* **Basic model:**

Let’s define E as the sum of per capita education spending between age 3 and age 26. Denote CE (x) as the per capita overall education expenditure at age x. Note that CE(x)=CGE(x)+CFE(x) whereby CGE represents per capita public education spending and CFE represents per capita private education spending respectively.

Therefore

We hypothesize that the following elasticity equation describes the relationship between per capita education consumption, average per capita labor income during prime working age of 30 to 49 (YL30-49), and total fertility rate (TFR).

Note that this representation constrains the coefficient of to be 1 by normalizing total education spending on average prime working age labor income per capita. One could also unconstrain by using properties of logarithms to the equation shown below:

* Constrained Model

In the constrained model, the dependent variable in our regression model is the normalized education spending (public, private, and overall). By normalizing consumption in education, the coefficient estimate of ln(YL) was forced to be 1 and the only explanatory variable is Child Dependency Ratio (measures the relative size of dependent children in the population—the ratio of population under the age of 24 to population between the age of 25 and 59) or Total Fertility Rate (measures the average number of children born to a woman during her reproductive life).

Mathematically, we are estimating in the two equations below where E represents all three types of education consumption as mentioned above:

+ Equation(1) &

+ Equation(2)

In Table 1, the regression results are reported:

Table 1

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | (1) | (2) | (3) | (4) | (5) | (6) |
|  | ln\_norm\_cge | ln\_norm\_cfe | ln\_norm\_ce | ln\_norm\_cge | ln\_norm\_cfe | ln\_norm\_ce |
| ln\_cdr | -1.250\*\*\* | 0.423 | -0.722\*\*\* |  |  |  |
|  | (-9.25) | (1.24) | (-7.06) |  |  |  |
|  |  |  |  |  |  |  |
| ln\_tfr |  |  |  | -1.158\*\*\* | 0.160 | -0.672\*\*\* |
|  |  |  |  | (-7.66) | (0.47) | (-6.21) |
|  |  |  |  |  |  |  |
| \_cons | 0.456\*\*\* | -0.494\*\* | 0.948\*\*\* | 1.444\*\*\* | -0.640 | 1.522\*\*\* |
|  | (7.49) | (-3.22) | (20.58) | (10.29) | (-2.02) | (15.14) |
| r2 | 0.698 | 0.0402 | 0.574 | 0.613 | 0.00587 | 0.510 |

t-statistic in parentheses

\* *p* < 0.05, \*\* *p* < 0.01, \*\*\* *p* < 0.001

Note that

1. The coefficients of ln(CDR) and ln(TFR) are both statistically significant for the natural logarithms of public education expenditure and overall education spending. They also have the expected negative sign. The model explains over 60% of the variations in public education expenditure and overall education spending.
2. Normalized public education spending is elastic to changes in CDR and TFR: a 1% increase in CDR leads to a 1.25% decrease in normalized public education spending; a 1% increase in TFR leads to a 1.158% decrease in normalized overall education consumption.
3. The coefficient estimates of CDR and TFR on private education consumption do not come out to be statistically significant even at 5% level. However, since only 39 observations are included in the model, the lack of statistical significance does not represent a huge issue.
4. Disregarding statistical significance, a 1% increase in CDR leads to a 0.423% increase in private educational expenditure and a 1% increase in TFR leads to a 0.160% increase in private educational expenditure.
5. Overall, a given percentage increase in CDR leads to a bigger change in educational spending than that in TFR.

Figure 1(a) Equation (1) CDR Figure 1(b) Equation (2) TFR

 

Given the unexpected positive sign of the coefficient estimates for CDR and TFR on private education spending, we suspect that this could be caused by the fact that in European countries, education spending is highly concentrated in the public sector and private consumption in education would be significantly smaller compared to public consumption. To check this, we have also analyzed the residuals by drawing scatterplot of residuals versus predicted value of the dependent variable—natural logarithms of education spending. Notably, the majority of the 10 European countries included in the analysis have negative residuals as expected—powerful enough to cause the line of best fit to have a positive slope.

