Does Demography Change Wealth Inequality?

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     - Be able to explain the increasing heterogeneity within cohorts

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     - be able to explain the increasing heterogeneity within cohorts

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   - The model must:

     - be able to explain the increasing heterogeneity between cohorts → **life cycle saving behavior**

     - be able to explain the increasing heterogeneity within cohorts → **Intergenerational wealth transfers (i.e., bequests)**

Demographic model

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  Generational gap (l) $\Rightarrow$ Age difference between the parent and the child

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- **Demography**:
  Modeling the population dynamic processes realistically

Fertility rates:	$m(x)$
Mortality rates:	$\mu(x)$
Survival prob.:	$S(x) = \exp\left\{-\int_0^x \mu(a)da\right\}$
Dyn. cohort size:	$\begin{cases} N(0,l) = m(l)\int_0^\omega N(l,\ell)d\ell & \text{(births)} \\ \frac{\partial N(x,l)}{\partial x} = -\mu(x)N(x,l) & \text{(deaths)} \end{cases}$

Demographic relationships:

- **Household saving behavior** $\rightarrow$ Linking parents with children
  - Surviving children/heirs $$n(x) = \int_0^x S(x-l)m(l)dl,$$
  - Household size (consumers) $$h(x) = 1 + \int_{x-A}^x S(x-l)m(l)\delta(x-l)\frac{S(l)}{S(x)}dl,$$

where A is the age at leaving the household and $\delta(x)$ is the adult EAC units at age x

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- **Transmission of wealth** $\rightarrow$ heirs at age $x \sim \text{Pois}\,(\lambda = n(x))$
  - Prob. of no children $$\theta(x) = \exp\{-n(x)\},$$
  - Fraction of wealth $$\eta(x) = \frac{1-\theta(x)}{n(x)},$$ figure

Life Cycle Savings/Wealth inequality

- Accumulation of wealth over the life cycle

$$\frac{\partial k(x,l)}{\partial x} = \begin{cases} [r + \theta(x)\mu(x)]k(x,l) + B(x,l) & \text{for } x < A, \\ [r + \theta(x)\mu(x)]k(x,l) + B(x,l) + y(x) - c(x,l) & \text{for } A \leq x < \omega. \end{cases} \tag{1}$$

- Boundary conditions

$$k(0,l) = 0 \text{ and } k(\omega,l) = 0, \tag{2}$$

where

r	interest rate
A	first age at making decisions
ω	maximum longevity
$y(x)$	labor income (taken from the NTA database)
$c(x,l)$	household consumption

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- Expected bequest received

$$B(x,l) = \underbrace{\mu(x+l)\frac{S(x+l)}{S(l)}}_{\text{Prob. of dying}} \underbrace{k(x+l)\eta(x+l)}_{\text{Capital received}}, \quad (3)$$

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- Expected bequest received **(within cohort heterogeneity)** Example

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Wealth inequality (within cohort): Example

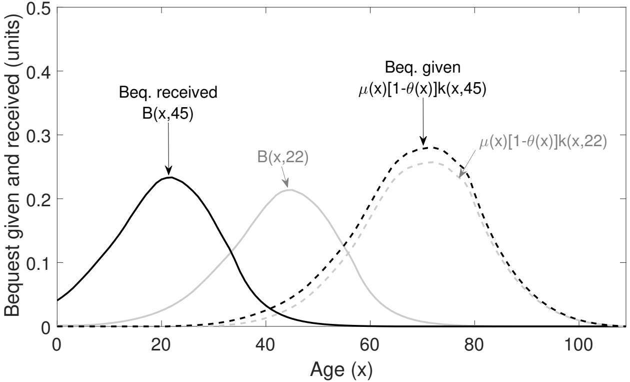

Figure 1: Per capita bequest given (dashed) and received (solid) by generational gap

Notes: Units relative to the average labor income ages 30 to 49. Both bequest profiles are derived using an annual interest rate of 3 percent, and fertility and mortality rates with an average TFR of 2.5 and a life expectancy of 65 years.

