Why Nations Become Wealthy:
The Effects of Adult Longevity on Saving

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Abstract

We analyze steady state and out-of-steady-state effects of the transition in adult longevity on the national saving rate using historical data and international panel data. The rise in adult life expectancy has a large and statistically significant effect on aggregate saving. The effects have been especially pronounced in East Asia because its mortality transition was very rapid. Gains in life expectancy are much more important than declines in child dependency. Population aging may not lead to lower saving rates in the future if life expectancy and the duration of retirement continue to increase.

Keywords: saving, investment, mortality, health, models, East Asia

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1. INTRODUCTION

One of the most salient features of modern economic development is the increase in wealth and capital. In the US, for example, the gross non-residential capital stock grew at 4.1 percent per annum as compared with annual GDP growth of 3.6 percent between 1820 and 1992. The experience of the UK was quite similar, while in Japan over a shorter period, 1890 to 1992, the annual rate of growth of capital exceeded the annual rate of growth of GDP by 1.4%. The ratio of the gross non-residential capital stock to GDP increased more than four-fold, from 0.71 to 3.02. (Maddison 1995). The extraordinary growth in wealth and capital was repeated, in a more condensed form, in the Newly Industrializing Economies. High rates of saving and investment in South Korea, Taiwan, Singapore, and several other Asian countries have led to rapid capital deepening.

Why did this occur? The demand side undoubtedly played an important role. Technological innovation led to new and better equipment and machinery. Structural change led to industrialization and the growth of the manufacturing and service sectors with accompanying investment. The possibility explored in this study, however, is that the modern rise in wealth was driven in large part by an important supply side factor – the increase in adult life expectancy.

The proportion of adult life lived after age 60 began to increase steadily in the West in the late 19th Century. In other parts of the world increases began much later – in the middle of the 20th Century. Retirement emerged as a significant feature of the lifecycle creating a powerful saving incentive. Institutional responses, the emergence of funded employment-based pension plans, the rise of the commercial financial service sector, the creation of tax incentives, and, in some countries, the establishment of funded public retirement systems reinforced the effects of increased longevity. Other responses, particularly the creation of transfer-based public pension programs in Latin America, Europe, and to a lesser extent in Japan and the US, undoubtedly undermined the incentive effects of a longer life span.

That rising life expectancy leads to higher saving is not a new idea. Yaari's (1965) seminal work established the micro-level theoretical foundation. Since then other scholars have explored the aggregate effects using steady-state models and simulation analysis (Lee, Mason et al. 2003). Previous empirical work supports the existence of an important link between saving and life expectancy (Yaari 1965; Davies 1981; Zilcha and Friedman 1985; Kuehlwein 1993; Leung 1994; Borsch-Supan 1996; Schieber and Shoven 1996; Bloom, Canning et al. 2003; Kageyama 2003).

The theoretical analysis described briefly in Section 2 employs an overlapping generations model which extends previous work to a dynamic context. A unique implication of the model is that the aggregate saving rate is influenced by both the level and the rate of change of adult life expectancy. Given the current level of mortality, countries experiencing rapid mortality transitions will have higher saving rates. The underlying logic behind this result is straight-forward. If a rapid mortality transition country is playing catch-up, the wealth required to support a longer retirement must be accumulated over a shorter period of time. Thus, saving rates must be elevated during the catch-up period.

The empirical analysis relies on two different approaches. Section 3 takes an historical perspective by looking at data for seven countries for which we can track saving and mortality trends over all or a substantial part of the entire demographic transition. In the sub-group of Western countries, adult life expectancy changed very slowly or not at all until the middle or end of the 19th Century. Thereafter, life expectancy changed at a pace that was remarkably constant and varied little from one country to the next. The sub-group of Asian countries began their mortality transitions later, went through a catch-up period when adult longevity increased rapidly, followed by a period of steady increase at a rate similar to that found in the West.

The difference in the demographic transitions between the West and East Asia offers a useful opportunity to compare the implications of our theoretical model with the experiences of these two groups of countries. Some of the idiosyncratic features of the saving trends are not explained by our model, but there is broad consistency. The increase in adult mortality was accompanied by a rise in aggregate saving rates in most countries. Rapid mortality transition was clearly accompanied by elevated saving rates.

In Section 4, we estimate the saving model using aggregate cross-national data. The evidence is consistent and robust in its support of the hypothesis that an increase in old-age survival leads to higher saving rates. In a sub-sample consisting of Western and East Asian countries, the rate of increase in old-age survival also has a positive effect on saving. In other parts of the developing world, however, we do not find evidence that the rate of change in old-age survival has an effect on saving. Why different patterns persist is an issue requiring further exploration and some possibilities are discussed below.

Two additional features of the analysis are important. First, several recent studies conclude that changes in age structure, especially the decline in the youth dependency ratio, accounted for high saving rates especially in East Asia (Kelley and Schmidt 1996; Higgins and Williamson 1997). Simulation studies (Lee, Mason et al. 2000; 2001a; 2001b) and empirical work based on household survey data (Deaton and Paxson, 2000) ascribe a substantially less important role for age structure. The empirical analysis presented here offers some reconciliation of these views. Once we control for adult life expectancy and its rate of change, the youth dependency ratio has a smaller effect than previously estimated. The decline in youth dependency accounts for about one-quarter of the increase in saving rates in East Asia, while changes in adult survival account for about three-quarters of the increase.

Second, while some studies conclude that population aging will lead to substantial declines in aggregate saving rates, we do not find this to be the case. So long as adult survival continues to rise–as it has for many decades – population aging will not drive saving rates lower.

2. THE THEORY[i]

Changes in adult survival influence aggregate saving in two ways in the lifecycle model employed here. First, there is a behavioral effect. The expected duration of retirement rises as the survival rate increases. Thus, individuals will consume less and save more during their working years in order to support more expected years of consumption and greater dis-saving during retirement. Second, there is a compositional effect as increases in the adult survival rate lead to an increase in the share of retirees in the adult population. Given that retirees are saving at a lower rate than workers, the compositional effect of an increase in adult survival is to reduce aggregate saving.

The net effect on saving is considered in steady-state and in dynamic settings. We summarize the dynamics in two ways: first, by considering the effect of a one-time increase in survival and, second, by considering the effect of continuing increases in survival. Considering these alternatives brings a clearer understanding of how the mortality transition – as it is actually evolving – will influence aggregate saving rates.

The behavioral and compositional effects of adult mortality are analyzed using a two-period overlapping generations model. The advantage of this approach is its relative simplicity. A disadvantage is its neglect of another important demographic change – the changes in youth dependency driven by trends in fertility and child mortality. Previous theoretical work has already explored the effects of the number of children on saving in a continuous-age, steady-state framework (Mason 1987) and in a dynamic OLG framework (Higgins 1994). The empirical analysis presented below considers the effects of youth dependency relying on the Higgins OLG model. This allows a simpler and more focused theoretical analysis here on changes in adult mortality.

Consider a population consisting of two generations of adults. Each person lives for up to two periods – the first period as a working, prime-age adult and the second period as a retiree. All individuals survive their working period, and q_t survive to the end of their retirement period. The remaining members of the population (1-q_t) die at the end of the first period of life. In the OLG framework q_t is the probability of reaching retirement age, the expected years lived during retirement, and the ratio of retirement years to working years for the average member of the population.[ii] Individuals cannot foresee whether they will survive, but they know the value of q_t for the population. Costless annuities are available so that individuals protect themselves against longevity risk by purchasing an annuity. Individuals know the interest rate that the annuity will pay.

The consumer's optimization problem is to maximize lifetime utility, assuming constant relative risk aversion, $V_t = \frac{c_{1,t}^{\;1-q}}{1-q} + d\, q_t \frac{c_{2,t+1}^{\;1-q}}{1-q}$, given the lifetime budget constraint: $w_t A_t = c_{1,t} + \frac{q_t}{1+r_{t+1}} c_{2,t+1}$; $c_{1,t}$ is consumption while a prime-age adult and $c_{2,t+1}$ is consumption while elderly; d is the discount factor, defined as $d=1/(1+ ?)$, where ? is the discount rate; $1/?$ is the intertemporal elasticity of substitution; r_{t+1} is the interest rate; A_t is labor-augmenting technology; and w_t is the wage per unit of effective labor.

Kinugasa (2004) shows that the per capita savings of prime-age adults and retirees are:

$$
\begin{aligned}
s_{1,t} &= \Psi_t A_t w_t = \frac{q_t d^{\frac{1}{q}} A_t w_t}{q_t d^{\frac{1}{q}} + (1+r_{t+1})^{\frac{q-1}{q}}} \\
s_{2,t} &= -s_{1,t-1} = -\Psi_{t-1} A_{t-1} w_{t-1} = \frac{q_{t-1} d^{\frac{1}{q}} A_{t-1} w_{t-1}}{q_{t-1} d^{\frac{1}{q}} + (1+r_t)^{\frac{q-1}{q}}}
\end{aligned}
\tag{1}
$$

where $?_t$ is the share in wage income of saving by prime-age adults. An increase in the adult survival rate has an unambiguous positive effect on $?_t$ and, hence, on per capita saving by prime age adults. An increase in the adult survival rate has an unambiguous negative effect on per capita saving by retirees. The response of the combined saving of workers and retirees to changes in survival depends on additional features of the macro-economy to which we now turn.

Gross domestic product (Y_t) is produced by a Cobb-Douglas production function with labor-augmenting technological growth, i.e., $Y_t = K_t^{\,f} L_t^{\,1-f}$, where f is the share of capital in GDP, $0 < f < 1$. $L_t = A_t N_{1,t}$ is the aggregate labor supply measured in efficiency units. $N_{1,t}$ is the population of prime-age adults and A_t is the technology index. The growth rate of the population of prime-age adults from time t-1 to time t is n_t-1 and, hence, $N_{1,t} = n_t N_{1,t-1}$. The technological growth rate from time t-1 to time t is g_t-1. Hence, the relationship between the total lifetime labor income of prime-age adults and pensioners is given by $w_{t-1} A_{t-1} N_{1,t-1} = w_t A_t N_{1,t} / g_t n_t$. Using lower case letters to represent quantities per unit of effective worker, output per effective worker is $y_t = k_t^f$ and the capital-output ratio is k_t^{f-1}. The depreciation rate (x) is assumed to be constant; hence, depreciation as a share of GDP is $x k_t^{f-1}$.

Total gross national saving is the sum of the saving of adults ($S_{1,t}$), the saving of the elderly ($S_{2,t}$) and depreciation (?K_t). Dividing by Y_t yields the gross national saving rate at time t:

$$\frac{S_t}{Y_t} = (1-f)\left(\Psi(q_t, k_t) - \Psi(q_{t-1}, k_{t-1}) \frac{1}{g_t n_t} \right) + x\, k_t^{1-f} \qquad (2)$$

The term $(1-f)$ is the share of labor income in GNP, $\Psi_t(q_t, k_t)$ is saving by current workers as a share of current labor income, $-\Psi(q_{t-1}, k_{t-1})\frac{1}{g_t n_t}$ is saving by current retirees as a share of current labor income, and $x\, k_t^{1-f}$ is depreciation as a share of current GNP.