* Unconstrained Model

In the unconstrained model, is no longer grouped together with education spending and it has become an independent variable itself for us to analyze. As a result, both education spending and must be converted to US dollars using PPP exchange rate for comparisons among countries. There are two options for PPP exchange rate—GDP PPP or Private Consumption PPP (NTA database has the latter in the system). Both PPP conversion rate have been used for analysis just for comparison as we would like to know whether the coefficient estimates would come out to be different.

Table 2

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | (1) | (2) | (3) | (4) | (5) | (6) |
|  | ln\_cge\_ppp\_gdp | ln\_cfe\_ppp\_gdp | ln\_ce\_ppp\_gdp | ln\_cge\_ppp\_pc | ln\_cfe\_ppp\_pc | ln\_ce\_ppp\_pc |
| ln\_yl\_ppp\_gdp | 1.375\*\*\* | 0.546\* | 1.170\*\*\* |  |  |  |
|  | (17.40) | (2.54) | (17.63) |  |  |  |
| ln\_tfr | -0.462\* | -0.683 | -0.357\* | -0.442\* | -0.731 | -0.378\* |
|  | (-2.43) | (-1.32) | (-2.24) | (-2.39) | (-1.44) | (-2.36) |
| ln\_yl\_ppp\_pc |  |  |  | 1.367\*\*\* | 0.542\* | 1.151\*\*\* |
|  |  |  |  | (18.68) | (2.69) | (18.11) |
| \_cons | -2.552\*\* | 4.199 | -0.289 | -2.461\*\* | 4.225 | -0.0840 |
|  | (-3.00) | (1.82) | (-0.41) | (-3.13) | (1.95) | (-0.12) |
| r2 | 0.963 | 0.481 | 0.963 | 0.967 | 0.511 | 0.965 |

t-statistic in parentheses

\* *p* < 0.05, \*\* *p* < 0.01, \*\*\* *p* < 0.001

Table 2 reports the results for the unconstrained model where the demographic variable measuring dependency here is TFR.

We have noticed that:

1. Coefficient estimates for ln(TFR) are negative with absolute values less than 1 across all six specifications. This suggests that after unrestraining YL, education spending has switched from being elastic to being inelastic to changes in TFR. A 1% increase in TFR leads to a 0.462% decrease in public education spending and 0.357% decrease in overall spending when using GDP PPP conversion.
2. Similar to Table 1, ln(TFR) only has statistically significant impact on public educational spending and overall educational spending, yet the level of statistical significance has dropped to 5% level from 0.1% level. The estimates do not differ dramatically depending on whether GDP PPP or Private Consumption PPP was used.
3. Coefficient estimates for ln(YL\_ppp) are larger than 1 for ln(CGE) and ln(CE) and are highly statistically significant.
4. The model explains over 95% of the variation in public and overall education spending, but only 50% of the variation in private expenditure.

Table 3

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | (1) | (2) | (3) | (4) | (5) | (6) |
|  | ln\_cge\_ppp\_gdp | ln\_cfe\_ppp\_gdp | ln\_ce\_ppp\_gdp | ln\_cge\_ppp\_pc | ln\_cfe\_ppp\_pc | ln\_ce\_ppp\_pc |
| ln\_yl\_ppp\_gdp | 1.313\*\*\* | 0.655\* | 1.107\*\*\* |  |  |  |
|  | (12.69) | (2.29) | (12.91) |  |  |  |
| ln\_cdr | -0.586\* | -0.308 | -0.494\* | -0.549\* | -0.398 | -0.530\* |
|  | (-2.33) | (-0.44) | (-2.37) | (-2.26) | (-0.59) | (-2.56) |
| ln\_yl\_ppp\_pc |  |  |  | 1.316\*\*\* | 0.630\* | 1.086\*\*\* |
|  |  |  |  | (13.84) | (2.37) | (13.42) |
| \_cons | -2.381\* | 2.634 | -0.0263 | -2.374\*\* | 2.820 | 0.175 |
|  | (-2.53) | (1.01) | (-0.03) | (-2.78) | (1.18) | (0.24) |
| r2 | 0.962 | 0.459 | 0.964 | 0.966 | 0.488 | 0.966 |

t-statistic in parentheses

\* *p* < 0.05, \*\* *p* < 0.01, \*\*\* *p* < 0.001

Table 3 reports the results for the unconstrained model where the demographic variable measuring dependency here is CDR.