Optimal decisions: Preferences

- Assuming no subjective discounting, the expected utility of a household head born in year τ, whose parent is I years older (*generational gap*), is

$$EU(c) = \int_A^\omega \frac{S(x,\tau)}{S(A,\tau)} \left\{ U\left(\frac{c(x,\tau,I)}{h(x,\tau)}\right) + \alpha\mu(x,\tau)U\left(\eta(x,\tau)k(x,\tau,I)\right) \right\} dx. \tag{4}$$

where

$U(.)$	Isoelastic functions U (that satisfy the Inada conditions: $U' > 0$, $U'' < 0$, with U being continuously differentiable, $U'(0) = \infty$, and $U'(\infty) = 0$)
$\alpha \geq 0$	Degree of altruism towards children
$\eta(x,\tau)k(x,\tau,I)$	Amount of wealth bequeathed to each offspring
$\frac{S(x,\tau)}{S(A,\tau)}\mu(x,\tau)$	The expected age at which the bequest is given

Labor income profile (NTA)

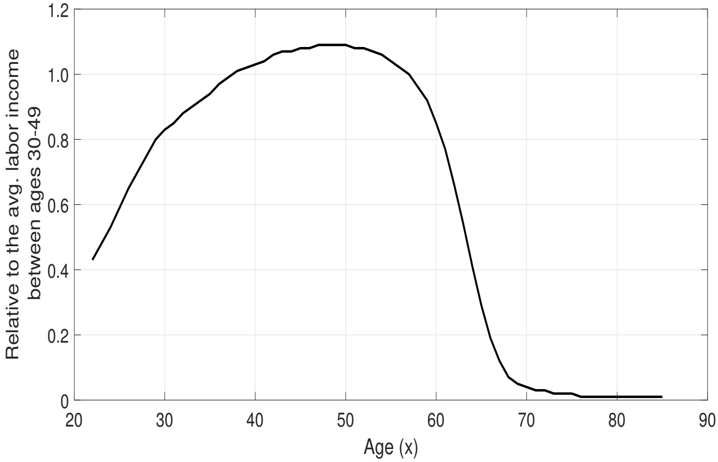

Figure 2: Labor income per capita in USA, 2003

Source: www.ntaccounts.org.

Age wealth profiles

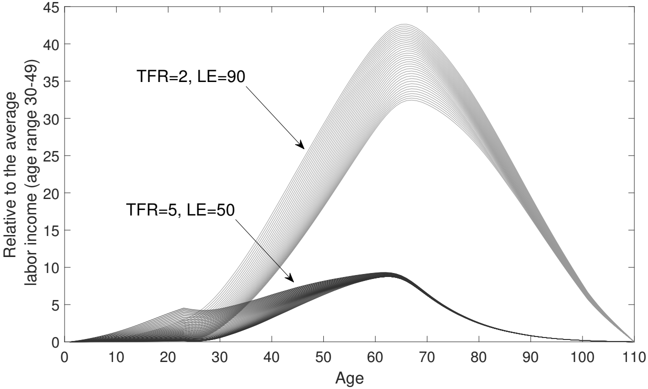

Figure 3: Wealth profiles for two different birth cohorts back

Impact of demography on wealth inequality

- **Demographic simulations**:

  Impact of alternative **life expectancies** (LE) and **total fertility rates** (TFR)

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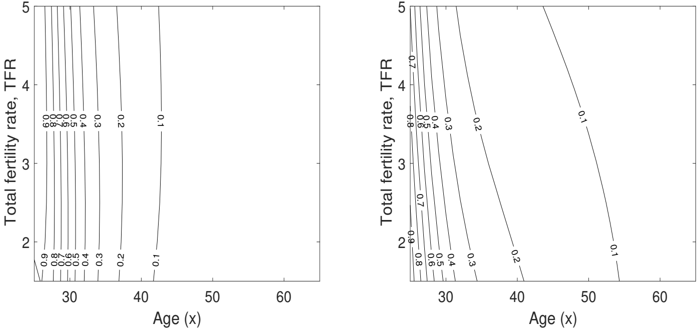
  Impact of alternative **life expectancies** (LE) and **total fertility rates** (TFR)