(a) Steady-state rate of growth effects

The steady-state gross national saving rate is:

$$\left(\frac{S}{Y}\right)^* = (1-f)\Psi(q^*, k^*)\left(\frac{gn-1}{gn}\right) + x\, k^{*1-f} \qquad (3)$$

where the * superscript denotes equilibrium values. Standard and well-known implications of the lifecycle model (Modigliani and Brumberg 1954) follow directly from equation (3). First, the net saving rate is zero if the economy is not growing (*gn=1*). If the economy is not growing, the lifetime earnings of retirees and workers are equal and, hence, the dis-saving by retirees will exactly balance the saving by workers. Second, the partial effect of an increase in the GDP growth rate is equal to the mean age of earning less the mean age of consumption, as in the variable rate-of-growth model (Mason (1981, 1988) and Fry and Mason (1982)). As shown in Kinugasa, ? is the difference between the mean ages of consumption and earning.[iii]

An important issue to clarify at this point is that the effect of the rate of growth of the population of children, not included in the theoretical model, will be very different than the rate of growth of the population of prime age adults. If the child population is growing rapidly, workers will be supporting many children and, hence, saving will be depressed. The effects of child dependency are adequately addressed in the current literature (Mason 1988; Higgins 1994). The effects of child dependency are discussed more thoroughly and estimated below.

(b) The effect of adult survival in steady-state and in transition

The effect of changes in adult survival on saving depends on whether or not the capital-labor ratio and interest rates are endogenous. In a small open economy, the equilibrium capital-labor ratio and interest rates are determined by global economic conditions. A rise in domestic saving – and factors that influence the saving rate – will have no effect on domestic investment nor on domestic interest rates. In a closed economy, however, saving and investment are equal and the rate of interest is endogenously determined by the interplay of the supply of capital by households and the demand for capital by firms. The effect on saving of an increase in adult survival in each of these environments is considered in turn.

(i) Saving in a small open economy

The effect of an increase in adult survival on the steady state saving rate in a small open economy depends on the rate of growth of income. If the economy is growing, $gn>1$, the steady state saving rate rises with adult survival. In an economy with negative economic growth, the steady state saving rate declines with an increase in adult survival. The steady state saving rate given by equation (3) holds with k^* exogenously determined. An increase in adult survival leads to a rise in the share of labor income saved by prime age adults, i.e., $\partial\Psi^* / \partial q^* > 0$, but the dis-saving by retirees increases, as well. In a growing economy, the increase in saving by prime age adults dominates the decline in saving by retirees and the aggregate saving rate rises with adult survival. In a declining economy, the decline in saving by retirees dominates and the aggregate saving rate declines as adult survival rises.

The response of saving rates in a dynamic context is more complex. The current saving rate is increasing in the rate of GDP growth during the previous period, g_t, because in an economy with rapid aggregate economic growth during the preceding period the size of current workers, measured in terms of total lifetime earnings, will be large relative to current retirees. The effect of the rate of economic growth will depend on adult survival or, to be more precise, on the expected duration of retirement of those who are currently working.

In a dynamic context, however, adult survival is also changing. Consider a small open economy that is in equilibrium, but experiences an increase in the survival rate in period *t*. Prime age adults respond by increasing their saving rates, while saving by retirees is unaffected. The aggregate saving rate must rise in period *t*. The transitory increase in saving is independent of the rate of economic growth *gn*. In period *t+1,* dis-saving by retirees rise and the aggregate saving rate declines. In the absence of further changes in survival, a new equilibrium saving rate is established in period *t+1*. As shown above, the new equilibrium depends on the rate of economic growth. In a growing economy, it will be higher than the saving rate in period *t-1,* but lower than the saving rate in period *t*.

(ii) Saving in a closed economy

In a closed economy, an increase in the saving rate leads to greater investment, capital deepening, more rapid growth in wages, and a decline in the interest rate. The supply of capital follows directly from the saving model presented above, because the capital stock in period *t+1* is equal to total saving by prime-age adults in period *t*. Expressed as capital per effective worker, the supply of capital in year *t+1* depends on the wage per effective worker in year *t,* the share of that wage that is saved by prime-age adults, and the rate of growth:

$$k_{t+1} = \frac{y\,(r_{t+1}, q_t)\, w_t(k_t)}{g_{t+1} n_{t+1}} = S_t\,(\underset{?}{r_{t+1}}, \underset{+}{w_t}(k_t), \underset{+}{q_t})\,, \tag{4}$$

where S_t is the supply function of capital. The effect of the interest rate on the supply of capital is ambiguous.[iv] The wage is equal to the marginal product of an effective worker, $w_t = f(k_t) - k_t f'(k_t)$ and is increasing in capital per effective worker. An increase in the survival rate leads to an increase in the share of labor income saved by prime-age adults and, hence, capital per effective worker and the supply of capital.

The demand for capital, **D**, is governed by the marginal condition that the cost of capital equals the net return, i.e., $r_{t+1} = f'(k_{t+1}) - x$. That *f''<0* implies that the demand curve is downward sloping. The demand for capital is independent of the survival rate.

The effect of an increase in the survival rate from q^* to q' in period 1 is traced in Figure 1. The demand curve, **DD**, is unaffected. Workers in period 1 increase their saving because in period 2 they expect to live longer and, perhaps, because they expect lower interest rates to depress the rate of return on annuities. Thus, the supply curve, **SS**, shifts to the right in period 2. This leads to a rise in capital per worker and wages for the new generation of workers. As a result, the saving function shifts further to the right, in part, because of the expected further decline in interest rates. The process continues until a new equilibrium is established.[v] Unless the decline in interest rates leads to a substantial reduction in saving by prime-age adults – a possibility not born out by empirical research – an increase in adult survival in period t leads to capital deepening.

[Figure 1 about here.]

The effect of survival on aggregate saving is readily inferred from its effect on capital per effective worker. In a closed economy, saving is equal to investment. Gross national saving is the sum of changes in asset holdings of prime age adults and the elderly and depreciation, so that $S_t=S_{1,t}+S_{2,t}+?K_t= S_{1,t}-S_{1,t-1}+?K_t$. Gross investment is given by $I_t=K_{t+1}-(1-?)K_t$. The national saving rate is:

$$\frac{S_t}{Y_t}=\left(g_{t+1}n_{t+1}\frac{k_{t+1}}{k_t}-1+x\right)k_t^{1-f}. \tag{5}$$

In steady state, the relationship simplifies to:

$$\left(\frac{S}{Y}\right)^*=(gn-1+x)k^{*1-f} \tag{6}$$

From inspection of equation (6) an increase in the equilibrium capital-labor ratio and, hence, the capital-output ratio (k^{1-f}), leads to an increase in the equilibrium net saving rate ($(gn-1)k^{*1-f}$) in a growing economy. The gross saving rate increases if the depreciation rate plus the rate of growth is positive.

During transition, as is clear from equation (5), the saving rate is elevated above the equilibrium level depending on the rate of capital deepening (k_{t+1}/k_t). As with the open economy case, a one-shot increase in the survival rate leads to a large increase in the saving rate for one generation followed by a decline in the saving rate to a steady-state level as a new equilibrium is established. In an economy with positive labor-augmenting growth, the net saving rate will be higher than in the initial equilibrium, but lower than during the transition period.

(c) Simulation results

Figure 2 compares the simulated saving rates in a small open economy and a closed economy produced by an increase in adult survival from 0.4 in year *t-1* to 0.5 in year *t*.[vi] The initial impact is large in both cases. The response is somewhat muted in the closed economy because the declines in interest rates lead to reduced saving rates among prime-age adults. A new equilibrium is established in period *t+1* in the open economy, but the adjustment is more gradual in the closed economy as described above. The equilibrium saving rate in the closed economy is greater than in the open economy, because of capital deepening in the closed economy. Higher net saving is required to sustain a higher capital-output ratio. Greater depreciation leads to an increase in gross saving beyond the increase in net saving.

[Figure 2 about here]

One would not be likely to observe the simulated saving paths shown in Figure 2. Adult survival rates trend upward at a relatively constant rate in many countries – as we will show below. Figure 3 presents simulated saving rates assuming that adult survival increases by 0.1 per period starting from 0.4 in year *t-1*. Otherwise, parameters are identical to those employed in the simulations presented in Figure 2. The onset of adult mortality decline leads to a secular rise in saving rates that continues as long as adult survival rates continue to increase. Saving rates appear to be very nearly linear in adult survival after period *t-1* in the open economy case and after period *t* in the closed economy case.

[Figure 3 about here]

3. HISTORICAL PERSPECTIVES ON OLD-AGE SURVIVAL AND SAVING

The modern mortality transition began in the West. Early gains were concentrated at young ages, but by 1900 old-age survival rates were rising steadily in Sweden, the United Kingdom, Italy, and the US – the four Western countries we examine below. The mortality transition began much later outside of the West. The three Asian populations for which we have relatively complete historical data – Japan, Taiwan, and India – did not experience significant gains in old-age survival until the middle of the 20th Century. When the mortality transition began, however, it was very rapid as these Asian countries caught or, in the case of Japan, surpassed the West.

Historical mortality transitions are of great interest here because of their implications for long-run trends in national saving rates. Previous empirical research – and the analysis presented in Section 4 – relies on data that cover a relatively small portion of the mortality transition. This is unfortunate given the long-term nature of the processes under consideration. The distinctive experiences of Asia and the West, however, provide an opportunity to assess long-term effects of mortality change on saving.

We begin with an examination of the mortality transitions of seven countries. The most widely available measures, life expectancy at birth and the crude death rate, are inappropriate because, early in the mortality transition, they are driven by changes in infant and child mortality. Whether the resultant increase in child dependency observed in many countries affected the accumulation of wealth is an issue considered below. The emphasis at this point, however, is on adult mortality.

The measure of mortality used here and in the world panel analysis is the expected years lived during old-age relative to the expected years lived during the working years. This measures in very direct fashion how mortality decline influences the expected duration of retirement relative to the expected number of working years. It directly measures the influence of mortality on population age structure. It also reflects mortality change that occurs at any adult age. Ages 30 and 60 are used to delineate the working ages (30-59) and the old ages (60+). This choice may be puzzling to some, but it is based on recent empirical estimates of the economic lifecycle in a small group of developing and industrialized countries that find that individuals do not begin to produce as much as they consume until their late twenties and that by their late fifties or early sixties they are consuming more than they are producing (Lee, Lee et al. 2005). Thus, individuals are only beginning to save toward retirement in their thirties and later and are beginning to rely on accumulated wealth beginning at about age 60.[vii]

In six of the countries – all but India – we are able to construct an old-age survival index (q), also used in Section 4.[viii] The old-age survival index is the expected years lived after age 60 per expected year lived between the ages of 30 and 60 given the age specific death rates during the year of observation. The value of q ranges from less than 0.2 expected years lived after age 60 per expected year lived between the ages of 30 and 60 in Taiwan *circa* 1900 to close to 0.8 in current day Japan.

Historical data for Sweden allow us to trace the transition in old-age survival from the mid-18th Century. The 250 years of data can be described remarkably well as consisting of a pre-transition period during which old-age survival was virtually stagnant and a transition period during which old-age survival increased steadily. In 1751 adults could expect to live about one-third of a year after age 60 for every year lived between the ages of 30 and 60. Between 1761 and 1876 the old-age survival index increased at an annual rate of only 0.0006. Between 1876 and 2001 the old-age survival index increased four times as rapidly - at an annual rate of 0.0026. Allowing for three short-run mortality crises – famine in 1772-73, the Finnish War in 1808-09, and the Spanish flu epidemic of 1918 – a piece-wise linear regression with one break-point at 1876 explains 93 percent of the variance in adult mortality (Table 1).[ix]

[Table 1 about here]

The old-age survival transitions of the three other Western countries can be characterized in equally simple fashion. For the United Kingdom, old-age survival increased at an annual rate of 0.0009 between 1841 and

1900 and at a rate of 0.0024 between 1900 and 1998, with 96 percent of the variance explained by the piece-wise linear model with a single break point. The available data for Italy and the United States do not extend into the pre-transition period. Old-age survival in Italy from 1872 to 2000 and in the United States from 1900 to 2001 can be explained as consisting of a single transition period with old-age survival increasing by 0.0029 years per year in Italy and by 0.0033 years per year in the United States.

