau = \bar{\tau}$ and $\varrho = \bar{\varrho}$, we have that*

$$\left.\frac{\partial k}{\partial \lambda}\right|_{\varrho=\bar{\varrho}} = \left.\frac{\partial k}{\partial \lambda}\right|_{\tau=\bar{\tau}} \ \ and \ \ \left.\frac{\partial e}{\partial \lambda}\right|_{\varrho=\bar{\varrho}} = \left.\frac{\partial e}{\partial \lambda}\right|_{\tau=\bar{\tau}}. \tag{31}$$

Proof Relegated to Appendix A. □

More complete annuity markets increase savings but have an ambiguous effect on the education decision. Again, the ambiguity comes from the fact that the direct effect of increasing annuitization on the interest rate—which reduces educational investments, c.f. Eq. 26b—may be offset by the indirect effect of rising capital—which decreases the interest rate and, thereby, increases education. Furthermore, the effect of λ on capital and education is the same in both social security scenarios. This is so because the adjustment of the contribution or replacement rate does not interact with λ.

However, how the level of λ interacts with the derivatives of k and e with respect to s, γ^N, and ω is more interesting. Unfortunately, due to the algebraic complexity of the problem, it is not possible to obtain clear results for these cross-derivatives. However, as is shown in Appendix A, a higher λ makes it more likely that $\partial k/\partial s < 0$. Further results on the importance of annuity markets are illustrated in our numerical simulations, cf., in particular, the discussion in Subsection 3.3.2.

3.3 Numerical results

As stated in the previous subsection, there are cases in which the signs of the derivatives are ambiguous. For these cases, we here present results from numerical simulations of our model to illustrate the sources for this ambiguity. Obviously, our stylized two-period model fails to capture many relevant aspects. This exercise is therefore an illustration only and is not meant to provide exact quantitative results of population aging on the economy. We first investigate the case with perfect annuity markets and then the case without annuity markets. Furthermore, we redo the calculations for both scenarios with constant contribution and constant replacement rates.

3.3.1 Perfect annuity markets

In this subsection, we focus on the case with perfect annuity markets ($\lambda = 1$) where the direct effects of changing survival rates on the education decision is strongest and, consequently, the effects of changing survival are likely to be ambiguous, cf. our previous discussion in Subsection 3.1 and Appendix A. Furthermore, the case with perfect annuity markets, although empirically doubtful, makes our results directly comparable to the perpetual youth model of Kalemli-Ozcan et al. (2000).

We take the periodicity of the model such that each generation covers a maximum of 40 years. Agents are assumed to become economically active at the actual age of 20. Correspondingly, the maximum age agents can reach is 100. Our calibration targets for some of the population parameters are for averages of the three core European countries France, Germany, and Italy.[7] For the survival rate, s, we take as calibration target the remaining life expectancy at the age of 20, LE_{20}, which is currently (in 2004) 68 years. As survival in our model is certain in the first period of life, the survival rate is given by $s = LE_{20}/40 - 1.0 = 0.69$. We calibrate γ^N using the implied γ^N to match the old-age dependency ratio of 44%. Accordingly, we set $\gamma^N = 1.5574$.[8] The long-run growth rate of productivity in European countries is roughly 0.015 (Barro and Sala-i-Martin 2003) annually, so that $\gamma^A = 1.015^{40} = 1.81$. We set the discount factor $\beta = 0.99^{40} = 0.67$ by reference to other studies, e.g., Hurd (1990).

The most critical parameters are ω and ψ and ξ. First, we calibrate ψ to the medium value of the estimates reported in Browning et al. (1999), which is 0.6. Second, there is no direct empirical counterpart of ω because it just reflects an auxiliary variable in our model that simplifies the exposition. To calibrate this parameter, we use the share of agents obtaining higher education as the calibration target.[9] Since the timing of the model is such that the first (and economically passive) period is 20 years, education can also be viewed as the share of people investing into higher education (university and post-graduate education). We construct aggregate indices using data from OECD (2008).[10] The procedure is as follows: We compute the average graduation age of a typical student for the two university (or equivalent) diploma categories (types A and B). Then, we use this number to compute how many years a

[7] Our population data are based on the Human Mortality Database (2008).

[8] The alternative would be to calibrate γ^N with the gross growth rate of the working age population ratio. This would require setting $\gamma^N = 1.06$. The implied oadr is then 0.66, and hence, this alternative would overestimate the actual old-age dependency.

[9] Alternative calibration targets are, e.g., the fraction of the old (age 60 and older) in the population who work, which is 5.4% in the data. In our model, this implies $\omega = 0.12$ and $e = 0.0077$ (0.31 years of education). The choice of this alternative measure does not change our conclusions (results available upon request).

[10] The data we use can be found in Tables A1.1a, A1.3a, and X1.1c. See also the same publication for more detailed information on the educational systems and definitions.

Table 1 Calibration target for time in tertiary education

Type of diploma	Graduation age		Share in population		Weighted years
	B	A	B	A	B+A
France	22.0	24.5	0.11	0.15	0.775
Germany	22.0	25.5	0.09	0.14	0.950
Italy	22.5	26.0	0.01	0.12	0.601
Total					0.795

Graduation age refers to the average within the particular type of diploma. The country weights (France 0.31, Germany 0.40 and Italy 0.39) are given by the relative population size in 2006 computed from the Human Mortality Database (2008)

person spends in tertiary education in excess of the economic starting age (which is set to 20). For example, the "average" French student (see Table 1) is obtaining a type-A diploma at the age of 24.5 and a type-B diploma at the age of 22. We then weight the "excess years" (4.5 and 2) by the population weights (0.11 and 0.15) to obtain years of tertiary education of a representative French agent (0.775). Then, we weight the country-specific years by the population of the three countries to compute years of education for the "representative European" (0.874). As a last step, we divide this number by the duration of one period (40 years) to convert it into the model-specific equivalent and use it as a calibration target. Hence, our target for e is $e = 0.01988$. Third, we calibrate ξ endogenously to match the ratio of peak life cycle wages to the wage rate at labor market entry, which is 1.6 (Attanasio 1999). Since we set $h_0 = 1$, this is the data equivalent to human capital holdings of the old, h_1, and our calibration target requires $\xi = 6.30$. Parameters are summarized in Table 2.

As our discussion of the analytical results in Subsection 3.1 shows, the most critical parameters in the case of perfect annuitization ($\lambda = 1$) are s, ω, ξ, ψ. We therefore consider a range of alternative specifications around the benchmark specification in Table 2 for all these parameters. The graphs have $\omega \in (0, 1)$ on the horizontal axis. The different lines are always drawn for a tuple from $\{\xi \otimes s\}$ for selected values for ξ and s, where the intermediate values

Table 2 Calibration parameters

	$\psi = 0.6$	$\psi = 0.3$
Firm sector		
Capital share, α	0.3	0.3
Technological progress, γ^A	1.81	1.81
Household sector		
Discount factor, β	0.67	0.67
Average productivity of human capital investments, ξ	6.30	1.94
Coefficient in human capital production function, ψ	0.6	0.3
Fraction of the old working, ω	0.36	0.94
Social Security		
Replacement rate, ϱ	0.6	0.6
Demographics		
Birth rate, γ^N	1.56	1.56
Survival rate, s	0.69	0.69