- **Measuring wealth inequality**

  - **within birth cohorts**: $c_C[\boldsymbol{k}(x)] = \frac{\sqrt{\mathrm{V}_C[\boldsymbol{k}(x)]}}{\mathrm{E}_C[\boldsymbol{k}(x)]}$
  - **whole population**: $c_N[\boldsymbol{k}] = \frac{\sqrt{\mathrm{V}_N[\boldsymbol{k}]}}{\mathrm{E}_N[\boldsymbol{k}]}$

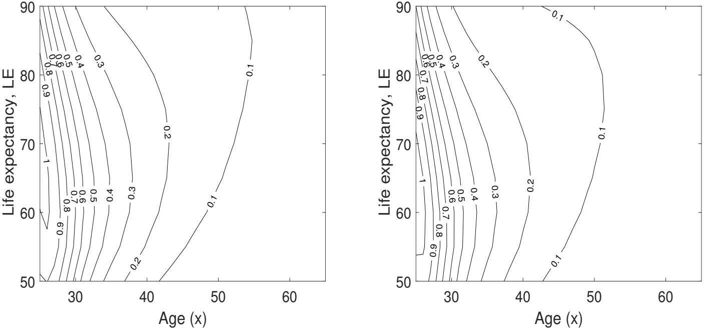


(a) Fixed LE=50 (b) Fixed LE=80

Figure 4: Impact of changes in life expectancy (LE) and fertility (TFR) on financial wealth inequality at selected ages

- $\uparrow$ age $\Rightarrow$ $\downarrow$ inequality & $\downarrow$ TFR $\Rightarrow$ $\uparrow$ inequality

Wealth inequality within cohorts



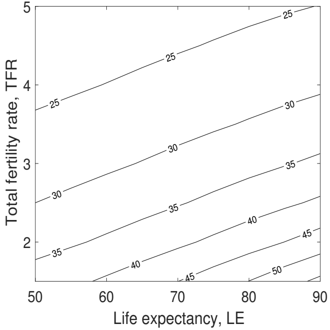
(a) Fixed TFR=1.5

(b) Fixed TFR=3.0

Figure 4: Impact of changes in life expectancy (LE) and fertility (TFR) on financial wealth inequality at selected ages

- ↑ age ⇒ ↓ inequality & ↓ LE ⇒ ↑ ↓ inequality

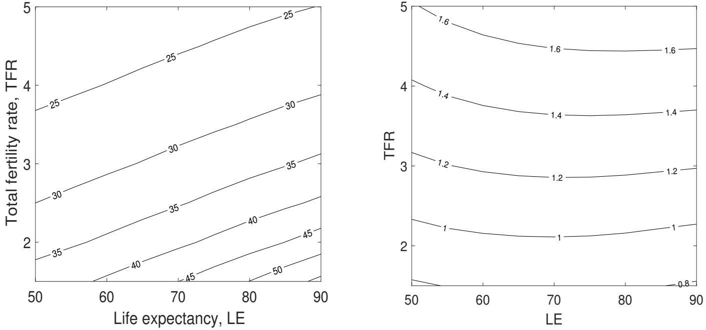
Population wealth inequality



(a) Mean-age of the population

Figure 5: Impact of changes in life expectancy (LE) and fertility (TFR) on financial wealth inequality

Population wealth inequality



(a) Mean-age of the population

(b) Financial wealth inequality

Figure 5: Impact of changes in life expectancy (LE) and fertility (TFR) on financial wealth inequality

Conclusions

- A decline in fertility raises wealth inequality within cohorts but it reduces inequality at the population level (across cohorts)

- Increases in life expectancy result in a non-monotonic effect on wealth inequality by age and across cohorts

Thank you!