That the gains in these four countries have been remarkably constant during the 20th Century has important – and unfortunate – implications for testing the dynamic saving model. In the absence of time series variation in the rate at which old-age survival is increasing, estimates of the effect of the rate of change in old-age survival will depend entirely on cross-country differences. Even though there are small year-to-year fluctuations and instances of more significant fluctuations, e.g., the flu epidemic of 1918, short-term fluctuations may have little or no effect on long-term expectations. In our model, it is long-term expectations that matter. To add to the difficulties, the cross-national differences among the four Western countries are quite modest. The historical experience of the West is useful, however, to the extent that we explore the effect on saving of the shift from pre-transition to transition. We return to this issue below.

The mortality experience in East Asia is quite distinctive judging from the relatively complete data for Japan and Taiwan. Their pre-transition periods lasted until much later, but were followed by a significant catch-up period during which old-age survival increased quite rapidly. Having closed the mortality gap with the West, the gains in adult mortality have slowed. Recent mortality gains in Taiwan are similar to those found in the West while Japan continues to experience larger gains in old-age survival (Table 1).

Indian life expectancy at age 30 also increased very gradually until 1951. For the next forty years substantial gains were achieved. The 1990s saw a marked slow-down. The pattern is similar to that found in Japan and Taiwan although direct comparison is not possible.

To what extent are the saving trends in these seven countries consistent with the predictions of our saving model? Three implications of the model can be examined. First, prior to the transition in old-age survival, saving rates would have been relatively low. Second, constant increases in old-age survival during the transition would have produced relatively constant increases in saving rates. Third, East Asian countries would have experienced relatively elevated saving rates during their period of rapid transition. Of course, these patterns would consistently emerge only if the trends in old-age survival dominated a myriad of other potentially important factors.

First, were pre-transition saving rates low? Estimates for five pre-transition populations are available and presented in Table 2. In all cases, the pre-transition saving rates are low as compared with the saving rates that followed, but the UK saving rate during pre-transition is only slightly less than its transition saving rate and Taiwan's pre-transition saving rate is quite high as compared with the other countries.

[Table 2about here]

Second, are saving and old-age survival correlated? The observed saving rates are plotted against the observed survival values in Figure 4 for each of the countries.[x] The US stands out as an exception with a negative simple correlation between saving rates and old-age survival. In the United Kingdom, the positive correlation is modest (0.37). In the other five countries, the correlation between the two variables ranges from 0.64 upward.

[Figure 4 about here]

With the exception of Sweden, the correlation between q and saving disappears or turns negative at higher survival rates. The theoretical model offers two possible explanations for this phenomenon. One is that a slowdown in the rate of change in mortality, as occurred in Japan and in Taiwan, would cause saving rates to drop below the regression lines. A second is that a decline in the rate of economic growth would have a similar effect. Institutional change – especially the adoption of pay-as-you-go pension programs – looms large as a potential explanation.

The third question is whether rapid changes in survival rates (observed mostly in the countries of East Asia) were associated with higher saving rates. As a simple analytic device we compare the countries at three benchmark old-age survival rates – 0.4, 0.5, and 0.6 – observed for the six countries for which we have values (Figure 5). There is a clear positive association between the national saving rate and the change in the old-age survival rates over the subsequent decade. The simple correlation between the variables ranges from 0.58 to 0.72. The effects are essentially identical for q equal to 0.4 and 0.5 and somewhat attenuated for 0.6. These results provide modest but consistent support for the dynamic model of adult survival and saving.

[Figure 5 about here]

4. ANALYSIS OF WORLD PANEL DATA 1965-1999

Estimates of national saving rates, life expectancy at birth, and other variables that may influence saving are available for 76 countries in 1965-69 increasing to 94 countries in 1995-99.[xi] In this section we present analysis of national saving rates employing these data. Using the world panel data offers advantages over the historical analysis. First, we can move to a multivariate framework that explicitly incorporates the role of other factors in determining national saving rates. Second, we can explore whether the Western/East Asian distinctions drawn in the historical analysis can be generalized to other countries of the world. There are also disadvantages that are discussed below.

(a) Model specification

The empirical model incorporates the two effects of old-age survival derived from the OLG model presented in Section 2: the steady-state effect, which interacts with the rate of economic growth, and the transitory effect, which depends on the rate at which old-age survival is increasing. A linear approximation of the OLG model implies that both the steady-state effect (b_1) and the transitory effect (b_2) are positive in equation (7):

$$(S/Y)_t = b_0 + b_1 q_t \cdot Y_{gr,t} + b_2 \Delta q_t + b_3 D1_t \cdot Y_{gr,t} + b_4 Y_{gr,t} + b_5 PRI_t + e_t \qquad (7)$$

where *S/Y* is the national saving rate, q is the old-age survival index, Y_{gr} is the growth rate of GDP, ? q is the change in the old-age survival index during the previous five year period, *D1* is the youth dependency ratio, and *PRI* is the relative price of investment goods, and subscript t means year t.[xii]

The basic empirical model incorporates three other saving determinants that have been explored in previous studies. The first is the effect of youth dependency. As youth dependency varies over the demographic transition, aggregate saving can be influenced in a variety of ways of which two seem particularly important. First, an increase in the number of children may have a direct effect on current household consumption because of the costs of additional children. The direction of the effect will depend, however, on whether or not a decline in the number of children is associated with a decline in total consumption by children. If the price of children relative to adults is rising and the demand for children is price inelastic, expenditures on children will increase as the number of children declines. In this instance, youth dependency would have a positive effect on saving. If the demand for children is price elastic or if the number of children is changing for reasons unrelated to changes in relative prices, youth dependency will have a negative effect on saving (Mason 1987).

Changes in youth dependency may also influence saving because children provide old-age support either through familial support systems or through public pension and health care systems. To the extent that children are seen as a substitute for pension assets, youth dependency will have a negative effect on saving. The specification of the youth dependency effect follows the variable rate-of-growth (VRG) model (Fry and Mason 1982; Mason 1981, 1987; Kelley and Schmidt 1996).

The second effect included in the basic model is the rate-of-growth effect, that is a feature of the OLG model presented in Section 2 and life-cycle saving models in general (Modigliani and Brumberg 1954). In an economy experiencing more rapid GDP growth, the lifetime earnings of young cohorts are greater relative to the lifetime earnings of older cohorts. To the extent that lifecycle saving is used to shift resources from younger to older ages, saving is concentrated among younger cohorts. Hence, the rate of growth effect is typically positive, but can in principle be negative.[xiii] The effect of GDP growth is variable, equal to $b_1 q_t + b_3 D1_t + b_4$, because the old-age survival rate and the youth dependency ratio influence the life-cycle profile of consumption. The coefficient β_4 has no economic interpretation in isolation and, hence, may be positive or negative depending on the effects of q and $D1$.

The third determinant included in the empirical model is the relative price of investment goods (*PRI*) following Taylor (1995) and Higgins and Williamson (1997). The *PRI* captures the effect of changing interest rates. An increase in the interest rate will lead to an increase or decrease in saving by prime-age adults and by the elderly depending on the relative strengths of the substitution and wealth effects. Thus, the effect of the *PRI* on the national saving rate is an empirical issue.

A potentially important extension of the model, considered below, allows for a transitory youth dependency effects following Higgins (1994) and Williamson and Higgins (2001). A drop in youth dependency would induce prime age adults to save more during their working years in anticipation that they would rely less on familial or public transfers during their retirement years. Current retirees would be unaffected as they are simply dis-saving the (small) assets they accumulated in the expectation that they would be supported by the many children they chose to bear. Hence, aggregate saving would rise steeply. In the next period, however, the higher saving by the new generation of prime age adults would be offset by the higher dis-saving by the new generation of retirees. Hence, saving would decline from the transitory peak to a steady-state peak that would be higher than saving in period *t*, but lower than saving in period *t+1*. Just as is the case for survival, the saving rate would depend on the current level of youth dependency and its rate of change.

(b) Variables, definitions, the sample, and estimation methods

Except as noted below, all variables are taken directly from or constructed using data from the World

Development Indicators (WDI 2003). The saving rate (S/Y_t) is the average gross national saving rate for the five-year period *t* to *t+4*. The rate of growth of income is measured by the growth rate of real GDP during the preceding five-year period. The relative price of investment goods (PRI_t) is taken directly from the Penn World Table (PWT) and is the average value for the period *t* to *t+4*. The youth dependency ratio ($D1_t$) is the ratio of the population 0-14 to the population 15-64.

The old-age survival index (q) is the ratio of expected years lived after age 60 (T_{60}) to expected years lived between ages 30 and 60 ($T_{30} - T_{60}$) given contemporaneous age-specific death rates. T_x, total number of years lived after age *x*, is a standard life table value. Life tables are not available for many countries in many years. Hence, life expectancy at birth from the World Development Indicators (World Bank 2003) were used in conjunction with Coale-Demeny model life tables (Coale and Demeny 1983) to construct estimates of q.[xiv] The change in the old-age survival index is the average increase over the preceding five-year period, i.e., $\Delta q_t = q_t - q_{t-5}$.

The full sample consists of 566 observations for 76 countries in 1965-69 increasing to 94 countries in 1995-99. Estimates for Western and East Asian countries and for Other Developing Countries are presented separately. Because the national saving rates may affect economic growth, previous empirical studies have relied on two-stage least squares (2SLS) method to deal with the endogeneity problem. We follow standard practice and present both ordinary least squares (OLS) and 2SLS estimates. OLS estimates are a useful complement to 2SLS estimates for two reasons. First, the rate of economic growth is for the five-year period preceding the saving rate rather than a contemporaneous measure. This may mitigate, although not eliminate, the endogeneity problem. Second, it is difficult to find strong instruments for economic growth and using weak instruments often does more harm than good (Bound et al., 1995).

For the first stage variables we follow Higgins and Williamson (1997) and use the lagged values of the national investment rate, the labor force growth rate, the price of investment goods, the consumer price index, real GDP per worker, real GDP per capita, and a measure of openness. The rationale for including the investment rate and the labor force growth rate follows directly from the neo-classical growth model. The price variables are included to capture the effects of inflation on productivity growth. Per capita income measures are employed to incorporate the possible effects of convergence. Openness influences economic growth by creating a more competitive economic environment. All estimates include year and regional dummies, as well. Below we report the results of an over-identifying restriction test useful to judge the overall exogeneity of the instruments. We alsouse a Hausman test to assess whether the OLS and 2SLS estimates are statistically distinguishable.

(c) Results

Estimates of the basic model, equation (7), are presented in Table 3 OLS and 2SLS estimates are both presented and they are generally similar. The instrument variables in the first stage regression are jointly significant at the one percent level in all cases. The Hausman test implies that the GDP growth rate is potentially endogenous. For the non-Western/non-East Asia sample, the overidentifying restriction test indicates that we cannot reject the hypothesis that the instruments for economic growth areentirely exogenous. But the other samples do not pass the overidentifying restriction test, and thus parameters estimates are influenced to some extent by the endogeneity problem. The OLS and 2SLS parameter estimates differ substantially in magnitude as is evident in Table 3. Thequalitative results are similar, however, and for the sake of brevity we will limit our discussion to the 2SLS results.