We have noticed that:

1. Coefficient estimates for ln(CDR) are negative with absolute values less than 1 across all six specifications. This suggests that after unrestraining YL, education spending has switched from being elastic to being inelastic to changes in CDR. A 1% increase in CDR leads to a 0.586% decrease in public education spending and 0.494% decrease in overall spending when using GDP PPP conversion.
2. Similar to Table 1, ln(CDR) only has statistically significant impact on public educational spending and overall educational spending, yet the level of statistical significance has dropped to 5% level from 0.1% level. The estimates do not differ dramatically depending on whether GDP PPP or Private Consumption PPP was used.
3. Coefficient estimates for ln(YL\_ppp) are larger than 1 for ln(CGE) and ln(CE) and are highly statistically significant. In this model though, ln(YL\_ppp) also affects ln(CFE) in a statistically significant manner at 5% level.

Comparing the result in Table 2 and 3, one also notices that in general, a 1% increase in CDR tends to have a larger impact in the percentage change in education spending than a 1% increase in TFR does.

Figure 2 (a) Figure 2 (b)

 

Figure 2 (c) Figure 2 (d)

 

In Figure 2 above, we plotted the residuals of ln(CFE) on the predicted value of ln(CFE) by the respective models. Again, we have observed that majority of the European countries have negative residuals.

* Unconstrained Model with Europe Dummy

Upon noticing that European countries tend to have negative residuals, especially when predicting ln(CFE), our model is further augmented by including the Europe dummy variable.

Table 4 (a): using GDP PPP

|  |  |  |  |
| --- | --- | --- | --- |
|  | (1) | (2) | (3) |
|  | ln\_cge\_ppp\_gdp | ln\_cfe\_ppp\_gdp | ln\_ce\_ppp\_gdp |
| ln\_yl\_ppp\_gdp | 1.351\*\*\* | 0.863\*\*\* | 1.193\*\*\* |
|  | (0.0872) | (0.199) | (0.0730) |
| Europe | 0.108 | -1.444\*\*\* | -0.107 |
|  | (0.162) | (0.370) | (0.136) |
| ln\_tfr | -0.451\* | -0.826 | -0.368\* |
|  | (0.192) | (0.438) | (0.161) |
| \_cons | -2.372\* | 1.787 | -0.467 |
|  | (0.898) | (2.053) | (0.753) |
| *N* | 39 | 39 | 39 |
| *R*2 | 0.963 | 0.638 | 0.964 |

Standard errors in parentheses

\* *p* < 0.05, \*\* *p* < 0.01, \*\*\* *p* < 0.001

Table 4 (b): using Private Consumption PPP

|  |  |  |  |
| --- | --- | --- | --- |
|  | (1) | (2) | (3) |
|  | ln\_cge\_ppp\_pc | ln\_cfe\_ppp\_pc | ln\_ce\_ppp\_pc |
| ln\_yl\_ppp\_pc | 1.348\*\*\* | 0.840\*\*\* | 1.172\*\*\* |
|  | (0.0810) | (0.188) | (0.0702) |
| Europe | 0.0928 | -1.419\*\*\* | -0.0991 |
|  | (0.159) | (0.370) | (0.138) |
| ln\_tfr | -0.433\* | -0.869 | -0.388\* |
|  | (0.187) | (0.435) | (0.162) |
| \_cons | -2.315\*\* | 2.006 | -0.239 |
|  | (0.831) | (1.933) | (0.720) |
| *N* | 39 | 39 | 39 |
| *R*2 | 0.967 | 0.656 | 0.965 |

Standard errors in parentheses

\* *p* < 0.05, \*\* *p* < 0.01, \*\*\* *p* < 0.001

Note that:

1. Average labor income has statistically significant impact on public, private, as well as overall education expenditure. CGE and CE are labor income elastic whereas CFE is labor income inelastic.
2. The Europe dummy is negative and statistically significant only for ln(CFE). This suggests that European countries on average spend approximately 1.4% less than non-European countries in our sample.
3. CGE and CE are inelastic to change in TFR. A 1% increase in TFR leads to about 0.44% decrease in public education spending and a 0.37% decrease in overall education spending.
4. R squares for CGE and CE are above 0.95—suggesting that our model estimates the variation in the dependent variable reasonably well.