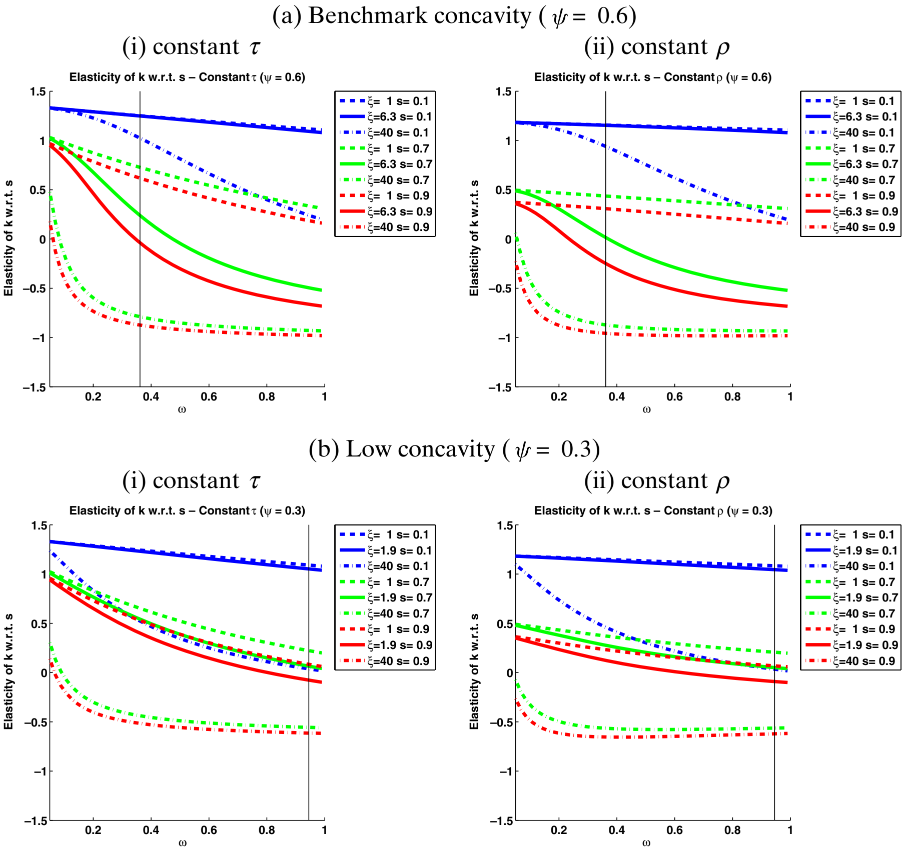

Fig. 1 Elasticity of k with respect to s (**a**, **b**)

(solid lines) are from the benchmark calibration. Finally, in order to address the sensitivity of our results with respect to the concavity of the education technology, we redo all calculations for $\psi = 0.3$.[11] We recalibrate the model when we change the value of ψ. The vertical black line is the calibrated value of ω. Observe that, with lower concavity of the education technology (lower ψ), the calibrated value of ω is increased substantially to match the same target. Instead of reporting the rather uninformative numbers for the derivatives, in the figures, we show elasticities that are better comparable across calibrations.

The effect of changing survival rates on the capital stock are displayed in Fig. 1. As claimed in Proposition 4, the sign is ambiguous. The sign is more likely to be negative for high survival rates, high marginal productivity of education (ψ and ξ), and high labor market participation in the second

[11] For the sake of brevity, simulation results with varying α and β are not displayed but are available upon request.

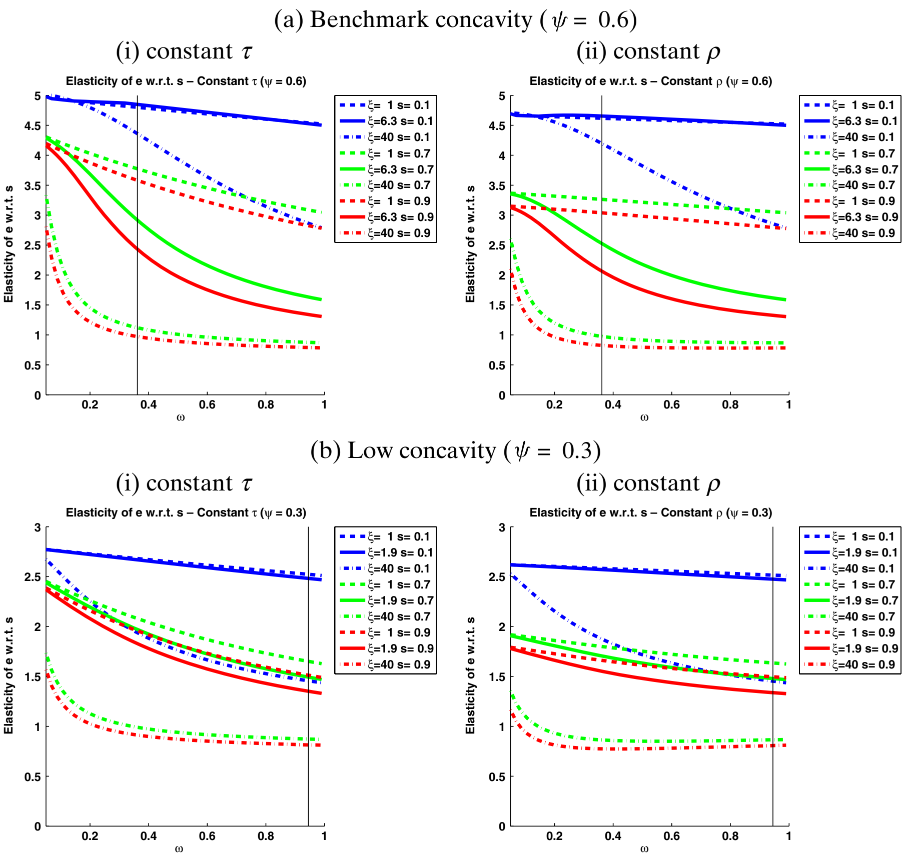

Fig. 2 Elasticity of e with respect to s (**a**, **b**)

period (high ω). Obviously, the higher the marginal product of education (as determined by ξ and ψ), the more agents will invest into education and the less they will work and save. The effect of ω goes in the same direction since it is reinforcing the effect of education.

Figure 2 shows the elasticity of education with respect to the survival rate. Although we show in Proposition 4 that the sign cannot be determined unambiguously, the elasticity is always positive in our simulations. Rising survival rates always increase educational attainment. The simulations also show that the elasticity is smaller for high values of ξ and higher survival rates. The curvature of the human capital production function ψ has only a minor influence.

Finally, Fig. 3 shows how education varies with the time spent on the labor market in the second period. Although the sign cannot be determined analytically, the simulations show that education always increases if ω increases. Thus, the direct effect of a rising ω is not overturned by a general equilibrium