[Table 3about here]

Old-age survival consistently has a large, statistically significant positive effect on national saving rates for the full sample and for the two sub-samples. The change in survival has a statistically significant positive effect on national saving in the West/East Asia (W/EA) sample, but not in the other sub-sample. The youth dependency effect is not statistically significant for the full sample or for the two sub-samples. The effect of

PRI has a large positive effect in the W/EA and a smaller negative effect elsewhere, but the effects are only marginally significant. The rate of growth effect evaluated at the mean value of *q* and *D1* is consistently positive with a value that ranges from 1.4 in W/EA to 0.75 in the non-W/EA sample. The regional dummy variables are statistically significant – positive for East Asia and negative for the non-West, non-East Asian countries. Thus, important regional differences in saving rates remain after controlling for demographic characteristics.

In analysis that is summarized only briefly here, we explored two other issues. First, we re-estimated the models presented in Table 3 to capture any transitory effects of youth dependency on saving by including the change in the youth dependency ratio. We did not find a significant transitory effect of youth dependency, nor did the inclusion of the change in youth dependency have any important effect on other aspects of the estimates. Old-age survival has a strong positive effect in all estimates; the change in old-age survival has a positive effect in W/EA but not elsewhere. The effects of other variables are similar to those reported in Table 3.

Second, we investigated the possibility that the effect of demographic variables depended on whether the economy was open or closed. As shown in Figures 2 and 3, openness influences the short-term and long-term effects of increased survival in a relatively complex fashion. Hence, the analysis was repeated (a) by sub-dividing the sample into countries that were relatively open and those that were relatively closed; or (b) by interacting the demographic variables with a measure of openness. Openness was measured using the Chinn-Ito (2005) *de jure* measure of financial openness. We found no statistically significant evidence that the effects of demographic variables were influenced by the degree of openness.

Two aspects of the results warrant emphasis. First, the results imply that the demographic transition has had an important effect on aggregate saving rates. Two different kinds of exercises show this to be the case. The first exercise is to calculate that share of the actual increase in saving rates in East Asia are explained by demographic variables. We find that demographic variables explain about half of the actual increase in saving rates in East Asia. Although substantial, the estimated effects are smaller that found in recent aggregate analyses. In the studies by Higgins and Williamson and Kelley and Schmidt changes in age structure explain virtually all of the increase in saving rates in East Asia (Kelley and Schmidt 1996; Higgins and Williamson 1997). Here the age structure effects are of similar magnitude to those found by Deaton and Paxson, while the effects of longevity would be part of the cohort effect in the Deaton and Paxson analysis (Deaton and Paxson 2000). These results bridge the gap, to some extent, between the large effects found in aggregate level analyses and the much smaller effects found in micro-level analyses.

The second exercise uses UN projections project saving rates. Projections for the West and East Asia allowing only the old-age survival ratio to change are presented in Figure 6. Projections allowing both the old-age survival ratio and the youth dependency ratio to vary, while holding all other variables constant, are presented in Figure 7.

[Figure 6 about here]

[Figure 7 about here]

The combined effect of the rise in adult mortality and the decline in youth dependency is to increase the aggregate saving rate between 1955-60 and 2045 by 14.6 percentage points in East Asia, by 8.6 percentage points in the West, and by 11.4 percentage points in the remaining countries based on the 2SLS estimates. Improvements in adult mortality accounted for a little more than three-quarters of the increase in each region while the decline in the youth dependency ratio accounted for a little less than one-quarter of the increase.

The second important feature of the results is the implication for population aging and aggregate saving. There is widespread concern, though limited empirical support, that population aging will lead to a decline in aggregate saving rates. The empirical results presented here do not support that conclusion. If old-age survival rates continue to increase, as is widely expected, our empirical results imply that saving rates will continue to rise. This empirical finding is consistent with the Lee, Mason and Miller (2003) simulations and suggests that economic growth may not slow as much with population aging as is anticipated in some quarters.

5. RESERVATIONS

An important unanswered question in this analysis is why no dynamic effect of old-age survival is found in the non-W/EA sample. One possibility is that drawbacks with the world panel data are responsible. The first limitation of the data is its relatively short time frame. For most economic analysis thirty years of data are more than adequate, but thirty years is only the length of a single generation. In a sense, we have a single observation of the OLG model presented in Section 2. The population of prime age adults/workers of 1965 is the old-age population of 1995.

A second difficulty, discussed in Section 3, is that the 1965-99 period may not be well-suited to testing our theoretical saving model. The countries of the West enjoyed similar and relatively constant increases in old-age survival; hence, analyzing variation across countries or across time is unlikely to shed much light on the role of increases in old-age survival. The Asian experience may be more fertile ground to analyze as suggested in Section 3. What about the rest of the developing world? Until the mid-1980s the gains in life expectancy were relative constant across the world. That the simple correlation between life expectancy at birth for the first half of the 1960s and the second half of the 1960s was 0.997 for the 176 countries for which the UN reports estimates illustrates the point.

Two groups of countries experienced significant departures from their historical trends beginning in the mid-1980s primarily because of two important events – the break-up of the Soviet empire and the emergence of the HIV/AIDS epidemic in sub-Saharan Africa. Regional conflicts also played a role in the Middle East and Africa. If the saving model is applicable to these mortality crises, saving rates should have declined. Evidence on this point is limited, but Gregory et al. (1999) conclude that the rise of mortality rates among middle-aged Russian men depressed their saving rates.

A third and related issue is that the measures of mortality in the international panel data are not ideal.

Although estimates of life expectancy at birth are reported by the World Bank, these values are often based on very incomplete information. We rely on model life tables to transform life expectancy at birth into the old-age survival ratio described in the previous section. Whether this is appropriate in all circumstances and, in particular, is reliable under severe mortality crises is a question to which we do not have a satisfactory answer. Moreover, the measure of the speed of mortality decline is for five-year periods. It may be that expectations about increases in old-age survival evolve much more slowly. Perhaps even generation length changes in old-age survival – a measure more consistent with our theoretical model – would be more appropriate.

It is not our intention to suggest that absence of a dynamic effect in the non-W/EA sample must surely be the fault of the data rather than the model. More fundamental explanations are possible. Higher saving rates are not the only possible response to an increase in the number of years lived at old age. One possibility is that consumption at old age may decline. Another is that the elderly may choose to increase their labor force participation, although the opposite is the case as an empirical matter with the recent exception of the US and a few other countries. Still another response is that transfer systems, either public or familial, may expand to meet the needs of a growing elderly population. Further work on these topics is clearly needed. [xv]

6. CONCLUSIONS

This is not the first paper to conclude that the increased duration of life could lead to higher saving rates. At the individual or household level, it is obvious that higher saving rates are a likely response to an increased duration of retirement. The effect on aggregate saving is more complex, however, because there are both behavioral responses and age composition effects at play. An important contribution of this paper is to show that the aggregate saving rate depends on both the level and the rate at which adult survival is increasing.

The empirical results provide strong and consistent evidence that an increase in the portion of adult life lived at old ages leads to an increase in saving rates. This finding holds for all samples and specifications. The historical evidence suggests and the international panel data confirms that the increase in adult survival was a major impetus to the rise in the wealth of many nations.

The evidence for the dynamic effect, i.e., the effect of the rate of change in old-age survival, is more fragile. When analysis is confined to the West/East Asia sample we find a statistically significant effect. Moreover, the long-run historical patterns available for countries from East Asia and the West are consistent with the dynamic effect. In other parts of the contemporary developing world, however, we do not find any support for a dynamic effect.

There are two additional important implications of the empirical analysis. The first is that the demographic transition had a strong positive effect on aggregate saving rates, but that most of the effect was a response to improvements in old-age survival rather than to changes in youth dependency. This result may help to reconcile the divergent findings of studies based on the analysis of aggregate data and household survey data. The second important implication is that population aging will not lead to a decline in saving rates. Any compositional effects associated with population aging are outweighed by the behavioral effects of continued increases in longevity.

Endnotes

[i] Theoretical results are summarized here. More details are available in Kinugasa, 2004.

[ii] The interpretation of q becomes important in the empirical analysis.

[iii] In this model, the mean age of consumption is $(c_{1,t} + 2q_t c_{2,t+1})/(c_{1,t} + q_t c_{2,t+1})$ and the mean age of earning is 1.

[iv] In the simulation results presented below we assume that an increase in the interest rate leads to greater saving. Qualitative results hold unless an increase in interest rates have a large negative effect on saving – a possibility not supported by empirical evidence. For details see Kinugasa 2004.

[v] The economy converges in a non-oscillatory pattern to the steady state under plausible parameter values. See Kinugasa (2004) for details.

[vi] The parameters for the simulation model have been chosen with an eye to maintaining consistency with previous studies, e.g., Barro and Sala-i-Martin (1995) and Higgins (1994). Productivity growth and the rate of population growth are both set to 1.5% per year. The population growth rate for the world for 1820-1998 was somewhat slower at 1.0% per year (Maddison 1995). For the 1950-2000, the population growth rate was somewhat faster at 1.75% per year (United Nations Population Division 2005). We assume that the intertemporal elasticity of consumption is 1.3, following Higgins. This is an important parameter in the closed economy model because it implies that a higher interest rate will lead to a modest increase in saving rates. We have used a discount rate of 0.02 per annum as have Barro and Sala-i-Martin as compared with a discount rate of 0.025 in Higgins given our view that a low discount rate is more consistent with observed age-profiles of consumption that are relatively flat in most countries. We use standard values for the depreciation rate (5%) and the elasticity of output with respect to capital (1/3) – the same values used by Barro and Sala-i-Martin and

Higgins. We part company with Higgins by assuming that a generation length is 30 years rather than 25. This assumption is based on recent estimates of the economic lifecycle showing that a generation length of 30 years is a more realistic representation (Lee, Lee et al. 2005).

[vii] Life tables used in the analysis here, and in other studies, are period tables based on current age-specific mortality rates. Thus, they describe the mortality experience of a synthetic cohort that lives its life subject to current mortality conditions. Using cohort specific life tables would conform more closely to the theoretical model. Projected cohort life tables are not generally available, but they could be constructed using projected age-specific survival rates from the UN, for example. Given the simple methods used to project life expectancy, it seems doubtful that projected values contain additional information beyond that already captured by the two measures of survival included in our analysis.

[viii] In India we analyze life expectancy at age 30, which is highly correlated with the old-age survival index.

[ix] The break points were assigned visually. A more precise iterative approach would improve the fit of the piece-wise linear approximations, but the analysis presented below would not be affected in any important way.

[x] In the regression analysis in Figure 4, we checked the trend stationarity of the variables by Augmented Dickey-Fuller test and find that some variables are not trend stationary.

[xi] In our world panel data, all variables are averages for the subsequent five-year period. So, saving rate in 1965 is calculated as the mean of saving rates from 1965 to 1969 and a variable in 1995 is the mean of the variable from 1995 to 1999.

[xii] The effect of openness is explored below.

[xiii] The saving mechanism can be used to shift consumption to younger ages, by accumulating credit card debt, for example, but constraints on indebtedness limit the importance of these transactions.

[xiv] See Kinugasa (2004) for additional details. The Coale-Demeny Model Life Table is an approximation that does not fit certain countries very well, e.g., countries with unusually high rates of mortality at prime-adult ages.

[xv] Our theoretical model and empirical analysis assume that demand side of capital is implicit. Therefore, the framework is more appropriate for capacity-constrained economies than demand-constrained economies.