The regression results of substituting CDR for TFR are reported in Table 5.

Table 5 (a) using GDP PPP

|  |  |  |  |
| --- | --- | --- | --- |
|  | (1) | (2) | (3) |
|  | ln\_cge\_ppp\_gdp | ln\_cfe\_ppp\_gdp | ln\_ce\_ppp\_gdp |
| ln\_yl\_ppp\_gdp | 1.309\*\*\* | 0.802\*\* | 1.125\*\*\* |
|  | (0.106) | (0.242) | (0.0859) |
| Europe | 0.0374 | -1.564\*\*\* | -0.184 |
|  | (0.170) | (0.389) | (0.138) |
| ln\_cdr | -0.570\* | -0.991 | -0.574\* |
|  | (0.265) | (0.607) | (0.215) |
| \_cons | -2.358\* | 1.660 | -0.141 |
|  | (0.959) | (2.192) | (0.777) |
| *N* | 39 | 39 | 39 |
| *R*2 | 0.962 | 0.630 | 0.965 |

Standard errors in parentheses

\* *p* < 0.05, \*\* *p* < 0.01, \*\*\* *p* < 0.001

Table 5 (b) using private consumption PPP

|  |  |  |  |
| --- | --- | --- | --- |
|  | (1) | (2) | (3) |
|  | ln\_cge\_ppp\_pc | ln\_cfe\_ppp\_pc | ln\_ce\_ppp\_pc |
| ln\_yl\_ppp\_pc | 1.313\*\*\* | 0.776\*\* | 1.103\*\*\* |
|  | (0.0976) | (0.227) | (0.0812) |
| Europe | 0.0258 | -1.550\*\*\* | -0.182 |
|  | (0.167) | (0.388) | (0.139) |
| ln\_cdr | -0.538\* | -1.059 | -0.608\*\* |
|  | (0.256) | (0.595) | (0.213) |
| \_cons | -2.358\* | 1.868 | 0.0632 |
|  | (0.872) | (2.024) | (0.725) |
| *N* | 39 | 39 | 39 |
| *R*2 | 0.966 | 0.648 | 0.967 |

Standard errors in parentheses

\* *p* < 0.05, \*\* *p* < 0.01, \*\*\* *p* < 0.001

Compare with Table 4, the absolute size of the Europe dummy is a little bigger using CDR than using TFR.

* Unconstrained model with Europe Dummy and Old Age Dependency Ratio (OADR)

In this model, we are adding the old age depedency ratio (OADR) to our model—defined as the ratio of the population between age 30 and 49 to that aged 30 and above. Table 6 represents the model with TFR and Table 7 represents the model with CDR. Results in Table 6 are discussed below.

Table 6 (a) using GDP PPP

|  |  |  |  |
| --- | --- | --- | --- |
|  | (1) | (2) | (3) |
|  | ln\_cge\_ppp\_gdp | ln\_cfe\_ppp\_gdp | ln\_ce\_ppp\_gdp |
| ln\_yl\_ppp\_gdp | 1.344\*\*\* | 0.967\*\*\* | 1.198\*\*\* |
|  | (0.106) | (0.240) | (0.0889) |
| Europe | 0.0961 | -1.271\*\* | -0.0985 |
|  | (0.191) | (0.433) | (0.160) |
| ln\_tfr | -0.448\* | -0.863 | -0.369\* |
|  | (0.196) | (0.443) | (0.164) |
| ln\_oadr | -0.0872 | 1.285 | 0.0601 |
|  | (0.720) | (1.630) | (0.603) |
| \_cons | -2.357\* | 1.563 | -0.478 |
|  | (0.920) | (2.083) | (0.771) |
| *N* | 39 | 39 | 39 |
| *R*2 | 0.963 | 0.645 | 0.964 |