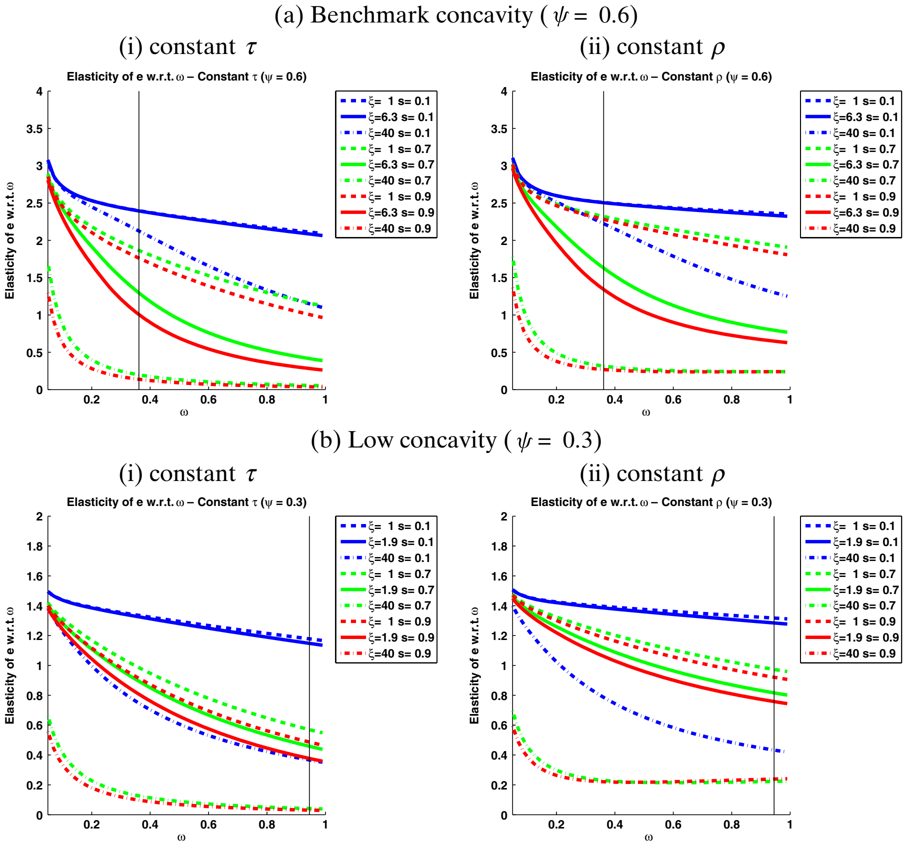

Fig. 3 Elasticity of *e* with respect to ω (**a**, **b**)

effect of rising interest rates. The factor having the largest influence is ψ, which governs the shape of the marginal productivity of schooling investment, and other parameters seem to have only a small effect on the behavior of the model. Not surprisingly, with more concavity of the human capital production function, the (positive) effect of increasing lifetime labor supply on the education decision increases.

3.3.2 No annuity markets

This subsection provides a sensitivity analysis with respect to changes in the degree of annuitization. We set $\lambda = 0$ (corresponding to an economy without annuity markets), recalibrate the model using the same calibration targets as above, and report the new parameters in Table 3, Appendix B. Since only the partial derivative $\partial k/\partial s$ changes its sign if we vary λ, we show only this result in Fig. 4. The other figures can be found in Appendix B.

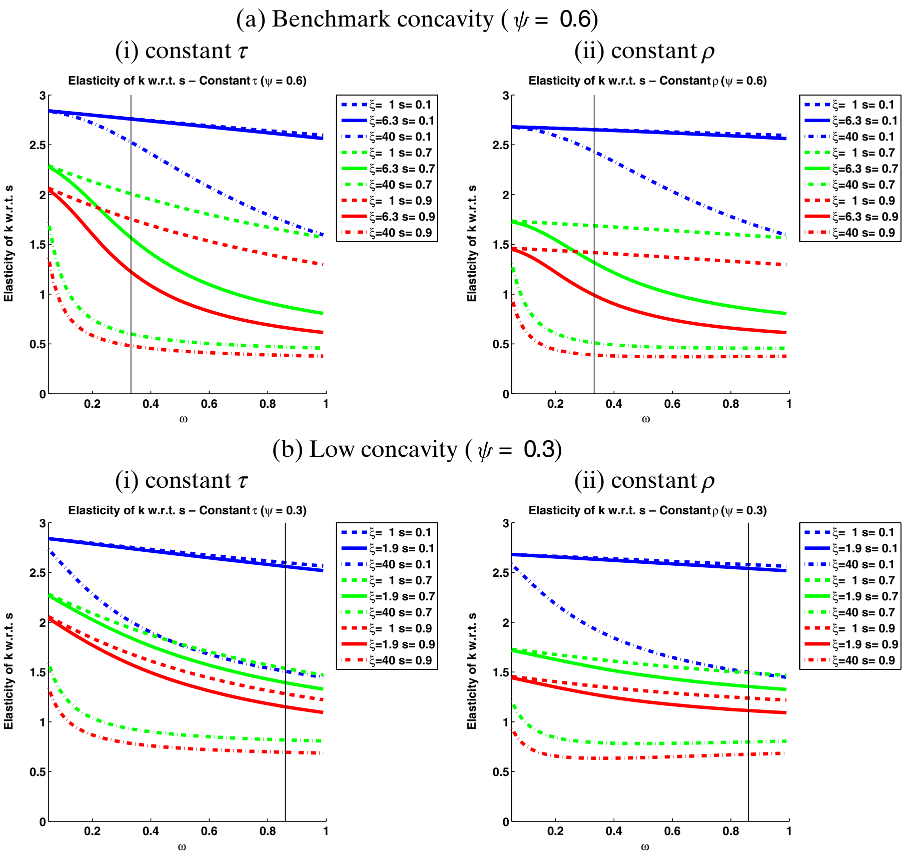

Fig. 4 Elasticity of k with respect to s: no annuity markets (**a**, **b**)

Indeed, with $\lambda = 0$, the reaction of the capital stock to changes in the survival rate is always positive, whereas for $\lambda = 1$, it may also be negative. Thus, the degree of completeness of annuity markets has an important effect on the reaction of the economy. The qualitative effects of changes in the population growth rate, γ^N, and lifetime labor supply, ω, are not affected by the choice of λ (see Appendix B).

4 Conclusion

This paper investigates the effects of a changing population structure on capital accumulation and educational investment in a tractable, two-period model in the spirit of Diamond (1965). We vary the population structure by three dimensions, first, by the fertility rate; second, by the survival rate; and, third, by the degree of old-age labor supply. We show that a decrease of the fertility rate and a corresponding increase of the old-age dependency ratio unambiguously increases the capital intensity and education if contribution rates to the pension system are held constant. An increase of the survival rate, on the other hand, does not unambiguously vary with these variables. Our analytical results and our numerical illustrations shed light on the sources of this ambiguity by highlighting the various and potentially offsetting interaction effects at work. Therefore, our tractable model is a useful laboratory for understanding the magnitudes of the effects found in applied quantitative work employing models with overlapping generations.

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Appendix

A Proofs

Proof of Proposition 1 We have that

$$k_{t+1} = \frac{K_{t+1}}{A_{t+1} L_{t+1}},$$

which, by Eq. 22, can be rewritten as

$$k_{t+1} \frac{L_{t+1}}{N_{t,0}} = \frac{a_{t,0}}{A_{t+1}} \tag{32}$$

We first work on the left-hand side (LHS) of Eq. 32. Using Eq. 23, we get

$$k_{t+1} \frac{L_{t+1}}{N_{t,0}} = k_{t+1} h_0 \left((1 - e_{t+1}) \gamma_{t+1}^N + \omega s_{t+1} (1 + g(e_t)) \right). \tag{33}$$

Next, we focus on the right-hand side (RHS) of Eq. 32). Using Eqs. 5, 9, and 20 in Eq. 14 and bringing the terms involving $a_{t,0}$ to the LHS of the resulting expression, we get

$$\begin{aligned} a_{t,0} &\left(1 + \frac{(1 - s_{t+1})(1 - \lambda)}{(1 + \beta s_{t+1}) \zeta_{t+1}} \right) \\ &= \frac{h_0}{1 + \beta s_{t+1}} \Bigg(\beta s_{t+1} (1 - e_t)(1 - \tau_t) w_t \\ &\qquad - \frac{w_{t+1}}{\zeta_{t+1}(1 + r_{t+1})} \left(s_{t+1} \omega (1 + g(e_t)) + \tau_{t+1} (1 - e_{t+1}) \gamma_{t+1}^N \right) \Bigg). \end{aligned}$$