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Table 1. Structural Changes of the Old-Age Survival Index

	Sweden	United Kingdom	Italy	United States	Japan	Taiwan	India
	1751-2002	1841-1998	1872-2000	1900-2001	1891-2001	1906-2002	1881-1997
t_1	1876	1900			1947	1940	1951
t_2					1989	1983	1989
t	0.0006***	0.0009***	0.0029***	0.0033***	0.0010***	0.0004	0.0392
	(0.0001)	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0010)	(0.0232)
$(t-t_1)d_1$	0.002***	0.0024***			0.0065***	0.0056***	0.3457***
	(0.0001)	(0.0001)			(0.0004)	(0.0011)	(0.0599)
$(t-t_2)d_2$					-0.0014***	-0.0024***	-0.3075*
					(0.0007)	(0.0003)	(0.1568)
Dm1773	-0.0231***	0.0004***	-0.1231***				
	(0.0027)		(0.0023)				
Dm1808	-0.1625***						
	(0.0063)						
Dm1918	-0.096***						
	(0.0040)						
Constant	0.2869***	0.3205***	0.2518***	0.2245***	0.3250***	0.2860***	22.7955***
	(0.0080)	(0.0049)	(0.0053)	(0.0195)	(0.0104)	(0.0322)	(0.8454)
Adjusted R^2	0.9251	0.9587	0.9471	0.9585	0.9872	0.9944	0.9628
N	252	158	129	48	59	43	20
P-value	0.0000	0.0000			0.0000	0.0000	0.0000
D.W.	0.5765	0.7026	0.397	0.0958	0.3897	0.3651	0.0338

Note. Dependent variables are adult survival index except for India. The dependent variable for India is life expectancy at age 30. The equation $q_t = b_0 + b_1 t + b_2 d_1 (t - t_1) + b_3 d_2 (t - t_2) + e_t$ is estimated, where $d_1 = 1$ if $t \geq t_1$ and 0 otherwise, and d_2=1 if $t \geq t_2$ and 0 otherwise. Dm1773 is a dummy variable equal to 1 for year 1773. Dm1808, Dm1918 are defined in similar fashion. *N* is number of observations. "P-value" is the p-value of F-test for the null hypothesis that $ß_1$, $ß_2$, and $ß_3$ are all zero. D.W. is Durbin-Watson statistics. The low Durbin-Watson statistics imply serial correlation. The augmented Dickey-Fuller test indicates that some variables are not trend stationary.

Table 2 Mean National Saving Rates in Pre-Transition and Transition Periods.

	Sweden	United Kingdom	Italy	United States	Japan	Taiwan	India[a)]
Pre-transition	1751-1875	1841-1899	<1872	<1900	1891-1947	1906-1939	1900-1950
Survival rate	0.31	0.34	na	na	0.35	0.24	24.0
Saving rate	7.3	11.6	na	na	15.7	22.1	6.9
Transition	1876-2000	1900-1998	1872-2000	1900-2001	1947-2000	1940-1999	1951-1997
Survival rate	0.53	0.49	0.47	0.57	0.60	0.54	37.7
Saving rate	16.6	13	17.7	17.7	32.6	26.1	17.7

Note. Life expectancy at age 30 is presented instead of survival rate in India.

Table 3 Estimated Saving Equation with Old-Age Survival Index and Youth Dependency

	(1) Whole World		(2) West and East Asia		(3) Non-West, Non East Asia	
	OLS	2SLS	OLS	2SLS	OLS	2SLS
$q*Y_{gr}$	8.2055 ***	14.8399 ***	8.3714 **	14.8129 ***	9.1327 ***	15.7251 ***
	(1.8606)	(2.6660)	(3.7148)	(3.5266)	(1.9979)	(2.9857)
dq	0.8615	0.5104	1.7954 **	1.8115 **	0.3284	-0.2538
	(0.5266)	(0.5752)	(0.8192)	(0.8519)	(0.5962)	(0.6823)
$D1*Y_{gr}$	0.7461	0.9319	-3.0912 ***	-1.7057	0.4379	1.0155
	(0.7639)	(0.9345)	(1.1164)	(1.1246)	(0.9871)	(1.2978)
Y_{gr}	-3.7274 ***	-6.5927 ***	-2.9073	-6.4693 ***	-3.6892 **	-6.8701 ***
	(1.3526)	(1.9655)	(2.4628)	(2.3958)	(1.5577)	(2.4026)
PRI	-0.0053 *	-0.0028	0.0422 **	0.0350 *	-0.0084 **	-0.0064 *
	(0.0032)	(0.0036)	(0.0207)	(0.0212)	(0.0035)	(0.0036)
y70	0.0100	0.0075	0.0093	0.0105	0.0069	0.0026
	(0.0181)	(0.0184)	(0.0161)	(0.0159)	(0.0251)	(0.0253)
y75	-0.0175	-0.0253	-0.0280 *	-0.0250	-0.0261	-0.0373
	(0.0166)	(0.0176)	(0.0159)	(0.0159)	(0.0233)	(0.0241)
y80	-0.0253	-0.0290	-0.0280	-0.0167	-0.0433 *	-0.0543 **
	(0.0165)	(0.0188)	(0.0198)	(0.0238)	(0.0222)	(0.0234)
y85	-0.0031	-0.0013	0.0001	0.0184	-0.0272	-0.0345
	(0.0163)	(0.0218)	(0.0173)	(0.0207)	(0.0226)	(0.0272)
y90	-0.0102	-0.0185	-0.0340 *	-0.0255	-0.0258	-0.0398
	(0.0161)	(0.0197)	(0.0175)	(0.0191)	(0.0225)	(0.0257)
y95	-0.0009	-0.0086	-0.0039	0.0196	-0.0339	-0.0598 **
	(0.0168)	(0.0235)	(0.0197)	(0.0231)	(0.0235)	(0.0285)
East Asia	0.0312 **	0.0189	0.1073 ***	0.0855 ***		
	(0.0137)	(0.0175)	(0.0165)	(0.0202)		
Other Countries	-0.1190 ***	-0.0915 ***				
	(0.0093)	(0.0125)				
Constant	0.1775 ***	0.1487 ***	0.1544 ***	0.1267 ***	0.0758 ***	0.0855 ***
	(0.0161)	(0.0245)	(0.0247)	(0.0306)	(0.0205)	(0.0305)
N	566	566	189	189	377	377
Adjusted R	0.4357	0.3945	0.4191	0.3848	0.1275	0.0852
P-values, Yg	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Pvalues, year dummies	0.2800	0.1160	0.0125	0.0006	0.2261	0.0954
Y_{gr} Effect	0.7203	1.1739	0.6967	1.4105	0.6177	0.7543
P-value, Hausman		0.0000		0.0037		0.0000

Note. The dependent variable is the national saving rate.

q: Adult survival index, Y_{gr}: GDP growth rate, $D1$: young dependency rate, $?q$: change in the adult survival rate, *PRI*: price of investment goods, yXX dummy variables for year XX, N: number of observation. "P-value, *Ygr* " is the p-value of F-test of the null hypothesis that the both coefficients of $q \cdot Y_{gr}$ and $D1 \cdot Y_{gr}$ are zero. "Pvalue, year dummies" is the p-value of F-test of the null hypothesis that all the year dummies are zero. " Y_{gr} Effect" is the partial effect of an increase in GDP growth. "P-value Hausman" is the p-value of Hausman test for the null hypothesis that economic growth rate is exogenous.

*** denotes significant at 1 % level, **denotes significant at 5 % level, and * denotes significant at 10 % level.

Figures in parentheses are standard errors.