Standard errors in parentheses

\* *p* < 0.05, \*\* *p* < 0.01, \*\*\* *p* < 0.001

Table 6 (b) using Private Consumption PPP

|  |  |  |  |
| --- | --- | --- | --- |
|  | (1) | (2) | (3) |
|  | ln\_cge\_ppp\_pc | ln\_cfe\_ppp\_pc | ln\_ce\_ppp\_pc |
| ln\_yl\_ppp\_pc | 1.342\*\*\* | 0.923\*\*\* | 1.168\*\*\* |
|  | (0.0969) | (0.224) | (0.0840) |
| Europe | 0.0814 | -1.267\*\* | -0.105 |
|  | (0.187) | (0.432) | (0.162) |
| ln\_tfr | -0.430\* | -0.909\* | -0.386\* |
|  | (0.191) | (0.442) | (0.166) |
| ln\_oadr | -0.0836 | 1.115 | -0.0444 |
|  | (0.693) | (1.600) | (0.600) |
| \_cons | -2.307\*\* | 1.900 | -0.235 |
|  | (0.846) | (1.953) | (0.733) |
| *N* | 39 | 39 | 39 |
| *R*2 | 0.967 | 0.661 | 0.965 |

Standard errors in parentheses

\* *p* < 0.05, \*\* *p* < 0.01, \*\*\* *p* < 0.001

We note that:

1. Average labor income has statistically significant impact on public, private, as well as overall education expenditure. CGE and CE are labor income elastic whereas CFE is labor income inelastic—even though it is almost unitary elastic with a size of 0.967 (GDP PPP) and 0.923 (private consumption PPP).
2. We again observe that on average, European countries spend less on private education than non-European countries—about 1.27 percentage points less.
3. TFR has negative and statistically significant impact on CGE and CE but not on CFE. As TFR increases by 1%, CGE decreases by 0.448% and CE decreases by 0.369% when using GDP PPP conversion rate.
4. OADR has no statistical significance on education spending. However, it still lies in the possible value range of labor income for CFE—suggesting that it is possible that they have the same effect on CFE.
5. Again, good R-square for CGE and CE, not so much for CFE.

Table 7 (a) using GDP PPP

|  |  |  |  |
| --- | --- | --- | --- |
|  | (1) | (2) | (3) |
|  | ln\_cge\_ppp\_gdp | ln\_cfe\_ppp\_gdp | ln\_ce\_ppp\_gdp |
| ln\_yl\_ppp\_gdp | 1.324\*\*\* | 0.908\*\* | 1.150\*\*\* |
|  | (0.114) | (0.256) | (0.0917) |
| Europe | 0.0712 | -1.324\*\* | -0.126 |
|  | (0.193) | (0.434) | (0.155) |
| ln\_cdr | -0.608\* | -1.260 | -0.638\*\* |
|  | (0.286) | (0.642) | (0.230) |
| ln\_oadr | 0.296 | 2.105 | 0.503 |
|  | (0.769) | (1.725) | (0.618) |
| \_cons | -2.331\* | 1.847 | -0.0961 |
|  | (0.973) | (2.182) | (0.782) |
| *N* | 39 | 39 | 39 |
| *R*2 | 0.963 | 0.646 | 0.966 |

Standard errors in parentheses

\* *p* < 0.05, \*\* *p* < 0.01, \*\*\* *p* < 0.001

Table 7 (b) using Private Consumption PPP

|  |  |  |  |
| --- | --- | --- | --- |
|  | (1) | (2) | (3) |
|  | ln\_cge\_ppp\_pc | ln\_cfe\_ppp\_pc | ln\_ce\_ppp\_pc |
| ln\_yl\_ppp\_pc | 1.326\*\*\* | 0.864\*\*\* | 1.122\*\*\* |
|  | (0.104) | (0.237) | (0.0861) |
| Europe | 0.0584 | -1.321\*\* | -0.132 |
|  | (0.189) | (0.432) | (0.157) |
| ln\_cdr | -0.577\* | -1.336\* | -0.668\*\* |
|  | (0.279) | (0.637) | (0.231) |
| ln\_oadr | 0.285 | 2.010 | 0.434 |
|  | (0.747) | (1.703) | (0.618) |
| \_cons | -2.314\* | 2.179 | 0.130 |
|  | (0.890) | (2.030) | (0.736) |
| *N* | 39 | 39 | 39 |
| *R*2 | 0.966 | 0.662 | 0.968 |

Standard errors in parentheses

\* *p* < 0.05, \*\* *p* < 0.01, \*\*\* *p* < 0.001