Bringing the term postmultiplying $a_{t,0}$ to the RHS, replacing r_t and w_t with their marginal products from Eq. 18, and dividing by A_{t+1} gives

$$\frac{a_{t,0}}{A_{t+1}} = \varphi_t \frac{h_0}{1+\beta s_{t+1}} \left(\beta s_{t+1}(1-e_t)(1-\tau_t)(1-\alpha)k_t^{\alpha} \frac{1}{\gamma^A} - \frac{1-\alpha}{\alpha \zeta_{t+1}} k_{t+1} \left(s_{t+1}\omega(1+g(e_t)) + \tau_{t+1}(1-e_{t+1})\gamma_{t+1}^N \right) \right), \quad (34)$$

where

$$\varphi_t = \frac{(1+\beta s_{t+1})\zeta_{t+1}}{(1+\beta s_{t+1})\zeta_{t+1} + (1-s_{t+1})(1-\lambda)}. \quad (35)$$

Next, use the equation above and combine it with Eq. 33 to get

$$k_{t+1} \left((1-e_{t+1})\gamma_{t+1}^N + s_{t+1}\omega(1+g(e_t)) + \varphi_t \frac{1-\alpha}{\alpha(1+\beta s_{t+1})\zeta_{t+1}} \left(s_{t+1}\omega(1+g(e_t)) + \tau_{t+1}(1-e_{t+1})\gamma_{t+1}^N \right) \right) = \varphi_t \frac{\beta s_{t+1}(1-\alpha)}{\gamma^A(1+\beta s_{t+1})}(1-e_t)(1-\tau_t)k_t^{\alpha}.$$

Multiply the above by $\alpha(1+\beta s_{t+1})$ and simplify to get

$$k_{t+1} \left((1-e_{t+1})\gamma_{t+1}^N \left(\alpha(1+\beta s_{t+1}) + \varphi_t \frac{(1-\alpha)\tau_{t+1}}{\zeta_{t+1}} \right) + s_{t+1}\omega(1+g(e_t)) \left(\alpha(1+\beta s_{t+1}) + \varphi_t \frac{1-\alpha}{\zeta_{t+1}} \right) \right) = \varphi_t \frac{\alpha(1-\alpha)\beta s_{t+1}}{\gamma^A}(1-e_t)(1-\tau_t)k_t^{\alpha}.$$

The expression for e_t immediately follows from replacing wages and interest rates by their respective counterparts from Eqs. 18a and 18b. Using $\hat{\rho} = \frac{1}{\beta s_{t+1}} - 1$ proves the claim in the proposition. □

Proof of Proposition 2 First, given that the function $g(e)$ satisfies the lower Inada condition with $\lim_{e\to 0} g'(e) \to \infty$, the solution with zero education is excluded for $\omega \in (0, 1]$. Second, having full educational investment (i.e., $e = 1$), labor supply and, thus, wage income of the young generation are zero. By the lower Inada condition of the utility function, we have that $c_{t,0} > 0$ for positive wages. Consequently, savings in the first period would be negative, as would be the capital stock of the economy. Thus, if there is an equilibrium with finite and positive capital stock, education will always be lower than unity.

To show that education always converges to the steady state solution, use Eq. 24a in Eq. 24b and rewrite the resulting expression as

$$e_t^{1-\psi} = \frac{s_{t+1}}{\zeta_{t+1}} \omega \xi \psi \frac{(1-\alpha)(1-\tau_{t+1})}{\left(\Gamma_1 \frac{1-e_{t+1}}{1-e_t} + \Gamma_2 \frac{1+g(e_t)}{1-e_t}\right)},$$

where

$$\Gamma_1 \equiv \left(\alpha(2+\hat{\rho}_{t+1}) + \varphi_t \frac{(1-\alpha)\tau_{t+1}}{\zeta_{t+1}}(1+\hat{\rho}_{t+1})\right) \gamma_{t+1}^N$$

$$\Gamma_2 \equiv \omega s_{t+1} \left(\alpha(2+\hat{\rho}_{t+1}) + \varphi_t \frac{1-\alpha}{\zeta_{t+1}}(1+\hat{\rho}_{t+1})\right)$$

$\Delta_t \equiv e_t - e_{t+1}(k^*)$ is defined as measuring the distance between e_t and e_{t+1}, which is ultimately a function of the steady state capital stock. Thus, Δ_t measures the change in education between t and $t+1$ outside the steady state. Rearranging gives

$$F(e_t, \Delta_t) = e_t^{1-\psi} - \frac{s_{t+1}}{\zeta_{t+1}} \omega \xi \psi \frac{(1-\alpha)(1-\tau_{t+1})}{\left(\Gamma_1 \frac{1-e_t+\Delta_t}{1-e_t} + \Gamma_2 \frac{1+g(e_t)}{1-e_t}\right)}. \tag{36}$$

Taking the derivative of e_t with respect to the distance to the steady state gives

$$\frac{\partial e_t}{\partial \Delta_t} = -\frac{\partial F/\partial \Delta_t}{\partial F/\partial e_t} < 0 \tag{37a}$$

$$\frac{\partial^2 e_t}{\partial \Delta_t^2} > 0. \tag{37b}$$

Therefore, if education is, e.g., below its new steady state level after an exogenous shock (i.e., $\Delta_t < 0$), e_t will always converge monotonically to the new steady state value. □

Proof of Proposition 3 Existence:
Using Eq. 22 and the assumption of constant population growth, we have

$$k_{t+1} = \frac{1}{\gamma^N \gamma^A} \tilde{a}_{t,0},$$

where $\tilde{a}_{t,0}$ is Eq. 22 divided by A_t to transform $a_{t,0}$ into savings per efficient worker. Define the function

$$d(w_t, r_{t+1}) = \gamma^A \gamma^N k_{t+1} - \tilde{a}(w_t(k_t), r_{t+1}(k_{t+1}), e_t(k_{t+1})), \tag{38a}$$

where $d(\cdot)$ is the change in the capital stock per effective worker. Given that we use log-utility, $e_t \in (0, 1)$, and a Cobb–Douglas production function, it holds that

$$0 < \tilde{a}(w_t, r_{t+1}, e_t) < \tilde{w}_t, \tag{38b}$$

$$0 < \frac{\tilde{a}(w_t, r_{t+1}, e_t)}{k_{t+1}} < \frac{\tilde{w}_t}{k_{t+1}}, \tag{38c}$$

where $\tilde{w}_t$ denotes wages scaled by the level of technology. All we have to show is that $d(\cdot)$ has opposite signs for k_{t+1} going to zero and infinity. Then, by continuity of $d(\cdot)$, there is at least one capital stock satisfying $d(\cdot) = 0$. This holds since

$$\frac{d(w_t, r_{t+1})}{k_{t+1}} = \gamma^N \gamma^A - \frac{\tilde{a}(w_t, r_{t+1}, e_t)}{k_{t+1}}, \tag{38d}$$

and, taking the limits, gives

$$\lim_{k_{t+1} \to \infty} \frac{d(w_t, r_{t+1})}{k_{t+1}} = \gamma^N \gamma^A > 0 \tag{38e}$$

$$\lim_{k_{t+1} \to 0} \frac{d(w_t, r_{t+1})}{k_{t+1}} = -\infty < 0 \tag{38f}$$

for sufficiently small k_{t+1}. For uniqueness, it is sufficient to show that $\partial d(w_t, r_{t+1})/\partial k_{t+1} > 0$ for all k, i.e., that for a given wage rate, $d(w_t, r_{t+1})$ is nondecreasing in the capital stock. Taking Eq. 25a and recalling that $\partial e/\partial k > 0$ establishes the result. By using Eq. 26b, it is clear that a unique solution for the capital stock automatically gives a unique e. □