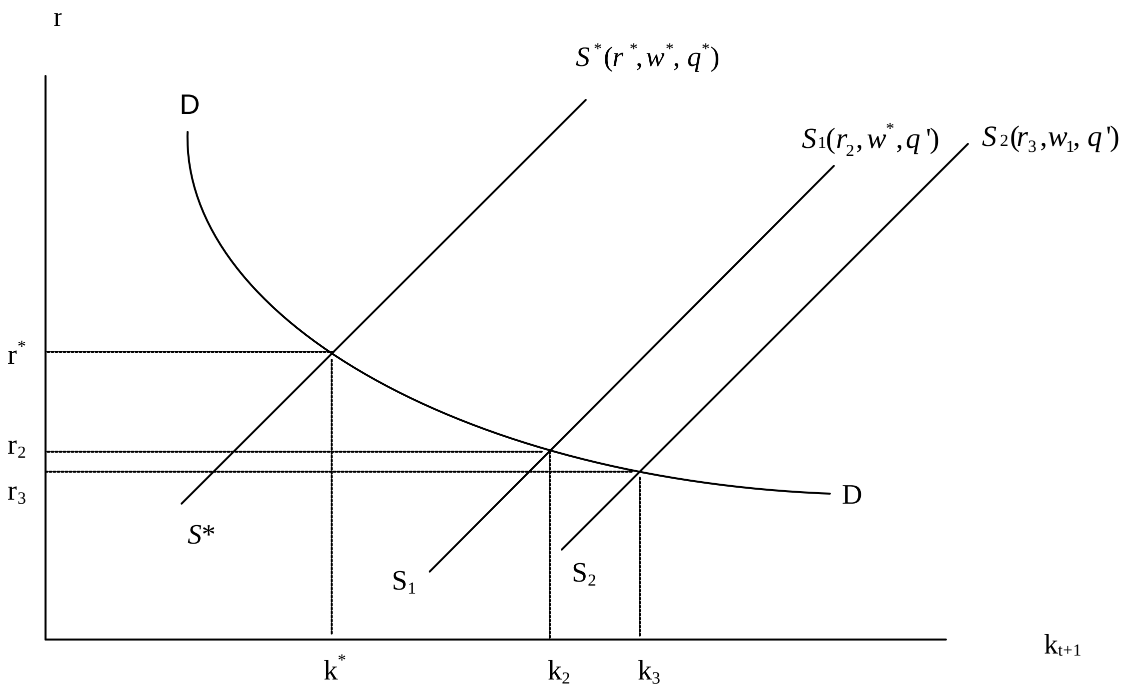


Figure 1 Demand and Supply of Capital in a Closed Economy

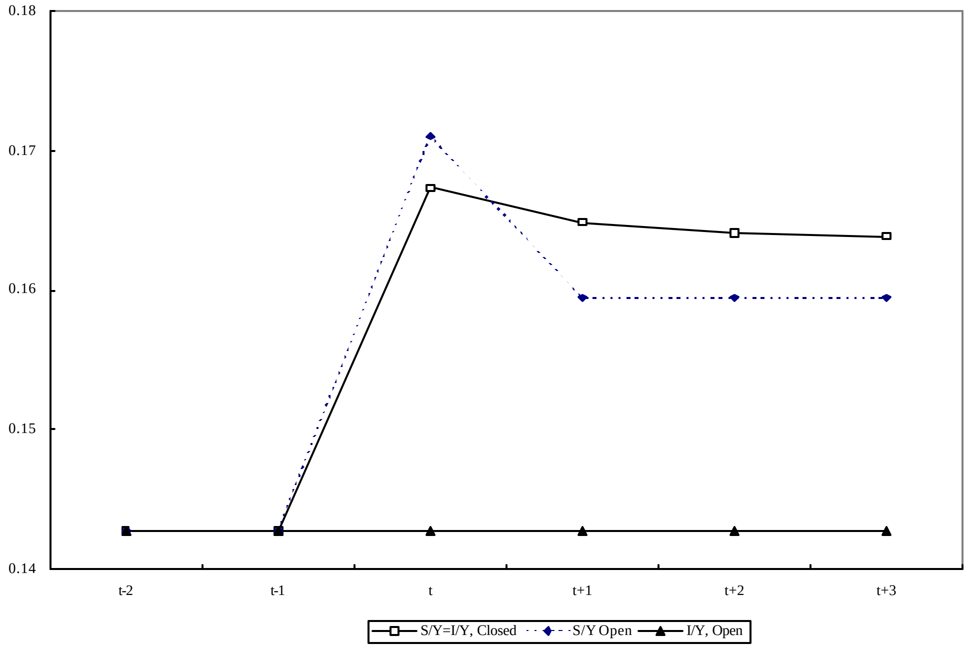


Figure 2 The Effect of an Increase in the Survival Rate at Time *t*

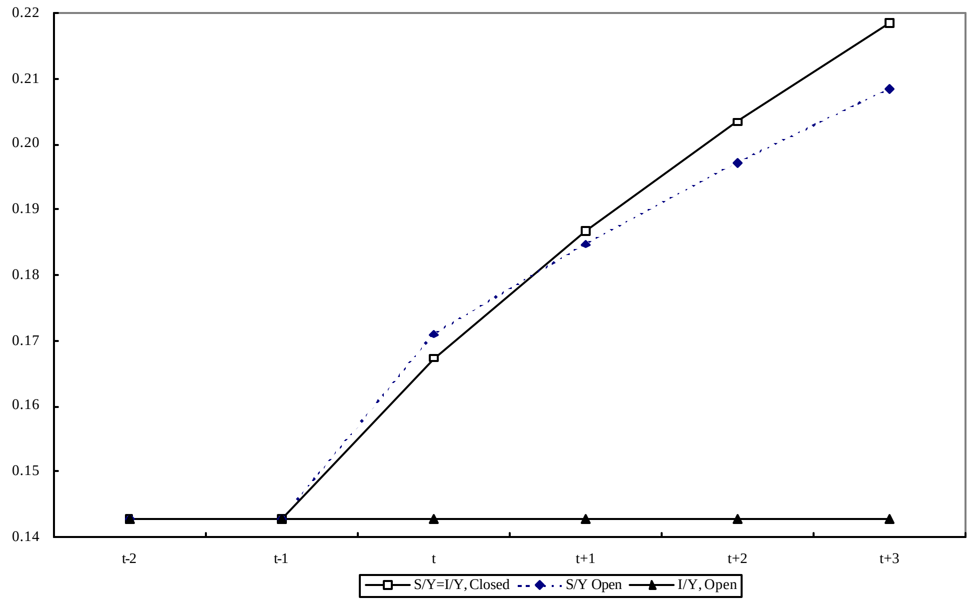


Figure 3 The Effect of a Continuing Increase in the Survival Rate

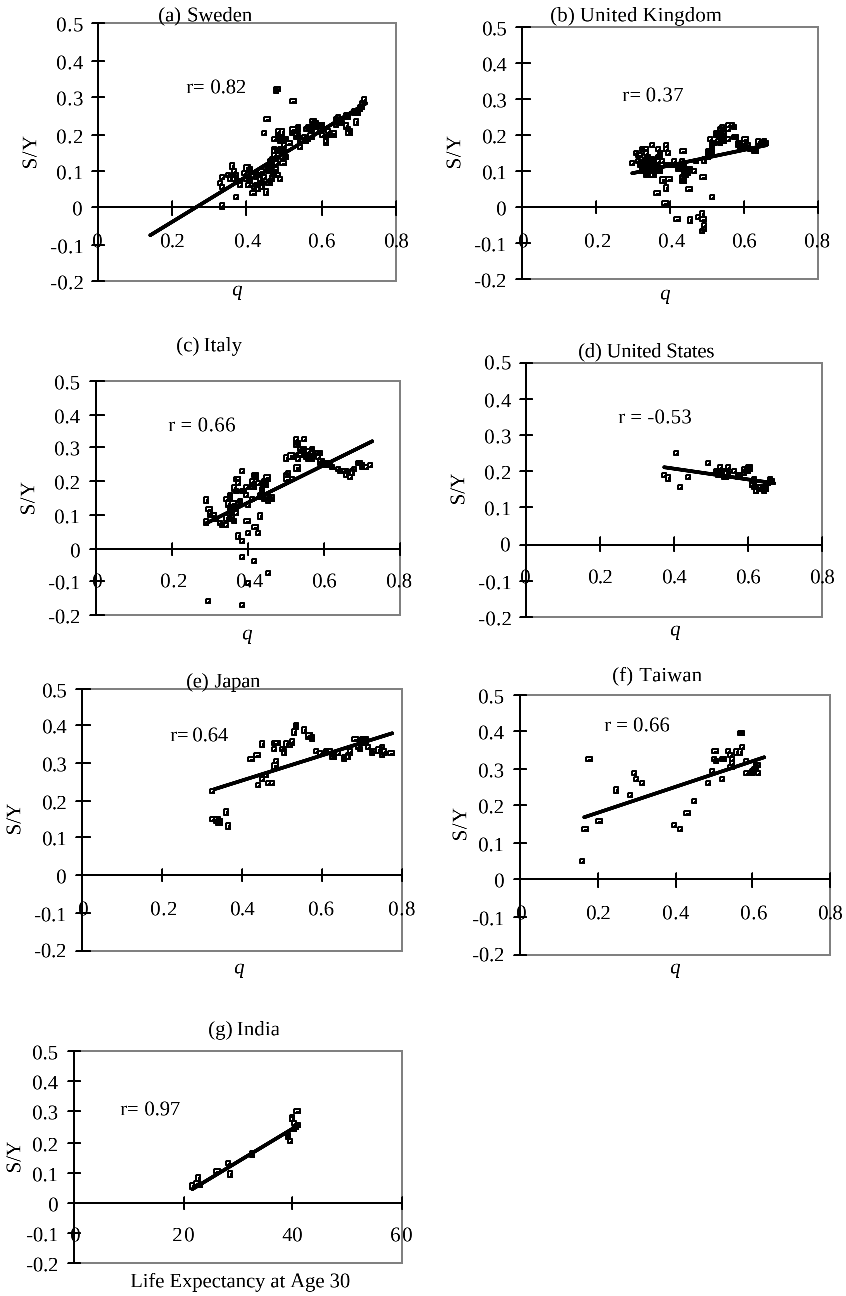


Figure 4 Adult Survival Index and the National Saving Rate

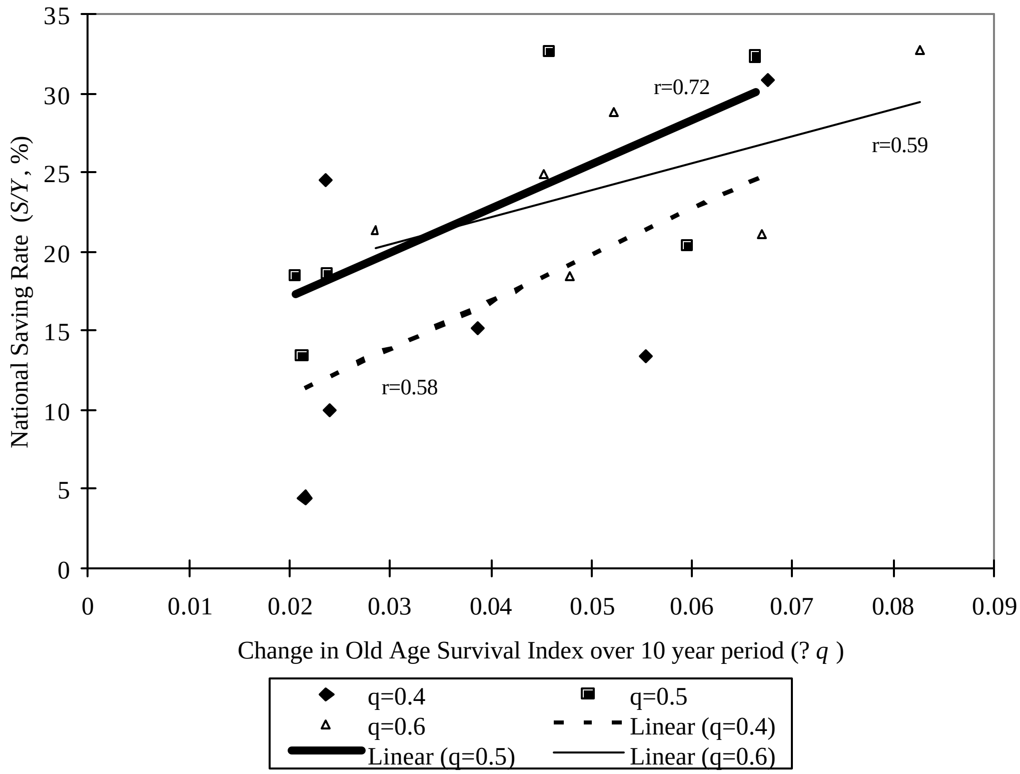
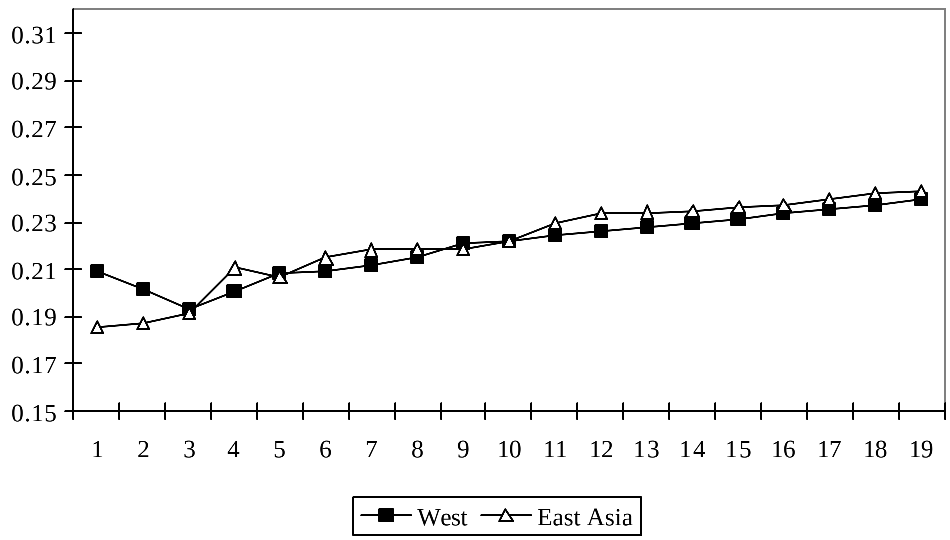


Figure 5 Changes in Old-Age Survival Index and the National Saving Rate

(a) OLS



(b) 2SLS

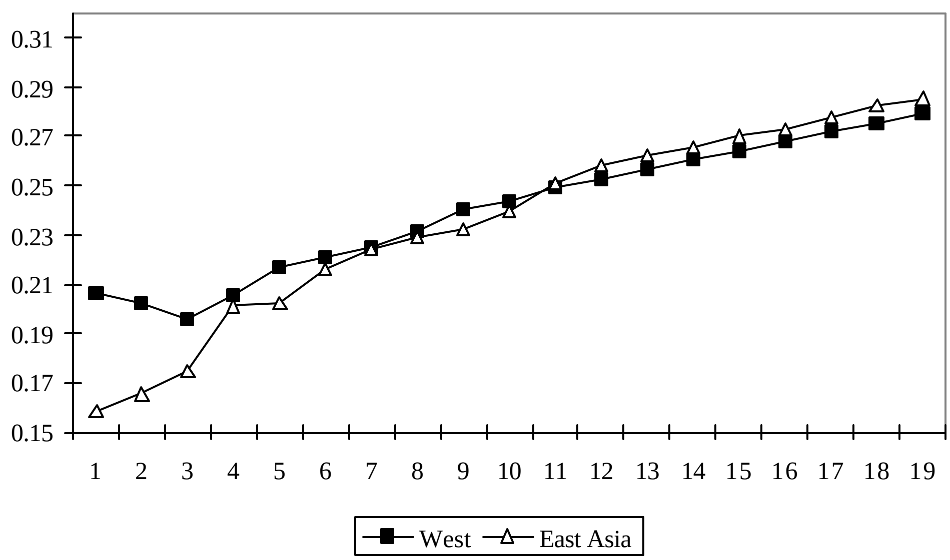


Figure 6 Projected Saving Rates of Western Countries and East Asia (Constant Youth Dependency, 1955-2050)