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Optimal decisions: Analytical solution

- The consumption path c that maximizes the expected utility (4) subject to the constraint (1) is the one that solves the Hamiltonian

$$\mathcal{H}(k, c, \lambda, x) = \tilde{S}U(c/h) + \alpha\mu\tilde{S}U(\eta k) + \lambda([r + \theta\mu]k + B + y - c), \tag{5}$$

where
λ is the adjoint variable related to k,
$\tilde{S}$ denotes the probability of survival conditional on being alive at age A.

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$$\mathcal{H}_c = \tilde{S}[h]^{-1}U'(c/h) - \lambda \stackrel{!}{=} 0. \tag{6}$$

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- Assuming $U(c) = \log(c)$ the dynamics of the adjoint variable and wealth are given by

$$\frac{\partial\lambda}{\partial x} = -[r + \theta\mu]\lambda - \alpha\mu\tilde{S}/k, \tag{7}$$

$$\frac{\partial k}{\partial x} = [r + \theta\mu]k + B + y - \tilde{S}/\lambda, \tag{8}$$

and the boundary conditions $k(0, \tau, l) = 0$ and $k(\omega, \tau, l) = 0$. figure

Household problem: Value Function

Each household head, whose father is l years older (*generational gap*), maximizes

$$\max_{c,k} \int_A^\omega \frac{S(x)}{S(A)} \left\{ U\left(\frac{c(x,l)}{h(x)}\right) + \alpha\mu(x)U\left(\eta(x)k(x,l)\right)\right\} dx. \quad (9)$$

where

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ω	maximum longevity
$c(x,l)$	household consumption
$k(x,l)$	financial wealth

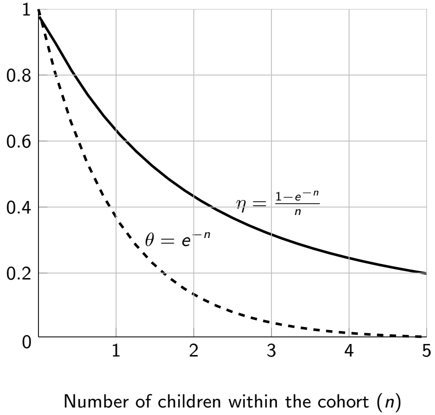

Figure 6: Fraction of annuitized wealth (θ) and fraction of wealth received according to the number of children within the cohort (η) back

- **Lifetime budget constraint**
  An individual whose parent is l years older is

$$\int_A^\omega e^{-rx} S(x) c(x,l) dx = \int_A^\omega e^{-rx} S(x) y(x) dx + T_B(0,l), \tag{10}$$

where $T_B(0,l)$ is the *bequest wealth* at birth

$$T_B(0,l) = \underbrace{\int_0^\omega e^{-rx} S(x) B(x,l) dx}_{\text{Bequest received}} - \underbrace{\int_0^\omega e^{-rx} S(x) \mu(x) [1-\theta(x)] k(x,l) dx}_{\text{Bequest given}}. \tag{11}$$

back

Wealth inequality (within cohort)

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- **Economic model**:
  Small-open economy, Yaari(1965)'s model with bequest motive

back

Family profiles

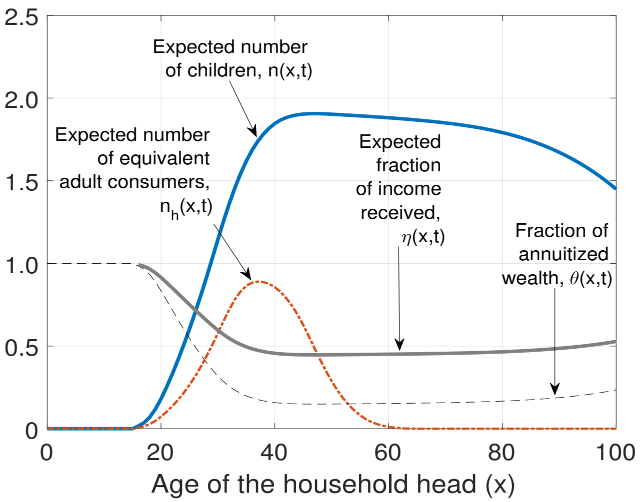

Figure 7: Family profiles

Inherited wealth profiles (cross-section)