Proof of Proposition 4 From Eq. 26, define

$$F_1(k, e; \gamma^N, s, \lambda, \omega) = \Omega(e, \gamma^N, s, \lambda, \omega)^{\frac{1}{1-\alpha}} - k = 0 \tag{39a}$$

$$F_2(k, e; \gamma^N, s, \lambda, \omega) = c \cdot \left(\frac{s}{\zeta}\right)^{\frac{1}{1-\psi}} \cdot k^{\frac{1-\alpha}{1-\psi}} - e = 0, \tag{39b}$$

where

$$\Omega(e, \gamma^N, s, \lambda, \omega) \equiv \frac{\varphi}{\phi}(1-\tau)\alpha(1-\alpha)\beta \tag{40}$$

$$c \equiv \left[\omega \xi \psi \frac{\gamma^A}{\alpha}\right]^{\frac{1}{1-\psi}} \tag{41}$$

and ϕ is as in Eq. 27a and φ is as in Eq. 27b.

1. For the case where $\tau = \bar{\tau}$, we can ignore that τ is related to γ^N and s by the steady state version of Eq. 21. The general problem with two implicitly defined endogenous variables can be written as

$$\begin{bmatrix} \frac{\partial k}{\partial X} \\ \frac{\partial e}{\partial X} \end{bmatrix} = - \begin{bmatrix} \frac{\partial F_1}{\partial k} & \frac{\partial F_1}{\partial e} \\ \frac{\partial F_2}{\partial k} & \frac{\partial F_2}{\partial e} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial F_1}{\partial X} \\ \frac{\partial F_2}{\partial X} \end{bmatrix} = -A^{-1} \begin{bmatrix} \frac{\partial F_1}{\partial X} \\ \frac{\partial F_2}{\partial X}, \end{bmatrix} \tag{42}$$

where X is any variable from the vector of exogenous variables $\{\gamma_N, s, \omega,\}$, and therefore,

$$\begin{bmatrix} \frac{\partial k}{\partial X} \\ \frac{\partial e}{\partial X} \end{bmatrix} = -|A|^{-1} \begin{bmatrix} \frac{\partial F_2}{\partial e} & -\frac{\partial F_1}{\partial e} \\ -\frac{\partial F_2}{\partial k} & \frac{\partial F_1}{\partial k} \end{bmatrix} \begin{bmatrix} \frac{\partial F_1}{\partial X} \\ \frac{\partial F_2}{\partial X}, \end{bmatrix} \tag{43}$$

and rearranging gives

$$\begin{bmatrix} \frac{\partial k}{\partial X} \\ \frac{\partial e}{\partial X} \end{bmatrix} = -|A|^{-1} \begin{bmatrix} \frac{\partial F_2}{\partial e}\frac{\partial F_1}{\partial X} & -\frac{\partial F_1}{\partial e}\frac{\partial F_2}{\partial X} \\ -\frac{\partial F_2}{\partial k}\frac{\partial F_1}{\partial X} & +\frac{\partial F_1}{\partial k}\frac{\partial F_2}{\partial X} \end{bmatrix} \tag{44}$$

Since $\tau = \bar{\tau}$, we get

$$\frac{\partial F_1}{\partial k} = -1 < 0 \tag{45a}$$

$$\frac{\partial F_1}{\partial e} = \frac{1}{1-\alpha}\Omega^{1/(1-\alpha)-1}\frac{\partial \Omega}{\partial e} < 0 \tag{45b}$$

$$\frac{\partial F_2}{\partial k} = c\left(\frac{s}{\zeta}\right)^{\frac{1}{1-\psi}} \frac{1-\alpha}{1-\psi} k^{\frac{1-\alpha}{1-\psi}-1} > 0 \tag{45c}$$

$$\frac{\partial F_2}{\partial e} = -1 < 0, \tag{45d}$$

whereby the sign in Eq. 45b follows from $\frac{\partial \Omega}{\partial e} < 0$. Consequently,

$$|A| = \underbrace{\frac{\partial F_1}{\partial k}\frac{\partial F_2}{\partial e}}_{=1} - \underbrace{\frac{\partial F_1}{\partial e}\frac{\partial F_2}{\partial k}}_{<0} > 0. \tag{46}$$

   a. To determine the effect of a changing population growth rate γ_N on k and e, we have to replace X by γ_N in Eq. 42, which gives

$$\frac{\partial F_1}{\partial \gamma^N} = \frac{1}{1-\alpha}\Omega^{1/(1-\alpha)-1}\frac{\partial \Omega}{\partial \gamma^N} < 0 \tag{47a}$$

$$\frac{\partial F_2}{\partial \gamma^N} = 0, \tag{47b}$$

whereby Eq. 47a follows from $\frac{\partial\Omega}{\partial\gamma^N} < 0$, cf. Eqs. 40 and 27a. To get an intuitive idea of what is determining the sign, note that we can write

$$\frac{\partial\Omega}{\partial\gamma^N} = \frac{\partial\varphi/\partial\gamma^N\phi - \varphi\partial\phi/\partial\gamma^N}{\phi^2} = -\frac{\varphi\partial\phi/\partial\gamma^N}{\phi^2} < 0 \tag{48}$$

since φ is independent of γ^N and ϕ is a positive function of γ^N, cf. Eqs. 26a and 27. Thus, γ^N has a direct effect on k but only an indirect effect on e via changing relative prices (this is the reason why $\partial F_2/\partial\gamma^N = 0$). Formally, we have

$$\frac{\partial k}{\partial\gamma^N} = -|A|^{-1}\left(\underbrace{\frac{\partial F_2}{\partial e}\frac{\partial F_1}{\partial\gamma^N}}_{>0} - \underbrace{\frac{\partial F_1}{\partial e}\frac{\partial F_2}{\partial\gamma^N}}_{=0}\right) < 0 \tag{49a}$$

$$\frac{\partial e}{\partial\gamma^N} = -|A|^{-1}\left(-\underbrace{\frac{\partial F_2}{\partial k}\frac{\partial F_1}{\partial\gamma^N}}_{<0} + \underbrace{\frac{\partial F_1}{\partial k}\frac{\partial F_2}{\partial\gamma^N}}_{=0}\right) < 0. \tag{49b}$$

b. To derive the analogous steps for differentiation of Eq. 39 with respect to s, replace the terms in Eq. 47 by

$$\frac{\partial F_1}{\partial s} = \frac{1}{1-\alpha}\Omega^{1/(1-\alpha)-1}\frac{\partial\Omega}{\partial s} \gtrless 0 \tag{50a}$$

$$\frac{\partial F_2}{\partial s} = ck^{\frac{1-\alpha}{1-\psi}}\frac{\partial\frac{s}{\zeta}}{\partial s} \geq 0. \tag{50b}$$

giving

$$\frac{\partial k}{\partial s} = -|A|^{-1}\left(\underbrace{\frac{\partial F_2}{\partial e}\frac{\partial F_1}{\partial s}}_{\gtrless 0} - \underbrace{\frac{\partial F_1}{\partial e}\frac{\partial F_2}{\partial s}}_{\leq 0}\right) \gtrless 0 \tag{51a}$$

$$\frac{\partial e}{\partial s} = -|A|^{-1}\left(-\underbrace{\frac{\partial F_2}{\partial k}\frac{\partial F_1}{\partial s}}_{\gtrless 0} + \underbrace{\frac{\partial F_1}{\partial k}\frac{\partial F_2}{\partial s}}_{\leq 0}\right) \gtrless 0. \tag{51b}$$