(a) OLS

(b) 2SLS

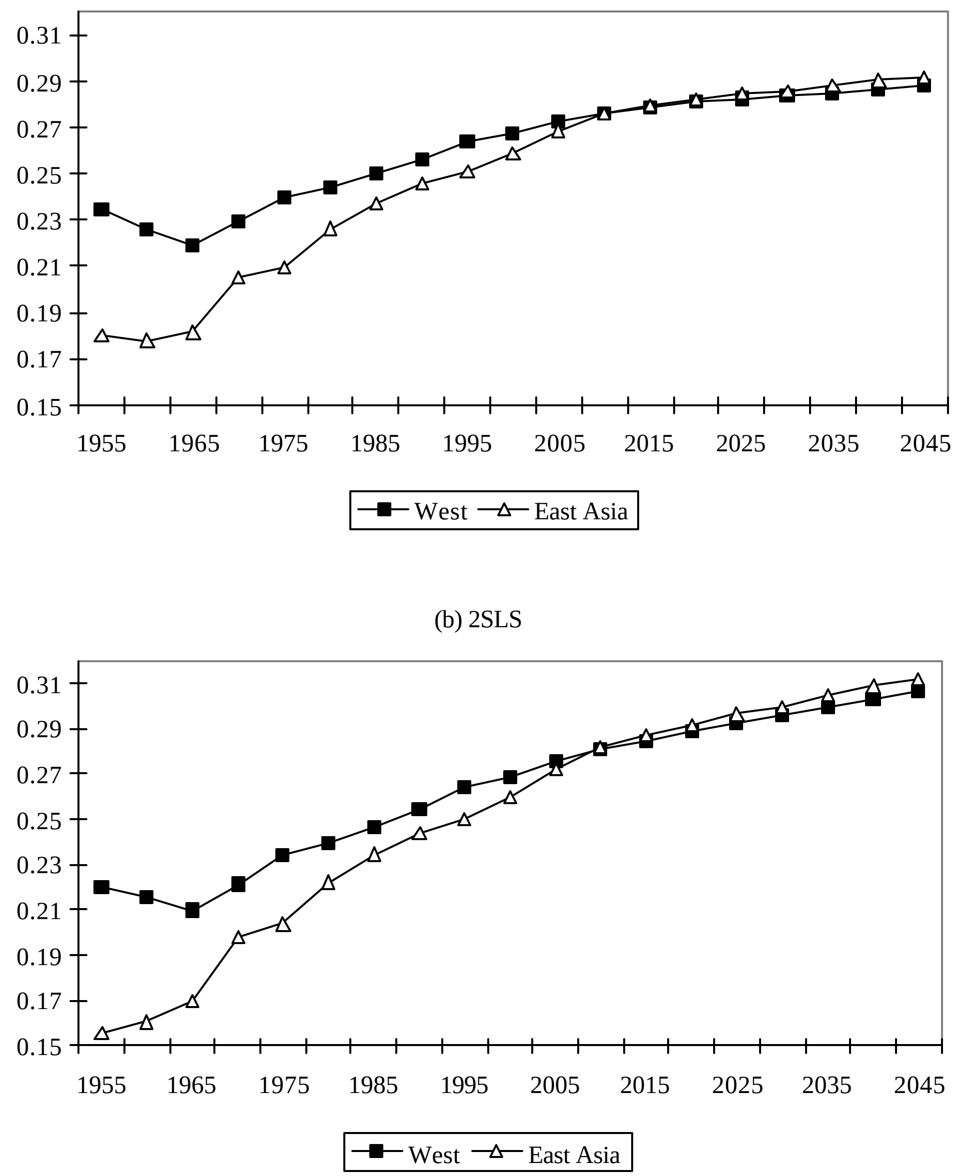


Figure 7 Projected Saving Rates of Western Countries and East Asia (Changing Youth Dependency, 1955-2050)

Appendix Details of the Overlapping Generations Model

Individuals live at most for two periods. With probability $1-q_t$ they die at the end of period 1 and with probability q_t at the end of period 2. The probabilities are known but individuals have no information about their own survival. The consumer's optimization problem is to maximize lifetime utility, assuming constant relative risk aversion, $V_t = \frac{c_{1,t}^{\;1-q}}{1-q} + d\,q_t \frac{c_{2,t+1}^{\;1-q}}{1-q}$, given the lifetime budget constraint. Consumption at age 1 in period t is $c_{1,t}$ and $c_{2,t+1}$ is consumption at age 2 during period t+1; d is the discount factor, defined as $d=1/(1+?)$, where ? is the discount rate; and $1/?$ is the intertemporal elasticity of substitution. Prime-age adults of age 1 earn $A_t w_t$. A_t is labor-augmenting technology and w_t is wage per unit of effective labor. Consumers divide their earnings between current consumption and saving for old age, so that:

$$c_{1,t} + s_{1,t} = A_t w_t \,, \tag{A. 1}$$

where $s_{1,t}$ is the saving of prime-age adults. Elderly adults are retired and consume what they saved while young. Insurance against longevity risk is available. An annuity is purchased at the beginning of age 1. If insurance companies are risk neutral and annuity markets are perfect, the rate of return to survivors is $[(1+r_{t+1})/q_t]$, where r_{t+1} is the riskless interest rate on saving. The return to annuities is $[(1+r_{t+1})/q_t]$. In this situation, returns to insurance are greater than a regular note. Thus, individuals save only in the form of insurance and consume:

$$c_{2,t+1} = \frac{1+r_{t+1}}{q_t} s_{1,t} \,. \tag{A. 2}$$

From (A.1) and (A. 2), the lifetime budget constraint of the individuals at age 1 is:

$$w_t A_t = c_{1,t} + \frac{q_t}{1+r_{t+1}} c_{2,t+1} \,. \tag{A.3}$$

The corresponding Lagrangian is:

$$L = \frac{c_{1\,t}^{\;1-?}}{1-?} + d\,q_t \frac{c_{2,t+1}^{\;1-?}}{1-?} - ?\left(c_{1,t} + \frac{q_t}{1+r_{t+1}} c_{2,t+1} - A_t w_t \right),$$

where ? is a Lagrangian multiplier. First order conditions are:

$$\frac{\partial L}{\partial c_{1,t}} = c_{1,t}^{\ -q} - l = 0 \tag{A.4}$$

$$\frac{\partial L}{\partial c_{2,t+1}} = d q_t c_{2,t+1}^{\ -q} - \frac{q_t}{1+r_{t+1}} l = 0 \tag{A.5}$$

$$\frac{\partial L}{\partial l} = -c_{1,t} - \frac{q_t}{1+r_{t+1}} c_{2,t+1} + A_t w_t = 0 \tag{A.6}$$

Substituting (A.4) and (A.5) into equation yields:

$$l = \frac{[1+q_t d^{\frac{1}{q}}(1+r_{t+1})^{\frac{1-q}{q}}]^q}{A_t^q w_t^{\ q}} \tag{A.7}$$

Substituting back for ? yields:

$$c_{1,t} = \frac{(1+r_{t+1})^{\frac{1-q}{q}} A_t w_t}{q_t d^{\frac{1}{q}} + (1+r_{t+1})^{\frac{1-q}{q}}} \tag{A.8}$$

$$c_{2,t+1} = \frac{d^{\frac{1}{q}}(1+r_{t+1}) A_t w_t}{q_t d^{\frac{1}{q}} + (1+r_{t+1})^{\frac{1-q}{q}}} \tag{A.9}$$

From the budget constraints in equations (A.1) and (A.2) and equations (A.8) and (A.9), we can calculate saving by prime age adults ($s_{1,t}$)and the elderly ($s_{2,t}$). Saving by prime age adults is:

$$s_{1,t} = \Psi_t A_t w_t = \frac{q_t d^{\frac{1}{q}} A_t w_t}{q_t d^{\frac{1}{q}} + (1+r_{t+1})^{\frac{q-1}{q}}} , \tag{A.10}$$

where ? $_t$ is the share of wage income saved by prime-age adults. The elderly consume assets accumulated while working plus interest income from the annuity. But the return on the annuity is part of their income, so: $s_{2,t} = \left(\frac{1+r_t}{q_t} - 1\right) s_{1,t-1} - c_{2,t} = -s_{1,t-1}$. Saving of the elderly is:

$$s_{2,t} = -s_{1,t-1} = -\Psi_{t-1} A_{t-1} w_{t-1} = \frac{q_{t-1} d^{\frac{1}{q}} A_{t-1} w_{t-1}}{q_{t-1} d^{\frac{1}{q}} + (1+r_t)^{\frac{q-1}{q}}} . \tag{A.11}$$

Equations (A.10) and (A.11) correspond to equations (1) in the text. From equation (A.11), the effect of an increase in the survival rate at time t on the share of wage income saved by prime-age adults is:

$$\frac{\partial \Psi_t}{\partial q_t} = \frac{d^{\frac{1}{q}}\left[q_t d^{\frac{1}{q}} + (1+r_{t+1})^{\frac{q-1}{q}}\right] - q_t d^{\frac{2}{q}}}{\left[q_t d^{\frac{1}{q}} + (1+r_{t+1})^{\frac{q-1}{q}}\right]^2} = \frac{d^{\frac{1}{q}}(1+r_{t+1})^{\frac{q-1}{q}}}{\left[q_t d^{\frac{1}{q}} + (1+r_{t+1})^{\frac{q-1}{q}}\right]^2} > 0.$$

Thus, an increase in survival rate increases the share of saving by prime-age adults.

Gross domestic product (Y_t) is produced by a Cobb-Douglas production function with labor-augmenting technological growth, i.e., $Y_t = K_t^f L_t^{1-f}$, where f is the share of capital in GDP, $0 < f < 1$. $L_t = A_t N_{1,t}$ is the aggregate labor supply measured in efficiency units. $N_{1,t}$ is the population of prime-age adults and A_t is the technology index. The population growth rate per generation is $n_t - 1$ and, hence, $N_{1,t} = n_t N_{1,t-1}$. The technological growth rate per generation is $g_t - 1$. Hence, the relationship between the total lifetime labor income of prime-age adults and pensioners is given by $w_{t-1} A_{t-1} N_{1,t-1} = w_t A_t N_{1,t} / g_t n_t$. Using lower case letters to represent quantities per unit of effective worker, output per effective worker is $y_t = k_t^f$ and the capital-output ratio is k_t^{f-1}. The depreciation rate (x) is assumed to be constant; hence, depreciation as a share of GDP is $x k_t^{f-1}$.