Intuitively, the ambiguity of $\frac{\partial e}{\partial s}$ results from the fact that, holding k constant, e is increasing in s as long as $\lambda > 0$ (direct effect), but the capital stock may increase or decrease in s for given education e. As

e increases in k monotonically, the ambiguity of $\frac{\partial k}{\partial s}$ translates into the ambiguity of $\frac{\partial e}{\partial s}$ (indirect effect of s on e).
Arguing formally, the ambiguity of $\frac{\partial k}{\partial s}$ comes from

$$\frac{\partial \Omega}{\partial s} = \frac{\partial(\varphi/\phi)}{\partial s} = \alpha(1-\alpha)\beta(1-\tau)\frac{\varphi'\phi - \varphi\phi'}{\phi^2} \gtrless 0, \tag{52}$$

where $\phi' = \partial\phi/\partial s$ and $\varphi' = \partial\varphi/\partial s$, cf. Eq. 26a. It can be shown that $\varphi' > 0$. Consequently, the sign of $\frac{\partial \phi}{\partial s}$ determines the sign of $\frac{\partial F_1}{\partial s}$ (and thus, $\frac{\partial \Omega}{\partial s}$), and therefore, the sign of Eq. 50a is unambiguous only if $\frac{\partial \phi}{\partial s} < 0$.
To see what determines the sign of ϕ', observe from Eq. 27a that s enters in three places: (1) s pre-multiplies the term $\omega\frac{1+g(e)}{1-e}$; (2) s decreases the effective discount rate $\hat{\rho}$; and (3) s increases the annuity factor, ζ, as long as $\lambda < 1$. Consequently, ϕ increases in s by effect 1, whereas it decreases in s by the effects 2 and 3. We can therefore study an upper bound of ϕ' by setting $\lambda = 1$ so that effect 3 is not at work. This helps to clarify the interaction at the cost of introducing a special case. Using $\hat{\rho} = \frac{1}{\beta s} - 1$ in Eq. 27a and taking the derivative of the resulting equation with respect to s gives

$$\frac{\partial \phi}{\partial s}\frac{1}{\gamma_A} = \omega\alpha\frac{1+g(e)}{1-e} - \frac{\gamma^N}{s^2\beta}\Big[1-(1-\alpha)\,(1-\tau)\Big] \gtrless 0,$$

which is ambiguous.[12] The right part of this equation consists only of exogenous variables. The left part involves the endogenous education decision e for which no closed-form solution is available. Thus, it is not possible to show analytically that the derivative has an unambiguous sign. However, constructing a few special cases clarifies under which conditions $\frac{\partial \phi}{\partial s} < 0$ may hold.

- For $\omega \to 0$, the left part converges to zero (e also converges to zero), and thus, $\frac{\partial \phi}{\partial s} < 0$.
- For $\omega = 1$, which implies that $\tau = 0$, we have

$$\frac{\partial \phi}{\partial s}\frac{1}{\gamma_A} = \alpha\left(\frac{1+g(e)}{1-e} - \frac{\gamma^N}{s^2\beta}\right) \gtrless 0.$$

[12] To see what happens for $\lambda \neq 1$, define $\mu \equiv \varphi/\xi$ and $\mu' \equiv \partial\mu/\partial s$. Then, the corresponding term is

$$\frac{\partial \phi}{\partial s}\frac{1}{\gamma^A} = \omega\frac{1+ge(e)}{1-e}\left(\alpha + \mu'\frac{1-\alpha}{\beta}\right) - \frac{\gamma^N}{s^2\beta}(\alpha + (1-\alpha)\mu\tau) + \mu'\frac{\gamma^N(1-\alpha)\tau}{s\beta},$$

where it is obvious that the last two terms are negative ($\mu > 0$ and $\mu' < 0$) but the sign of the term in the first bracket is ambiguous again. Thus, by setting $\lambda = 1$ (perfect annuity markets), which implies $\frac{\varphi}{\zeta} = 1$, we know that $\phi'|_{0\leq\lambda<1} < \phi'|_{\lambda=1}$ holds.

- For $\xi \to 0$ or $\psi \to 0$, we have that $e \to 0$, which means that

$$\frac{\partial \phi}{\partial s}\frac{1}{\gamma_A} = \omega\alpha - \frac{\gamma^N}{s^2\beta}\left[1-(1-\alpha)(1-\tau)\right] \gtrless 0.$$

Summarizing the arguments made so far, the sign of $\frac{\partial \phi}{\partial s}$ is negative (implying that k is increasing in s) if

- Returns to education are low (low ξ and/or ψ)
- The horizon over which the benefits can be reaped is short (low ω)
- The discount factor β is low (i.e., high discount rate)
- The population growth rate γ^N is high
- The survival probability s is low

c. Changing the planning horizon ω gives

$$\frac{\partial F_1}{\partial \omega} = \frac{\partial \Omega}{\partial \omega} < 0 \tag{53a}$$

$$\frac{\partial F_2}{\partial \omega} = \frac{1}{1-\psi}\omega^{\frac{\psi}{1-\psi}}\left(\xi\psi\frac{\gamma^A}{\alpha}\right)^{\frac{1}{1-\psi}}\left(\frac{s}{\zeta}\right)^{\frac{1}{1-\psi}} k^{\frac{1-\alpha}{1-\psi}} > 0, \tag{53b}$$

and therefore,

$$\frac{\partial k}{\partial \omega} = -|A|^{-1}\left(\underbrace{\frac{\partial F_2}{\partial e}\frac{\partial F_1}{\partial \omega}}_{>0} - \underbrace{\frac{\partial F_1}{\partial e}\frac{\partial F_2}{\partial \omega}}_{<0}\right) < 0 \tag{54a}$$

$$\frac{\partial e}{\partial \omega} = -|A|^{-1}\left(-\underbrace{\frac{\partial F_2}{\partial k}\frac{\partial F_1}{\partial \omega}}_{<0} + \underbrace{\frac{\partial F_1}{\partial k}\frac{\partial F_2}{\partial \omega}}_{<0}\right) \gtrless 0. \tag{54b}$$

Some intuition as to why the sign of $\partial e/\partial \omega$ is indeterminate can be gained by writing out Eq. 54b and inserting the derivatives from above, which gives

$$\frac{\partial e}{\partial \omega} = |A|^{-1}\left(\frac{s}{\zeta}\right)^{\frac{1}{1-\psi}}\frac{k^{\frac{1-\alpha}{1-\psi}}}{1-\psi}\left((1-\alpha)\frac{\partial \Omega}{\partial \omega}k^{-1} + \omega^{-1}\right).$$

Hence, the ambiguity is caused by the negative effect of rising labor market participation on the capital stock ($\partial\Omega/\partial\omega < 0$) and the positive counterbalancing effect of more education (ω^{-1}) due to a higher lifetime labor supply ω.

On the contrary, the reason why the sign of $\partial k/\partial\omega$ can always be determined is that the effects of ω on k and e work into the same direction. Writing out Eq. 54a and simplifying yields

$$\frac{\partial k}{\partial\omega} = |A|^{-1}\left(\frac{\partial\Omega}{\partial\omega} - \frac{1}{1-\psi}\omega^{-1}e\right) < 0,$$

where $\partial\Omega/\partial\omega < 0$ captures the direct effect of more labor and the second part captures the additional effect of changing education.

2. In case $\varrho = \bar{\varrho}$, there is a direct (d) and an indirect effect in the partial derivatives of Ω, $\frac{\partial\Omega}{\partial X} = (\frac{\partial\Omega}{\partial X})^d + \frac{\partial\Omega}{\partial\tau}\frac{\partial\tau}{\partial X}$. Observe from Eq. 21 that

$$\frac{\partial\tau}{\partial s} = \frac{\gamma^N\bar{\rho}(1-\omega)}{(s(\bar{\rho}(1-\omega)+\omega)+\gamma^N)^2} > 0 \tag{55}$$

$$\frac{\partial\tau}{\partial\gamma^N} = -\frac{s\bar{\rho}(1-\omega)}{(s(\bar{\rho}(1-\omega)+\omega)+\gamma^N)^2} < 0 \tag{56}$$

$$\frac{\partial\tau}{\partial\omega} = -\frac{s\bar{\rho}(s+\gamma^N)}{(s(\bar{\rho}(1-\omega)+\omega)+\gamma^N)^2} < 0 \tag{57}$$

Therefore, for given γ^N, s, and ω, the strength of the indirect effect increases in $\bar{\varrho}$. Note that changing the adjustment rule of the social security system affects only F_1 because there is no direct effect of τ on the education decision in steady state. Due to the additional indirect effect, it is not possible any more to determine the sign of the derivatives. We can only say whether the effects become smaller or larger, compared to the $\tau = \bar{\tau}$ case.

a. The difference between the two social security scenarios if γ^N changes and τ adjusts is given by

$$\frac{\partial F_1}{\partial\gamma^N} = \Omega^{1/(1-\alpha)-1}\alpha\beta\left(\frac{\partial\varphi/\phi}{\partial\gamma^N}(1-\tau) - \frac{\varphi}{\phi}\frac{\partial\tau}{\partial\gamma^N}\right), \tag{58}$$

with

$$\frac{\partial\phi}{\partial\gamma^N} = \gamma^A\left(\alpha(2+\hat{\rho}) + \varphi\frac{(1-\alpha)}{\zeta}(1+\hat{\rho})\left(\tau+\gamma^N\frac{\partial\tau}{\partial\gamma^N}\right)\right) > 0, \tag{59}$$

where the difference to the $\tau = \bar{\tau}$ scenario is only the term $\gamma^N\frac{\partial\tau}{\partial\gamma^N}$. Using Eq. 56 implies that

$$\left.\frac{\partial F_1}{\partial\gamma^N}\right|_{\rho=\bar{\rho}} > \left.\frac{\partial F_1}{\partial\gamma^N}\right|_{\tau=\bar{\tau}}, \tag{60}$$

which proves that

$$\left.\frac{\partial k}{\partial \gamma^N}\right|_{\varrho=\bar{\varrho}} > \left.\frac{\partial k}{\partial \gamma^N}\right|_{\tau=\bar{\tau}} \quad \text{and} \quad \left.\frac{\partial e}{\partial \gamma^N}\right|_{\varrho=\bar{\varrho}} > \left.\frac{\partial e}{\partial \gamma^N}\right|_{\tau=\bar{\tau}}. \tag{61}$$

b. To see how changes in the survival rate affect k and e with fixed replacement rate, we have to evaluate

$$\frac{\partial F_1}{\partial s} = \Omega^{1/(1-\alpha)-1}\beta\alpha\left(\frac{\partial\varphi/\phi}{\partial s}(1-\tau) - \frac{\varphi}{\phi}\frac{\partial\tau}{\partial s}\right). \tag{62}$$

The right part in the parentheses is obviously negative. To obtain the total effect, we have to evaluate $\frac{\partial(\varphi/\phi)}{\partial s}$. Since φ does not vary with τ, there is no indirect effect. Thus, we again only have to evaluate the change in ϕ, including now the change in the contribution rate τ. Again, differentiating Eq. 27a with respect to s gives

$$\frac{\partial\phi}{\partial s}\frac{1}{\gamma_A} = \omega\alpha\frac{1+g(e)}{1-e} - \frac{\gamma^N}{s^2\beta}\Big[1-(1-\alpha)\,(1-\tau)\Big] + (1-\alpha)\frac{\gamma^N}{s\beta}\frac{\partial\tau}{\partial s} < 0,$$

where we see that the derivative is identical to the case with $\tau = \bar{\tau}$, except for the last positive term. Using Eq. 52 and knowing that $\partial\phi/\partial s$ evaluated with the indirect effect is larger (smaller in absolute value) gives

$$\left.\frac{\partial\varphi/\phi}{\partial s}\right|_{\rho=\bar{\rho}} < \left.\frac{\partial\varphi/\phi}{\partial s}\right|_{\tau=\bar{\tau}} \quad \Rightarrow \quad \left.\frac{\partial F_1}{\partial s}\right|_{\rho=\bar{\rho}} < \left.\frac{\partial F_1}{\partial s}\right|_{\tau=\bar{\tau}}, \tag{63}$$

which implies that

$$\left.\frac{\partial k}{\partial s}\right|_{\varrho=\bar{\varrho}} < \left.\frac{\partial k}{\partial s}\right|_{\tau=\bar{\tau}} \quad \text{and} \quad \left.\frac{\partial e}{\partial s}\right|_{\varrho=\bar{\varrho}} < \left.\frac{\partial e}{\partial s}\right|_{\tau=\bar{\tau}}. \tag{64}$$

c. Differences between the two social security scenarios if ω changes are given by

$$\frac{\partial F_1}{\partial \omega} = \Omega^{1/(1-\alpha)-1}\beta\alpha\left(\frac{\partial\varphi/\phi}{\partial \omega}(1-\tau) - \frac{\varphi}{\phi}\frac{\partial\tau}{\partial \omega}\right). \tag{65}$$

Differentiating Eq. 27a with respect to ω gives

$$\begin{aligned}\frac{\partial\phi}{\partial\omega} = \gamma^A\Bigg(&\varphi\frac{(1-\alpha)}{\zeta}(1+\hat{\rho})\gamma^N\frac{\partial\tau}{\partial\omega}\\ &+s\left(\alpha(2+\hat{\rho}) + \varphi\frac{1-\alpha}{\zeta}(1+\hat{\rho})\right)\frac{1+g(e)}{1-e}\Bigg),\end{aligned} \tag{66}$$

where the difference is only the adjusting contribution rate $\frac{\partial\tau}{\partial\omega}$. Using Eq. 57, it holds that

$$\left.\frac{\partial F_1}{\partial\omega}\right|_{\rho=\bar{\rho}} > \left.\frac{\partial F_1}{\partial\omega}\right|_{\tau=\bar{\tau}} \tag{67}$$

proving that

$$\left.\frac{\partial k}{\partial \omega}\right|_{\varrho=\bar{\varrho}} > \left.\frac{\partial k}{\partial \omega}\right|_{\tau=\bar{\tau}} \quad \text{and} \quad \left.\frac{\partial e}{\partial \omega}\right|_{\varrho=\bar{\varrho}} > \left.\frac{\partial e}{\partial \omega}\right|_{\tau=\bar{\tau}}. \tag{68}$$