Total gross national saving is the sum of the saving of adults ($S_{1,t}$), the saving of the elderly ($S_{2,t}$) and depreciation (?K_t). Dividing by Y_t yields the gross national saving rate at time t:

$$\frac{S_t}{Y_t} = (1-f)\left(\Psi(q_t, k_t) - \Psi(q_{t-1}, k_{t-1})\frac{1}{g_t n_t}\right) + x k_t^{1-f}. \tag{A.12}$$

This equation corresponds to equation (2) in the text. The term $(1-f)$ is the share of labor income in GNP, $\Psi_t(q_t, k_t)$ is saving by current workers as a share of current labor income, $-\Psi(q_{t-1}, k_{t-1})\frac{1}{g_t n_t}$ is saving by current retirees as a share of current labor income, and $x k_t^{1-f}$ is depreciation as a share of current GNP. The steady-state gross national saving rate is:

$$\left(\frac{S}{Y}\right)^* = (1-f)\Psi(q^*, k^*)\left(\frac{gn-1}{gn}\right) + x k^{*1-f} \tag{A.13}$$

where the * superscript denotes equilibrium values. This equation corresponds to equation (3) in the text.

In a small open economy, capital is perfectly mobile. Residents can borrow and lend in the international capital market at the world interest rate. The assumption of a small open economy implies that the country is sufficiently small not to influence the world interest rate. According to the arbitrage condition, the marginal product of domestic capital is equal to the world interest rate; thus, $f k_t^{f-1} - x = r_{w,t}$. The capital stock per effective worker is:

$$k_t = \left(\frac{f}{r_{w,t} + x} \right)^{\frac{1}{1-f}}. \tag{A.14}$$

The aggregate capital stock is given by $K_t = A_t N_{1,t} k_t$. Equation (A.14) implies that capital per effective worker is determined solely by the world interest rate and the parameters of the production process. The rate of growth of the labor force does not affect k_t, nor does the survival rate affect k_t.

In the absence of international capital flows, domestic saving equals investment. Gross national saving is the sum of changes in asset holdings of prime age adults and the elderly and depreciation, so that $S_t = S_{1,t} + S_{2,t} + ?K_t = S_{1,t} - S_{1,t-1} + ?K_t$. Gross investment is given by $I_t = K_{t+1} - (1 - ?)K_t$. Because saving is equal to investment, $S_{1,t} = K_{t+1}$. The saving of the young is equal to the capital stock in the next period. Thus, the supply of capital is:

$$K_{t+1} = S_{1,t} = s_{1,t} N_{1,t}. \tag{A.15}$$

Noting that $K_{t+1} = A_{t+1} N_{1,t+1} k_{t+1}$, equation (A.15) can be rewritten as:

$$k_{t+1} = \frac{\Psi(r_{t+1}, q_t)\, w_t(k_t)}{g_{t+1} n_{t+1}} = S_t(r_{t+1}, w_t(k_t), q_t), \tag{A.16}$$

where S_t denotes the supply of capital per unit of effective labor. Equation (A.16) corresponds to equation (4) in the text.

Here, as a matter of convenience we set O_t and $?_t$ as: $\Omega_t = 1 + q_t d^{\frac{1}{q}} (1 + r_{t+1})^{\frac{1-q}{q}}$, and $X_t = q_t d^{\frac{1}{q}} (1 + r_{t+1})^{\frac{1-q}{q}}$, where O^{-1} is the share of labor income consumed in prime age and $?\, O^{-1}$ is the share saved by prime-age adults, so that $\Psi = X \Omega^{-1}$. From equation (A.16) the following equation is obtained:

$$(1-f) X_t (q_t, r(k_{t+1})) k_t^f = k_{t+1} g_{t+1} n_{t+1} \Omega_t (q_t, r(k_{t+1})). \tag{A.17}$$

If the survival rate is constant, capital per effective worker will approach the steady state:

$$k^* = \left[\frac{(1-f)\mathrm{X}(q^*, r(k^*))}{gn\Omega(q^*, r(k^*))} \right]^{\frac{1}{1-f}}. \qquad \text{(A.18)}$$

where k^* and q^* denote capital per unit of effective labor and the survival rate in steady state, respectively.

Assuming variables other than capital per effective worker at times t and $t+1$ are constant in equation (A.17), the relationship between k_t and k_{t+1} is described as:

$$\frac{dk_{t+1}}{dk_t} = \frac{(1-f)f'(k_t)\mathrm{X}_t\Omega_t^{-1}}{g_{t+1}n_{t+1} + [g_{t+1}n_{t+1}k_{t+1} - (1-f)f(k_t)](\frac{1-q}{q})\mathrm{X}_t\Omega_t^{-1}[1+f'(k_{t+1})-x]^{-1}f''(k_{t+1})}. \qquad \text{(A.19)}$$

In equation (A.19), gnk_{t+1} is saving and $(1-f)f(k_t)$ is earning per prime age adult at time t. Because saving is less than earning, $g_{t+1}n_{t+1}k_{t+1} < (1-f)f(k_t)$. Note that $f'' < 0$ and that 0<? O^{-1}<1 hold.

Substituting equation (A.18) into equation (A.19), the numerator of the right hand side of equation (A.19) reduces to $f\,gn$ in steady state. Thus, $0 < \frac{dk_{t+1}}{dk_t} < 1$ is satisfied at least around the steady state unless the intertemporal elasticity of substitution (1/?) is much less than 1. If $0 < \frac{dk_{t+1}}{dk_t} < 1$ holds in equation (A.19), the steady state is stable. Also, if the intertemporal elasticity of substitution is much less than 1, it is possible that the denominator of equation (A.19) is negative. For simplicity, we assume that $0 < \frac{dk_{t+1}}{dk_t} < 1$ holds; in other words, the economy converges in a non-oscillatory pattern.

Suppose the survival rate is constant in steady state. Then, in steady state, the equilibrium of the capital market in equation (A.17) is given by:

$$(1-f)\mathrm{X}(q^*, r(k^*))k^{*f} = k^* gn\Omega(q^*, r(k^*)). \qquad \text{(A.20)}$$

Calculating total differentials of both sides of equation (A.20) and simplifying the results yields the following expression of the relationship between capital per effective worker and the survival rate:

$$\frac{dk^*}{dq^*}=\frac{\Omega^{*-1}d^{\frac{1}{q}}[1+f'(k^*)-x]^{\frac{1-q}{q}}[(1-f)k^{*f}-k^*gn]}{gn+[gnk^*-(1-f)f(k^*)](\frac{1-q}{q})\mathrm{X}^*\Omega^{*-1}[1+f'(k^*)-x]^{-1}f''(k^*)-\mathrm{X}^*\Omega^{*-1}(1-f)f'(k^*)}. \tag{A.21}$$

Equation (A.18) and $0<\frac{dk_{t+1}}{dk_t}<1$ imply that the denominator of equation (A.21) is positive. Because saving of prime age adults is less than wage income, $(1-f)k^{*f}-k^*gn>0$ holds, so that the numerator is also positive. Therefore, $\frac{\partial k^*}{\partial q^*}>0$ holds.

Consider the effect of an anticipated one-time increase in life expectancy at time t and assume that q stays at q^* in subsequent periods. In equation (A.17), assuming that all variables other than k_{t+1} and q_t are constant, we can characterize the relationship between the changes of q_t and k_{t+1} as:

$$\frac{dk_{t+1}}{dq_t}=\frac{\Omega_t^{-1}d^{\frac{1}{q}}[1+f'(k_{t+1})-x]^{\frac{1-q}{q}}[(1-f)k^{*f}-k_{t+1}g_{t+1}n_{t+1}]}{g_{t+1}n_{t+1}+[g_{t+1}n_{t+1}k_{t+1}-(1-f)f(k_t)](\frac{1-q}{q})\mathrm{X}_t\Omega_t^{-1}[1+f'(k_{t+1})-x]^{-1}f''(k_{t+1})}. \tag{A.22}$$

The denominator of the right hand side of equation (A.22) is equal to that of equation (A.19). We assume that $0<\frac{dk_{t+1}}{dk_t}<1$ holds, so that the denominator of equation (A.22) is positive. As we discussed above, $(1-f)k^{*f}$ is wage earning at time t, and $k_{t+1}gn$ is the saving per prime-age adult. Because saving of the young does not exceed earning, $(1-f)k^{*f}-k_{t+1}g_{t+1}n_{t+1}>0$, the numerator is positive and $\frac{dk_{t+1}}{dq_t}>0$ holds.

A greater survival rate brings capital deepening ($\partial k^*/\partial q^*>0$). If the GDP growth rate is greater than the deprecation rate, the saving rate increases. If GDP is growing at a rate less than the rate of depreciation, an increase in q^* has a negative effect on the saving rate.

Gross national saving is the sum of changes in asset holdings of prime age adults and the elderly and depreciation, so that S_t=$S_{1,t}$+ $S_{2,t}$+ ?K_t= $S_{1,t}$ – $S_{1,t-1}$+ ?K_t. Gross investment is given by I_t= K_{t+1} –(1 – ?)K_t. GDP is expressed as $Y_t=K_t^{\,f}(A_tN_{1,t})^{1-f}$. Therefore, national saving rate is calculated as:

$$\frac{S_t}{Y_t}=\left(g_{t+1}n_{t+1}\frac{k_{t+1}}{k_t}-1+x\right)k_t^{1-f}. \tag{A.23}$$

Equation (A.23) corresponds to equation (5) in the text. In steady state, the relationship simplifies to:

$$\left(\frac{S}{Y}\right)^*=(gn-1+x)k^{*1-f} \tag{A.24}$$

Equation (A.24) corresponds to equation (6) in the text.

The effect of the national saving rate of a one-time, anticipated increase of the survival rate is $\frac{\partial(S_t/Y_t)}{\partial q_t}=\frac{g_{t+1}n_{t+1}}{k^{*f}}\frac{dk_{t+1}}{dq_t}>0$. An anticipated increase in the survival rate at time t leads to a higher national saving rate. At time t, saving by prime-age adults increases, which causes an increase in k_{t+1}. The supply of capital increases and the saving rate increases. The effect of an increase in the survival rate at time t on the national saving rate at time $t+1$ is: $\frac{\partial(S_{t+1}/Y_{t+1})}{\partial q_t}=k_{t+1}^{-f}\left[g_{t+2}n_{t+2}\left(\frac{\partial k_{t+2}}{\partial k_{t+1}}-\frac{k_{t+2}}{k_{t+1}}f\right)-(1-f)(1-x)\right]\frac{\partial k_{t+1}}{\partial q_t}$,

where $0<\partial k_{t+2}/\partial k_{t+1}<1$ and $\partial k_{t+1}/\partial q_t>0$ hold. For plausible parameter values, this expression is negative. The saving at time $t+1$ is the difference between the wealth at time $t+2$ and $t+1$. An increase in q_t induces capital deepening at time $t+1$, but the wealth of the previous period and depreciation are also higher than at time t. The net increase in wealth at time $t+1$ is less than that at time t. In a closed economy, the survival rate at time t also influences the saving rate at time $t+2$. This stands in contrast to the case of small open economy. The survival rate at time t has a negative effect on the saving rate after time $t+2$.[i]

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[i] $\frac{\partial(S_{t+2}/Y_{t+2})}{\partial q_t}=k_{t+2}^{-f}\left[g_{t+3}n_{t+3}\left(\frac{\partial k_{t+3}}{\partial k_{t+2}}-\frac{k_{t+3}}{k_{t+2}}f\right)-(1-f)(1-x)\right]\frac{\partial k_{t+2}}{\partial k_{t+1}}\frac{\partial k_{t+1}}{\partial q_t}$, $0<\partial k_{t+3}/\partial k_{t+2}<1$, $0<\partial k_{t+2}/\partial k_{t+1}<1$, and $\partial k_{t+1}/\partial q_t>0$ hold, therefore, in the right hand side of this equation, the value inside the bracket is negative for plausible parameters, so that the saving rate at time t has a negative effect on the saving rate at time $t+2$. In the same way, the effects of the survival rate at time t on the saving rate after time $t+2$ can be derived. q_t has a negative effect on the saving rate after time $t+2$