□

Proof of Proposition 5 The effect of the degree of annuitization (λ) on the capital stock and education decision is given by

$$\frac{\partial F_1}{\partial \lambda} = \frac{\partial \Omega}{\partial \lambda} > 0 \tag{69a}$$

$$\frac{\partial F_2}{\partial \lambda} = c \cdot s^{\frac{1}{1-\psi}} k^{\frac{1-\alpha}{1-\psi}} \frac{\partial \zeta^{-1}}{\partial \lambda} < 0. \tag{69b}$$

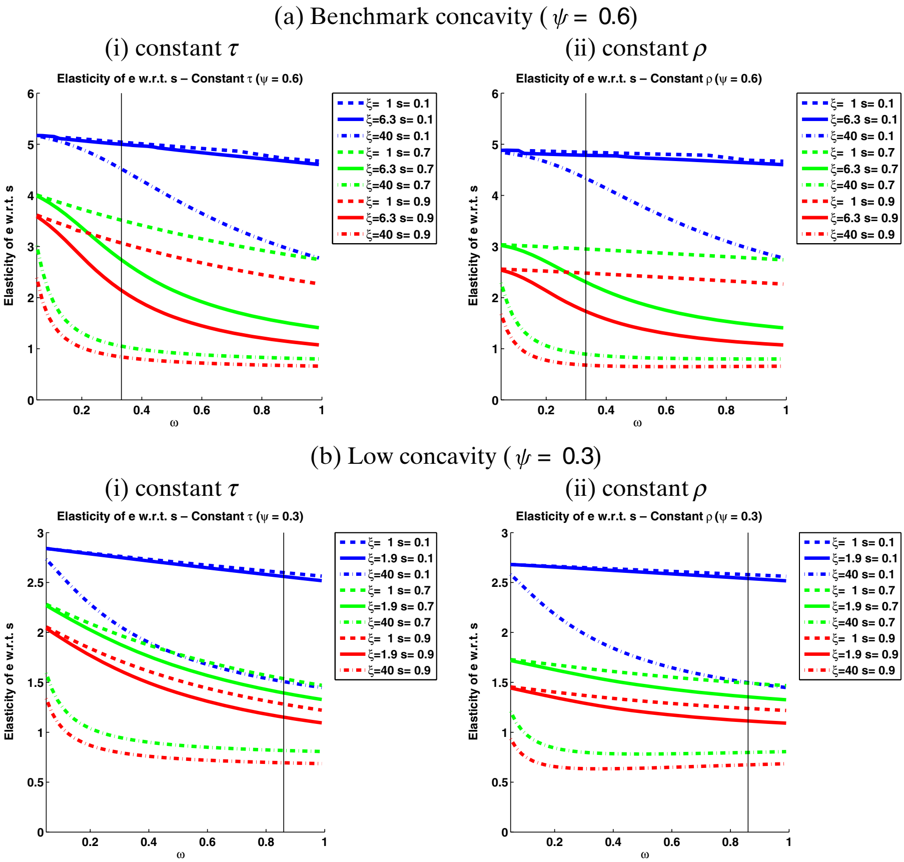

Fig. 5 Elasticity of *e* with respect to *s*: no annuity markets (**a**, **b**)

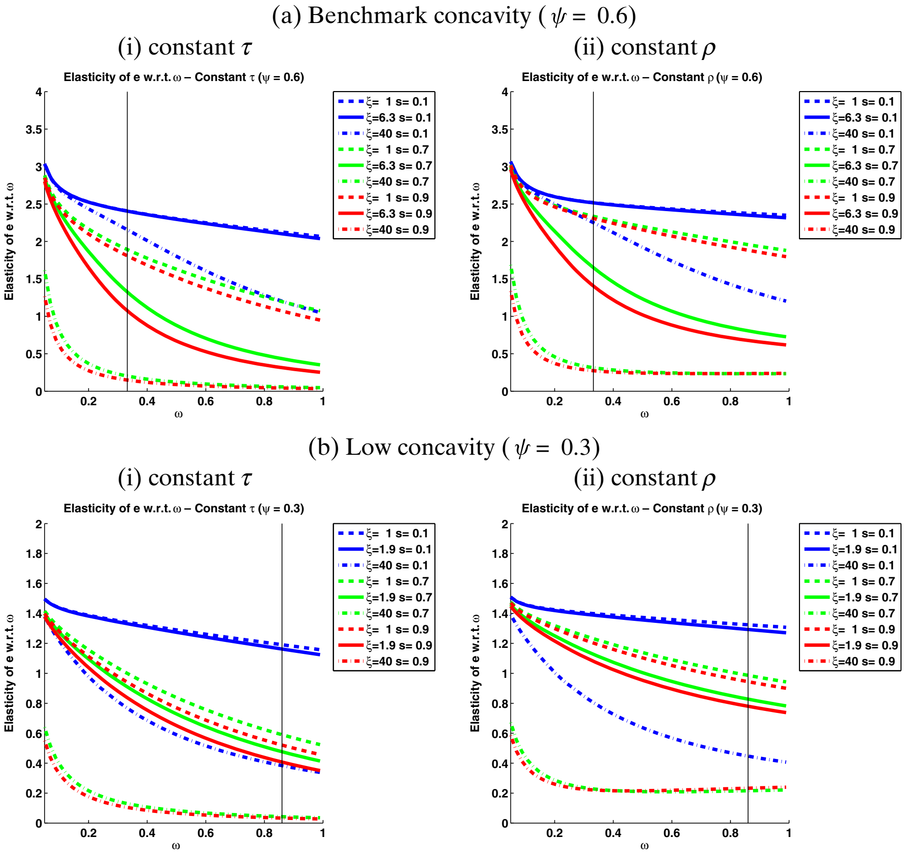

Fig. 6 Elasticity of e with respect to ω: no annuity markets (**a**, **b**)

Table 3 Calibration parameters: no annuity markets

	$\psi = 0.6$	$\psi = 0.3$
Firm sector		
Capital share, α	0.3	0.3
Technological progress, γ^A	1.81	1.81
Household sector		
Discount factor, β	0.67	0.67
Average productivity of human capital investments, ξ	6.30	1.94
Coefficient in human capital production function, ψ	0.6	0.3
Fraction of the old working, ω	0.33	0.86
Social Security		
Replacement rate, ϱ	0.6	0.6
Demographics		
Birth rate, γ^N	1.56	1.56
Survival rate, s	0.69	0.69

Replacing the terms in Eq. 47 with the ones from above gives

$$\frac{\partial k}{\partial \lambda} = -|A|^{-1}\left(\underbrace{\frac{\partial F_2}{\partial e}\frac{\partial F_1}{\partial \lambda}}_{<0} - \underbrace{\frac{\partial F_1}{\partial e}\frac{\partial F_2}{\partial \lambda}}_{>0}\right) > 0 \tag{70a}$$

$$\frac{\partial e}{\partial \lambda} = -|A|^{-1}\left(-\underbrace{\frac{\partial F_2}{\partial k}\frac{\partial F_1}{\partial \lambda}}_{>0} + \underbrace{\frac{\partial F_1}{\partial k}\frac{\partial F_2}{\partial \lambda}}_{>0}\right) \gtrless 0 \tag{70b}$$

Qualitatively, changing λ has the same effects in both social security scenarios because the availability of annuity markets does not interact with the adjustment of contribution or replacement rates. □

B Numerical results: no annuity markets

This appendix presents additional numerical results of our sensitivity analysis for the case of perfect annuity markets, cf. our discussion in Subsection 3.3.2. Calibration parameters are reported in Table 2. Results for the elasticities of e with respect to s and w are shown in the corresponding Figs. 5 and 6.

Figures 1, 2, 3, 4, 5, and 6, and Tables 1, 2, and 3.

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